

Identities using $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\sin^2 \theta + \cos^2 \theta = 1$

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Worked example

Simplify:

$$\sin^2 \frac{1}{3}y + \cos^2 \frac{1}{3}y$$

$$\cos^2(3\theta - 10) + \sin^2(3\theta - 10)$$

Your turn

Simplify:

$$\sin^2 3x + \cos^2 3x$$

1

Worked example

Given that $\cos \theta = \frac{12}{13}$, find the two possible values of $\sin \theta$

Your turn

Given that $\sin \theta = \frac{3}{5}$, find the two possible values of $\cos \theta$

$$\cos \theta = \pm \frac{4}{5}$$

Worked example

Simplify

$$5 \cos x (\cos x - 2) + 5(\sin^2 x + 2 \cos x)$$

Your turn

Simplify

$$3 \sin x (\sin x + 2) - 3(2 \sin x - \cos^2 x)$$

3

Worked example

Prove that $1 - \frac{\tan \theta \cos^3 \theta}{\sin \theta} \equiv \sin^2 \theta$

Your turn

Prove that $1 - \tan \theta \sin \theta \cos \theta \equiv \cos^2 \theta$

Proof

LHS

$$\equiv 1 - \tan \theta \sin \theta \cos \theta$$

$$\equiv 1 - \frac{\sin \theta}{\cos \theta} \sin \theta \cos \theta$$

$$\equiv 1 - \frac{\sin^2 \theta \cos \theta}{\cos \theta}$$

$$\equiv 1 - \sin^2 \theta$$

$$\equiv \cos^2 \theta$$

$$\equiv RHS$$

Worked example

Prove that $\tan \theta - \frac{1}{\tan \theta} \equiv \frac{1-2 \cos^2 \theta}{\sin \theta \cos \theta}$

Your turn

Prove that $\tan \theta + \frac{1}{\tan \theta} \equiv \frac{1}{\sin \theta \cos \theta}$

Proof

LHS

$$\begin{aligned} &\equiv \tan \theta + \frac{1}{\tan \theta} \\ &\equiv \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \\ &\equiv \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \\ &\equiv \frac{1}{\sin \theta \cos \theta} \\ &\equiv \text{RHS} \end{aligned}$$

Worked example

Simplify $10 - 10 \cos^2 \theta$

Your turn

Simplify $5 - 5 \sin^2 \theta$

$$5 \cos^2 \theta$$

Worked example

Prove that

$$\frac{\cos \theta - \cos^3 \theta}{\sin^3 \theta} \equiv \frac{1}{\tan \theta}$$

Your turn

Prove that

$$\frac{\sin \theta - \sin^3 \theta}{\cos^3 \theta} \equiv \tan \theta$$

$5 \cos^2 \theta$

Worked example

Simplify:

$$\frac{\cos 4\theta}{\sqrt{1 - \sin^2 4\theta}}$$

Your turn

Simplify:

$$\frac{\sin 2\theta}{\sqrt{1 - \sin^2 2\theta}}$$

$\tan 2\theta$

Worked example

Prove that

$$\frac{\sin^4 \theta - \cos^4 \theta}{\sin^2 \theta} \equiv 1 - \frac{1}{\tan^2 \theta}$$

Your turn

Prove that

$$\frac{\cos^4 \theta - \sin^4 \theta}{\cos^2 \theta} \equiv 1 - \tan^2 \theta$$

Proof

LHS

$$\begin{aligned} & \equiv \frac{\cos^4 \theta - \sin^4 \theta}{\cos^2 \theta} \\ & \equiv \frac{(\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta)}{\cos^2 \theta} \\ & \equiv \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta} \\ & \equiv \frac{\cos^2 \theta}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta} \\ & \equiv 1 - \tan^2 \theta \\ & \equiv \textit{RHS} \end{aligned}$$

Worked example

Prove that

$$\frac{\frac{\sin x}{\tan x}}{\sqrt{1 - \sin^2 x}} \equiv 1$$

Your turn

Prove that

$$\frac{\tan x \cos x}{\sqrt{1 - \cos^2 x}} \equiv 1$$

Proof

LHS

$$\begin{aligned} & \frac{\tan x \cos x}{\sqrt{1 - \cos^2 x}} \\ & \equiv \frac{\frac{\sin x}{\cos x} \cos x}{\sqrt{\sin^2 x}} \\ & \equiv \frac{\sin x}{\sin x} \\ & \equiv 1 \\ & \equiv RHS \end{aligned}$$

Worked example

Show that

$$\frac{\cos^3 x + \sin x - \sin^3 x}{\sin x + \cos x}$$

can be simplified to $\cos^k x$ where k is an integer

Your turn

Show that

$$\frac{\sin^3 x + \cos x - \cos^3 x}{\sin x + \cos x}$$

can be simplified to $\sin^k x$ where k is an integer

$$\sin^2 x$$

Worked example

Prove that

$$\frac{1}{\tan^2 \theta} \equiv \frac{1}{\sin^2 \theta} - 1$$

Your turn

Prove that

$$\tan^2 \theta \equiv \frac{1}{\cos^2 \theta} - 1$$

Proof

RHS

$$\begin{aligned} &\equiv \frac{1}{\cos^2 \theta} - 1 \\ &\equiv \frac{1}{\cos^2 \theta} - \frac{\cos^2 \theta}{\cos^2 \theta} \\ &\equiv \frac{1 - \cos^2 \theta}{\cos^2 \theta} \\ &\equiv \frac{\sin^2 \theta}{\cos^2 \theta} \\ &\equiv \tan^2 \theta \\ &\equiv LHS \end{aligned}$$

Worked example

Given that $\sin \theta = \frac{3}{7}$ and that θ is acute, find the exact value of $\cos \theta$

Your turn

Given that $\sin \theta = \frac{2}{5}$ and that θ is obtuse, find the exact value of $\cos \theta$

$$-\frac{\sqrt{21}}{5}$$

Worked example

Given that $\cos \theta = -\frac{5}{13}$ and that θ is obtuse, find the value of $\sin \theta$

Your turn

Given that $\cos \theta = -\frac{3}{5}$ and that θ is reflex, find the value of $\sin \theta$

$$-\frac{4}{5}$$

Worked example

Given that $\tan \theta = -\frac{5}{12}$ and that θ is reflex, find the value of $\sin \theta$ and $\cos \theta$

Your turn

Given that $\tan \theta = \frac{3}{4}$ and that θ is acute, find the value of $\sin \theta$ and $\cos \theta$

$$\sin \theta = \frac{3}{5}$$

$$\cos \theta = \frac{4}{5}$$