

Transformations of the unit square: Rotations, reflections, enlargements

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Worked example

Find a 2×2 matrix that represents:

- A reflection in the y -axis.

- A reflection in the line $y = x$

Your turn

Find a 2×2 matrix that represents:

- A reflection in the x -axis.

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- A reflection in the line $y = -x$

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

Worked example

Find a 2×2 matrix that represents:

- Rotation 90° anticlockwise about the origin
- Rotation 180° about the origin

Your turn

Find a 2×2 matrix that represents:

- Rotation 270° anticlockwise about the origin

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Worked example

Describe geometrically the effect of the following matrices:

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

Your turn

Describe geometrically the effect of the following matrices:

$$\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$$

Enlargement, centre origin, scale factor 4

Worked example

The transformation matrix $\begin{pmatrix} a & -2 \\ 1 & 1 \end{pmatrix}$ maps the point (2, 5) onto the point (3, b).
Work out the values of a and b.

Your turn

The transformation matrix $\begin{pmatrix} a & 2 \\ -1 & 1 \end{pmatrix}$ maps the point (3, 4) onto the point (2, b).
Work out the values of a and b.

$$a = -2, b = 1$$

Worked example

The matrix $\begin{pmatrix} -a & 3b \\ 2a & b \end{pmatrix}$ maps the point (2, 3) onto the point (-31, -1). Work out the values of a and b.

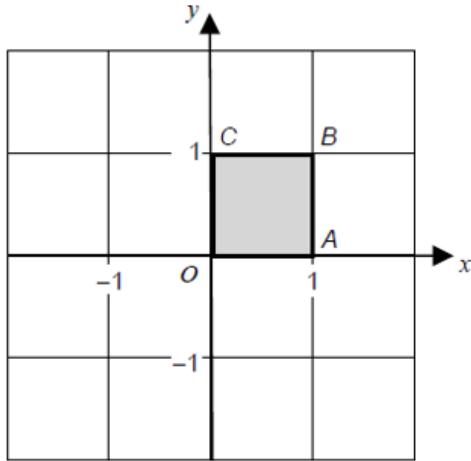
Your turn

The matrix $\begin{pmatrix} a & b \\ -a & 2b \end{pmatrix}$ maps the point (5, 4) onto the point (1, 17). Work out the values of a and b.

$$a = -1, b = \frac{3}{2}$$

Worked example

The diagram shows the unit square OABC.

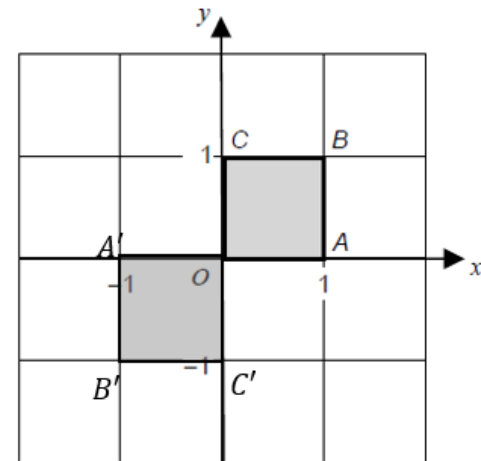


The image of OABC after transformation by the matrix $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ is OA'B'C'.

Draw and label OA'B'C'.

Your turn

The diagram shows the unit square OABC.



The image of OABC after transformation by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ is OA'B'C'.

Draw and label OA'B'C'.

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

Worked example

A triangle T has vertices $(1, 1)$, $(1, 2)$ and $(2, 2)$.

- a) Find the vertices of the image of T under the transformation given by the matrix $\mathbf{M} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$.
- b) Sketch T and its image, T' on a coordinate grid.
- c) Describe the geometric transformation.

Your turn

A triangle T has vertices $(1, 1)$, $(1, 2)$ and $(2, 2)$.

- a) Find the vertices of the image of T under the transformation given by the matrix $\mathbf{M} = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$.
- b) Sketch T and its image, T' on a coordinate grid.
- c) Describe the geometric transformation.

a) $(3, 2)$, $(3, 4)$ and $(6, 4)$

b) Sketch

c) The triangle has been stretched by a scale factor of 3 parallel to the x -axis and by a scale factor of 2 parallel to the y -axis