

Surds

[CONTENTS](#)

Worked example

Express in the form $a + b\sqrt{3}$:

$$(4 + \sqrt{3})(1 + 2\sqrt{3})$$

Your turn

Express in the form $a + b\sqrt{5}$:

$$(2 + \sqrt{5})(3 + 4\sqrt{5})$$

$$26 + 11\sqrt{5}$$

Worked example

Express in the form $a + b\sqrt{3}$:

$$(2\sqrt{3} - 5)^2$$

Your turn

Express in the form $a + b\sqrt{5}$:

$$(3\sqrt{5} - 2)^2$$

$$49 - 12\sqrt{5}$$

Worked example

Simplify $\frac{\sqrt{450}}{3}$ by writing it in the form $a\sqrt{b}$ where b is prime

Your turn

Simplify $\frac{\sqrt{288}}{4}$ by writing it in the form $a\sqrt{b}$ where b is prime

$$3\sqrt{2}$$

Worked example

Rationalise the denominator and simplify:

$$\frac{3}{5 + \sqrt{2}}$$

$$\frac{4}{1 + \sqrt{3}}$$

Your turn

Rationalise the denominator and simplify:

$$\frac{18}{1 + \sqrt{7}}$$

$$-3 + 3\sqrt{7}$$

Worked example

Rationalise the denominator and simplify:

$$\frac{3\sqrt{3} - 5}{2\sqrt{3} - 7}$$

Your turn

Rationalise the denominator and simplify:

$$\frac{5\sqrt{5} - 2}{2\sqrt{5} - 3}$$
$$4 + \sqrt{5}$$

Worked example

Rationalise the denominator and simplify:

$$\frac{(3 + \sqrt{2})(3 - \sqrt{2})}{\sqrt{7}}$$

$$\frac{(7 + \sqrt{5})(7 - \sqrt{5})}{\sqrt{11}}$$

Your turn

Rationalise the denominator and simplify:

$$\frac{(5 + \sqrt{3})(5 - \sqrt{3})}{\sqrt{22}}$$

$$\sqrt{22}$$

Worked example

Simplify, giving your answer in the form $a\sqrt{b}$:

$$\frac{\sqrt{54}}{3} + \frac{12}{\sqrt{6}}$$

Your turn

Simplify, giving your answer in the form $a\sqrt{b}$:

$$\frac{\sqrt{63}}{3} + \frac{21}{\sqrt{7}}$$

$$4\sqrt{7}$$

Worked example

Show that $\frac{1}{\frac{1}{\sqrt{3}}+1} = \frac{3}{2} - \frac{1}{2}\sqrt{3}$

Your turn

Show that $\frac{1}{1+\frac{1}{\sqrt{2}}} = 2 - \sqrt{2}$

$$= \frac{\sqrt{2}}{\sqrt{2} + 1}$$

$$= \frac{\sqrt{2}}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1}$$

$$= \frac{2 - \sqrt{2}}{2 + \sqrt{2} - \sqrt{2} - 1}$$

$$= \frac{2 - \sqrt{2}}{1}$$

$$= 2 - \sqrt{2}$$

Worked example

Simplify

$$(3 + \sqrt{2})^2 - (3 - \sqrt{2})^2$$

$$(4 + \sqrt{5})^2 - (4 - \sqrt{5})^2$$

Your turn

Simplify

$$(2 + \sqrt{3})^2 - (2 - \sqrt{3})^2$$

$$8\sqrt{3}$$

Worked example

Simplify

$$(\sqrt{3} + 5)(\sqrt{3} - 3)(\sqrt{3} + 2)$$

Your turn

Simplify

$$(\sqrt{5} + 3)(\sqrt{5} - 2)(\sqrt{5} + 1)$$

4

Worked example

Find the value of a and b :

$$(a - 3\sqrt{5})^2 = b - 42\sqrt{5}$$

Your turn

Express b and c in terms of a

$$(a - 2\sqrt{3})^2 = b - 20\sqrt{3}$$

$$a = 5, b = 37$$

Worked example

Find in its simplest form $a:b$,
given:

$$a = \sqrt{5} + \sqrt{c}$$

$$b = \sqrt{80} + \sqrt{d}$$

c and d are positive integers

$$c:d = 1:16$$

Your turn

Find in its simplest form $a:b$,
given:

$$a = \sqrt{7} + \sqrt{c}$$

$$b = \sqrt{63} + \sqrt{d}$$

c and d are positive integers

$$c:d = 1:9$$

$$a:b = 1:3$$