

Solving harder trigonometric equations in a given interval

eg: $2 \sin^2 x - 3 \cos x + 1 = 0$ and $\tan^2 x = \frac{1}{4}$

Worked example

Solve in the interval $0 \leq x \leq 360^\circ$:

$$2 \cos^2 x + 3 \cos x + 1 = 0$$

Your turn

Solve in the interval $0 \leq x \leq 360^\circ$:

$$2 \sin^2 x - 3 \sin x + 1 = 0$$

$$x = 30^\circ, 90^\circ, 150^\circ$$

Worked example

Solve in the interval $0 \leq x \leq 360^\circ$:

$$3 \sin^2 x = 2 \sin x + 1$$

Your turn

Solve in the interval $0 \leq x \leq 360^\circ$:

$$3 \cos^2 x = 1 - 2 \cos x$$

$$x = 70.5^\circ, 180.0^\circ, 289.5^\circ \text{ (1 dp)}$$

Worked example

Solve in the interval $0 \leq x \leq 360^\circ$:

$$5 \cos^2 x - 3 \sin^2 x = 4$$

Your turn

Solve in the interval $0 \leq x \leq 360^\circ$:

$$8 \sin^2 x - 7 \cos^2 x = 6$$

$$x = 84.6^\circ, 95.4^\circ, 256.0^\circ, 264.6^\circ \text{ (1 dp)}$$

Worked example

Explain why there are no solutions to the equation $3 \sin^2 x - 4 \cos^2 x = 5$

Your turn

Explain why there are no solutions to the equation $3 \sin^2 x + 4 \cos^2 x = 5$

$$\sin^2 x = -1$$

No real solutions

Or

$$\cos^2 x = 2$$

$$\cos x = \pm\sqrt{2} \text{ but } -1 \leq \cos x \leq 1$$

$\therefore$ No solutions

Worked example

Solve in the interval $0 \leq x < 360^\circ$:

$$\sin^2 x = 2 \sin x$$

Your turn

Solve in the interval $0 \leq x \leq 360^\circ$:

$$\cos^2 x = 4 \cos x$$

$$x = 0^\circ$$

Worked example

Solve in the interval $0 \leq x \leq 360^\circ$:

$$\cos^2 x - \cos x = 0$$

Your turn

Solve in the interval $0 \leq x < 360^\circ$:

$$\sin^2 x + 3 \sin x = 0$$

$$x = 0^\circ, 180^\circ$$

Worked example

Solve in the interval $0 < x < 360^\circ$:

$$\tan^2 x - 5 \tan x = 0$$

Your turn

Solve in the interval $0 < x < 360^\circ$:

$$\tan^2 x + 3 \tan x = 0$$

$$x = 108.4^\circ, 288.4^\circ \text{ (1 dp)}$$

Worked example

Solve in the interval $0 \leq \theta \leq 270^\circ$:

$$64 \sin^2 \theta = 1$$

Your turn

Solve in the interval $0 \leq \theta \leq 270^\circ$:

$$16 \sin^2 \theta = 1$$

$$\theta = 14.5^\circ, 165.5^\circ, 194.5^\circ$$

Worked example

Solve in the interval $0 \leq \theta \leq 180^\circ$:

$$5 \cos^2 \theta - 1 = 0$$

Your turn

Solve in the interval $0 \leq \theta \leq 180^\circ$:

$$3 \cos^2 \theta - 1 = 0$$

$$\theta = 54.7^\circ, 125.3^\circ \text{ (1 dp)}$$

Worked example

Solve in the interval $0 \leq \theta \leq 360^\circ$:
 $\tan^2 \theta = 25$

Your turn

Solve in the interval $0 \leq \theta \leq 360^\circ$:
 $\tan^2 \theta = 16$

$$\theta = 76.0^\circ, 104.0^\circ, 256.0^\circ, 284.0^\circ \text{ (1 dp)}$$

Worked example

Solve in the interval $0 \leq x \leq 360^\circ$:

$$2 \sin x = 5 \cos x$$

Your turn

Solve in the interval $0 \leq x \leq 360^\circ$:

$$5 \sin x = 2 \cos x$$

$$x = 21.8^\circ, 201.8^\circ \text{ (1 dp)}$$

Worked example

Solve in the interval $0 < \theta \leq 180^\circ$:

$$2 \sin^2 \theta - 3 \cos \theta = 3$$

Your turn

Solve in the interval $0 \leq \theta < 180^\circ$:

$$2 \cos^2 \theta + 3 \sin \theta = 3$$

$$\theta = 30^\circ, 90^\circ, 150^\circ$$

Worked example

Solve in the interval $0 \leq x \leq 360^\circ$:

$$5 \cos^2 x - 4 \sin x + 3 = 0$$

Your turn

Solve in the interval $0 \leq x \leq 360^\circ$:

$$6 \sin^2 x + 7 \cos x - 2 = 0$$

$$x = 114.8^\circ, 245.2^\circ \text{ (1 dp)}$$

Worked example

Solve in the interval $-180^\circ \leq x \leq 180^\circ$:

$$2 \sin^2 x - 9 \cos x = 3 \cos^2 x$$

Your turn

Solve in the interval $-180^\circ \leq x \leq 180^\circ$:

$$2 \cos^2 x + 9 \sin x = 3 \sin^2 x$$

$$x = -168.5^\circ, -11.5^\circ \text{ (1 dp)}$$