

Second derivative and maxima/minima

[CONTENTS](#)

Worked example

$$y = 5x^3 - 7x^2 - x + 4$$

Find the value of $\frac{d^2y}{dx^2}$ when $x = 2$

Your turn

$$y = 2x^3 - 3x^2 + x + 9$$

Find the value of $\frac{d^2y}{dx^2}$ when $x = 2$

18

Worked example

Find the stationary point on the curve with equation

$y = x^4 - 108x$, and determine whether it is a local maximum, a local minimum or a point of inflection.

Your turn

Find the stationary point on the curve with equation

$y = x^4 - 32x$, and determine whether it is a local maximum, a local minimum or a point of inflection.

$(2, -48)$ Local minimum

Worked example

Find the coordinates of the stationary points on the curve with equation $y = 4x^3 + 30x^2 + 48x - 3$ and use the second derivative to determine their nature

Your turn

Find the coordinates of the stationary points on the curve with equation $y = 2x^3 - 15x^2 + 24x + 6$ and use the second derivative to determine their nature

(1, 17) Local maximum

(4, -10) Local minimum

Worked example

Show that one of the stationary points of the curve with equation $y = x^3 - 3x^2 + 45x$ is $(-3, -159)$, and by testing the gradient of the curve either side of the stationary point, determine whether it is a maximum, a minimum or a point of inflection.

Your turn

Show that one of the stationary points of the curve with equation $y = x^3 + 3x^2 - 45x$ is $(3, -81)$, and by testing the gradient of the curve either side of the stationary point, determine whether it is a maximum, a minimum or a point of inflection.

Minimum

Worked example

a) Find the turning point of

$$y = -x^3 + 6x^2 - 12x - 5$$

b) Write $\frac{dy}{dx}$ in the form $a(x + b)^2$

c) Hence, explain why your turning point is a point of inflection.

Your turn

a) Find the turning point of

$$y = x^3 - 6x^2 + 12x - 3$$

b) Write $\frac{dy}{dx}$ in the form $a(x + b)^2$

c) Hence, explain why your turning point is a point of inflection.

a) (2,5)

$$b) \frac{dy}{dx} = 3(x - 2)^2$$

c) As $(x - 2)^2 \geq 0$ for all x , the gradient is positive except at $x = 2$.

Worked example

The curve $y = f(x)$ is such that $\frac{dy}{dx} = -x(x - 3)^2$. The stationary points of the curve are at $(0, 1)$ and $(3, -\frac{23}{4})$. Determine the nature of each stationary point. You **must** show your working.

Your turn

The curve $y = f(x)$ is such that $\frac{dy}{dx} = -x(x - 2)^2$. The stationary points of the curve are at $(0, \frac{4}{3})$ and $(2, 0)$. Determine the nature of each stationary point. You **must** show your working.

$(0, \frac{4}{3})$ is a local maximum because $\frac{d^2y}{dx^2} = -4 < 0$;
 $(2, 0)$ is a point of inflection because $\frac{d^2y}{dx^2} = 0$