

Proof

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Worked example

Prove that $4n - 3 + 2n + 7$ is a multiple of 2 for all real integers n

Prove that $4n - 3 + 2n - 9$ is a multiple of 3 for all real integers n

Your turn

Prove that $4n - 3 + 10n - 11$ is a multiple of 7 for all real integers n

$$\begin{aligned} &14n - 14 \\ &\equiv 7(2n - 2) \\ &\equiv 7k \end{aligned}$$

Worked example

Prove that the sum of five consecutive integers is a multiple of 5.

Your turn

Prove that the sum of three consecutive integers is a multiple of 3.

Let the first integer be n :

$$\begin{aligned} & n + n + 1 + n + 2 \\ & \equiv 3n + 3 \\ & \equiv 3(n + 1) \end{aligned}$$

Worked example

Prove that the product of two odd numbers is an odd number.

Your turn

Prove that the product of two even numbers is an even number.

Let even numbers be $2m$ and $2n$:

$$2m \times 2n$$

$$\equiv 4mn$$

$$\equiv 2(2mn)$$

Worked example

Prove algebraically that $n^2 - 2 - (n - 2)^2$ is always even, given n is an integer greater than 1

Your turn

Prove algebraically that $(2n + 1)^2 - (2n + 1)$ is an even number

$$\begin{aligned} & (2n + 1)^2 - (2n + 1) \\ \equiv & 4n^2 + 4n + 1 - 2n - 1 \\ \equiv & 4n^2 + 2n \\ \equiv & 2(2n^2 + n) \end{aligned}$$

Worked example

Prove that $(n - 1)^2 + n^2 + (n + 1)^2$ is two more than a multiple of 3 for all positive integer values of n

Your turn

Prove that $(n + 1)^2 - n^2$ is one more than a multiple of 2 for all positive integer values of n

$$\begin{aligned}(n + 1)^2 - n^2 \\ \equiv n^2 + 2n + 1 - n^2 \\ \equiv 2n + 1\end{aligned}$$

Worked example

Prove that $(2n + 3)^2 - (2n - 3)^2$ is a multiple of 8 for all positive integer values of n

Your turn

Prove that $(3n + 2)^2 - (3n - 2)^2$ is a multiple of 8 for all positive integer values of n

$$\begin{aligned} & (3n + 2)^2 - (3n - 2)^2 \\ \equiv & 9n^2 + 12n + 4 - (9n^2 - 12n + 4) \\ \equiv & 9n^2 + 12n + 4 - 9n^2 + 12n - 4 \\ \equiv & 24n \\ \equiv & 8(3n) \end{aligned}$$

Worked example

Prove algebraically that the difference between two different odd numbers is an even number.

Your turn

Prove algebraically that the difference between two different even numbers is an even number.

Let even numbers be $2m$ and $2n$:

$$\begin{aligned} &2m - 2n \\ &\equiv 2(m - n) \end{aligned}$$

Worked example

Prove that the product of four consecutive integers is always a multiple of 8

Your turn

Prove that the product of three consecutive integers is always a multiple of 6

Proof by showing at least one is a multiple of 2, and one will be a multiple of 3...

Worked example

Show that $(3n + 2)^3 + 1$ is divisible by 9 for all integer values of n

Your turn

Show that $(2n + 3)^3 + n^3$ is divisible by 9 for all integer values of n

Proof

Worked example

Prove that, for all positive values of n , $\frac{(n+3)^2 - (n-2)^2}{2n^2 + n} = \frac{a}{b}$ where a and b are integers or variables.

Your turn

Prove that, for all positive values of n , $\frac{(n+2)^2 - (n+1)^2}{2n^2 + 3n} = \frac{a}{b}$ and find the integers a and b

$$a = 1, b = n$$