

Optimisation

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Worked example

$$U = 81y + \frac{49}{y}, y > 0$$

Use calculus to show that U has a minimum value and work out the minimum value of U

Your turn

$$V = 49x + \frac{81}{x}, x > 0$$

Use calculus to show that V has a minimum value and work out the minimum value of V

$$\frac{dV}{dx} = 49 - \frac{81}{x^2} = 0$$

$$x = \frac{9}{7} \text{ as } x > 0$$

$$\frac{d^2V}{dx^2} = \frac{162}{x^3}$$

$$x = \frac{9}{7}: \frac{d^2V}{dx^2} = \frac{162}{\left(\frac{9}{7}\right)^3} = \frac{686}{9} > 0 \therefore \text{local minimum.}$$

$$x = \frac{9}{7}: V = 49\left(\frac{9}{7}\right) + \frac{81}{\frac{9}{7}} = 126$$

Worked example

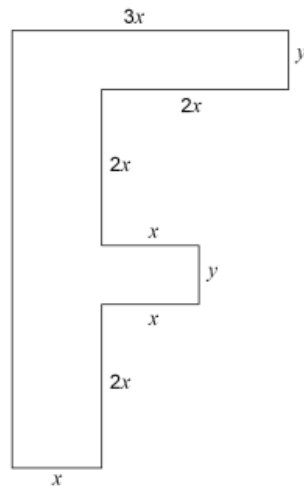
An F shape is made from rectangles.

All lengths are in centimetres.

The perimeter of the shape is 128 cm .

The area of the shape is $A \text{ cm}^2$

- Find an expression for y
- Hence find an expression for A .
- Use calculus to derive an expression for the rate of change of A as x varies.
- Hence work out the maximum area



Your turn

An F shape is made from rectangles.

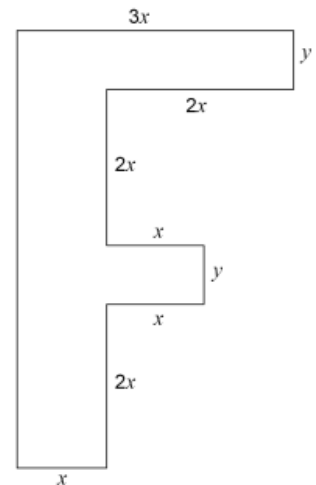
All lengths are in centimetres.

The perimeter of the shape is 64 cm .

The area of the shape is $A \text{ cm}^2$

- Find an expression for y
- Hence find an expression for A .
- Use calculus to derive an expression for the rate of change of A as x varies.
- Hence work out the maximum area

- $y = 16 - 4x$
- $A = 80x - 16x^2$
- $\frac{dA}{dx} = 80 - 32x$
- 100 cm^2



Worked example

A cuboid is to be made with volume 81 cm^3 .

The cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, $x \text{ cm}$.

The volume of the cuboid is 81 cm^3 .

- Show that the total length, L , of the twelve edges of the cuboid is given by $L = 12x + \frac{162}{x^2}$
- Given that x can vary, use differentiation to find the maximum or minimum value of L
- Justify that the value of L you have found is a minimum

Your turn

A cuboid is to be made from 54m^2 of sheet metal.

The cuboid has a horizontal base and no top.

The height of the cuboid is x metres.

Two of the opposite vertical faces are squares.

- Show that the volume, $V \text{ m}^3$, of the tank is given by

$$V = 18x - \frac{2}{3}x^3.$$

- Given that x can vary, use differentiation to find the maximum or minimum value of V .
- Justify that the value of V you have found is a maximum

a) Shown

b) $V = 36$

c) $\frac{d^2V}{dx^2} = -4x$; $x = 3$, $\frac{d^2V}{dx^2} = -12 < 0$