

Functions: Domain, Range, composite, inverse

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Worked example

$$f(x) = \frac{x - 3}{5x - 6}$$

State the value of x that cannot be in the domain of $f(x)$

Your turn

$$f(x) = \frac{3 - x}{6x - 5}$$

State the value of x that cannot be in the domain of $f(x)$

$$x = \frac{5}{6}$$

Worked example

$f(x)$ is a function with domain all values of x
 $f(x) = \sqrt{x^2 + 12x - a}$ where a is a constant
Work out the possible values of a

Your turn

$f(x)$ is a function with domain all values of x
 $f(x) = \sqrt{x^2 + 6x - a}$ where a is a constant
Work out the possible values of a

$$a \leq -9 \text{ or}$$

Worked example

$$f(x) = 23 - 5x \text{ with domain } -3 < x \leq 1$$

Work out the range of $f(x)$

Your turn

$$g(x) = 32 - 3x \text{ with domain } -5 \leq x < 2$$

Work out the range of $g(x)$

$$27 < g(x) \leq 47$$

Worked example

$$g(x) = x^2 - 6x + 5, \quad x \in \mathbb{R}$$

Determine the range of $g(x)$

Your turn

$$f(x) = x^2 - 4x + 7, \quad x \in \mathbb{R}$$

Determine the range of $f(x)$

$$f(x) \geq 3$$

Worked example

$$g(x) = 14 + 2x - x^2, \quad x \in \mathbb{R}$$

Determine the range of $g(x)$.

Your turn

$$f(x) = 21 + 4x - x^2, \quad x \in \mathbb{R}$$

Determine the range of $f(x)$.

$$f(x) \leq 25$$

Worked example

$$g(x) = x^2 + 6x + 5, \quad x \geq 2$$

Determine the range of $g(x)$.

Your turn

$$f(x) = x^2 + 4x + 3, \quad x \geq 1$$

Determine the range of $f(x)$.

$$f(x) \geq 8$$

Worked example

Determine the range of:

$$g(x) = \sin x, 180 \leq x < 360^\circ$$

$$h(x) = \tan x, 0 < x < 90^\circ$$

Your turn

Determine the range of:

$$f(x) = \cos x, 180 < x \leq 360^\circ$$

$$-1 < f(x) \leq 1$$

Worked example

The function $f(x)$ is defined for all x :

$$f(x) = \begin{cases} 9 & x < -3 \\ x^2 & -3 \leq x \leq 3 \\ 15 - 2x & x > 3 \end{cases}$$

Determine the range of $f(x)$.

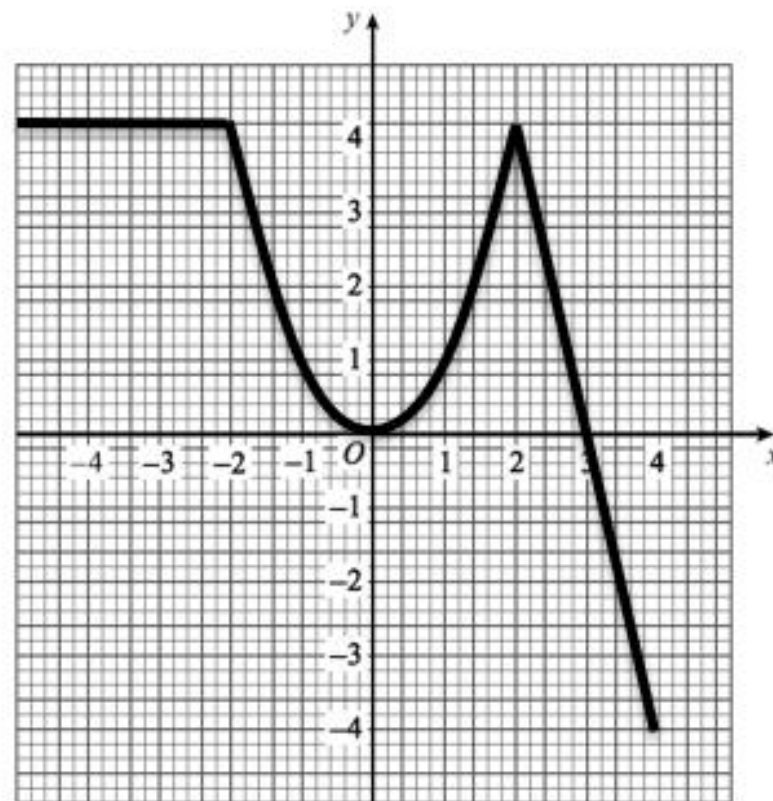
Your turn

The function $f(x)$ is defined for all x :

$$f(x) = \begin{cases} 4 & x < -2 \\ x^2 & -2 \leq x \leq 2 \\ 12 - 4x & x > 2 \end{cases}$$

Determine the range of $f(x)$.

$$f(x) \leq 4$$



Worked example

$g(x)$ is an increasing function with domain $1 \leq x \leq 5$ and range $3 \leq g(x) \leq 11$.

Construct a suitable function.

Your turn

$f(x)$ is a decreasing function with domain $4 \leq x \leq 6$ and range $7 \leq f(x) \leq 19$.

Construct a suitable function.

$$f(x) = 43 - 6x$$

Worked example

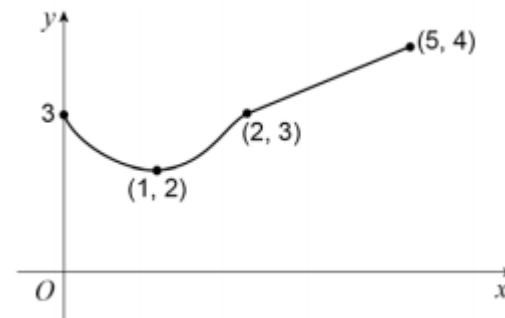
$$f(x) = \begin{cases} (x-a)^2 + b, & 0 \leq x < 3 \\ cx + d & 3 \leq x \leq 7 \end{cases}$$

The graph of $y = f(x)$ passes through the points $(0, 5)$, $(2, 1)$, $(3, 2)$ and $(7, 3)$. Find the values of a , b , c and d

Your turn

$$f(x) = \begin{cases} (x-a)^2 + b, & 0 \leq x < 2 \\ cx + d & 2 \leq x \leq 5 \end{cases}$$

The graph of $y = f(x)$ passes through the points $(0, 3)$, $(1, 2)$, $(2, 3)$ and $(5, 4)$. Find the values of a , b , c and d



$$a = 1, b = 2, c = \frac{1}{3}, d = \frac{7}{3}$$

Worked example

$$h(x) = 4x(x - 5)$$

Solve the equation

$$2h(x) = h(3x)$$

Your turn

$$h(x) = 5x(x - 4)$$

Solve the equation

$$3h(x) = h(2x)$$

$$x = 0, x = -4$$

Worked example

$$f(x) = 5x - 3$$

$$\text{Solve } f(x^2) = 42$$

$$\text{Solve } [f(x)]^2 = 49$$

Your turn

$$f(x) = 3x - 5$$

$$\text{Solve } f(x^2) = 43$$

$$x = \pm 4$$

$$\text{Solve } [f(x)]^2 = 81$$

$$x = \frac{14}{3}, x = -\frac{4}{3}$$