

# Basic differentiation

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## Worked example

Differentiate with respect to  $x$ :

$$x^2$$

$$x^3$$

$$x^4$$

## Your turn

Differentiate with respect to  $x$ :

$$x^5$$

$$5x^4$$

## Worked example

Differentiate with respect to  $x$ :

$$3x^2$$

$$-2x^3$$

$$5x^4$$

## Your turn

Differentiate with respect to  $x$ :

$$-3x^5$$

$$-15x^4$$

## Worked example

Differentiate with respect to  $x$ :

$$2x$$

$$3$$

$$4x - 5$$

## Your turn

Differentiate with respect to  $x$ :

$$6x - 7$$

$$6$$

## Worked example

Differentiate with respect to  $x$ :

$$\sqrt{x}$$

$$\sqrt[3]{x}$$

$$\sqrt[4]{x}$$

## Your turn

Differentiate with respect to  $x$ :

$$\sqrt[5]{x}$$

$$\frac{1}{5}x^{-\frac{4}{5}}$$

## Worked example

Differentiate with respect to  $x$ :

$$\frac{1}{x}$$

$$\frac{1}{x^2}$$

$$\frac{1}{x^3}$$

## Your turn

Differentiate with respect to  $x$ :

$$\frac{1}{x^4}$$

$$-4x^{-5} = -\frac{4}{x^5}$$

## Worked example

Differentiate with respect to  $x$ :

$$\frac{2}{x}$$

$$\frac{3}{4x^2}$$

$$\frac{6}{5x^3}$$

## Your turn

Differentiate with respect to  $x$ :

$$\frac{7}{8x^4}$$

$$-\frac{7}{2}x^{-5} = -\frac{7}{2x^5}$$

## Worked example

Differentiate with respect to  $x$ :

$$\frac{2}{3\sqrt{x}}$$

$$\frac{4}{7\sqrt[3]{x}}$$

$$\frac{5}{6\sqrt[4]{x}}$$

## Your turn

Differentiate with respect to  $x$ :

$$\frac{3}{5\sqrt{x}}$$

$$-\frac{3}{10}x^{-\frac{3}{2}} = -\frac{3}{10x\sqrt{x}}$$

## Worked example

Differentiate with respect to  $x$ :

$$y = 4x^3 + 3x^2 + 2x + 1$$

$$f(x) = x^3 - 2x^5 - 3x^{-2} - 2$$

## Your turn

Differentiate with respect to  $x$ :

$$y = 5x^4 - 2x^7 + 12345 - x^5$$

$$\frac{dy}{dx} = 20x^3 - 14x^6 - 5x^4$$

## Worked example

Differentiate with respect to  $x$ :

$$y = (5x - 3)^2$$

## Your turn

Differentiate with respect to  $x$ :

$$y = (3x - 5)^2$$

$$\frac{dy}{dx} = 18x - 30$$

## Worked example

Differentiate with respect to  $x$ :

$$y = x^4(x - 5)$$

$$f(x) = x^3(x + 2)$$

## Your turn

Differentiate with respect to  $x$ :

$$f(x) = x^2(x - 3)$$

$$f'(x) = 3x^2 - 6x$$

## Worked example

Differentiate with respect to  $x$ :

$$y = x^{\frac{1}{3}}(7x^{\frac{8}{3}} - 5x^{\frac{1}{3}})$$

## Your turn

Differentiate with respect to  $x$ :

$$f(x) = x^{\frac{1}{2}}(3x^{\frac{7}{2}} - 5x^{\frac{1}{2}})$$

$$f'(x) = 12x^3 - 5$$

## Worked example

Differentiate with respect to  $x$ :

$$y = \frac{(x + 3)^2}{x}$$

$$f(x) = \frac{(3x - 2)^2}{5x}$$

## Your turn

Differentiate with respect to  $x$ :

$$f(x) = \frac{(2x + 3)^2}{5x}$$

$$f'(x) = \frac{4}{5} - \frac{9}{5}x^{-2}$$

$$= \frac{4}{5} - \frac{9}{5x^2}$$

## Worked example

Differentiate with respect to  $x$ :

$$y = \frac{3x^4(2x^5 - 11x)}{x^2}$$

## Your turn

Differentiate with respect to  $x$ :

$$f(x) = \frac{2x^2(3x^3 - 7x)}{x}$$

$$f'(x) = 24x^3 - 28x$$

## Worked example

Find an expression for the rate of change of  $y$  with respect to  $x$ :

$$y = (x - 3)(x - 4)^2$$

## Your turn

Find an expression for the rate of change of  $y$  with respect to  $x$ :

$$y = (x + 1)(x + 2)^2$$

$$\frac{dy}{dx} = 3x^2 + 6x + 3 = 3(x + 1)^2$$

## Worked example

Find the gradient of the curve:

$$y = x^2 + 3x + 2 \text{ at } (4, 30)$$

$$y = 2x^3 - x + 5 \text{ at } (-1, 4)$$

## Your turn

Find the gradient of the curve:

$$y = 3x^2 - 2x + 1 \text{ at } (-2, 17)$$

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## Worked example

Find the coordinates of the point(s) where the gradient is 4:

$$y = x^2 - 8x + 3$$

$$y = 5x^2 - x + 7$$

## Your turn

Find the coordinates of the point(s) where the gradient is 3:

$$y = 3x^2 - 9x + 7$$

$$(2, 1)$$

## Worked example

Let  $f(x) = 4x^2 - 8x + 3$

- a) Find the gradient of  $y = f(x)$  at the point  $\left(\frac{1}{2}, 0\right)$
- b) Find the coordinates of the point on the graph of  $y = f(x)$  where the gradient is 8.
- c) Find the gradient of  $y = f(x)$  at the points where the curve meets the line  $y = 4x - 5$ .

## Your turn

Let  $f(x) = x^2 - 4x + 2$

- a) Find the gradient of  $y = f(x)$  at the point  $(1, -1)$
- b) Find the coordinates of the point on the graph of  $y = f(x)$  where the gradient is 5.
- c) Find the gradient of  $y = f(x)$  at the points where the curve meets the line  $y = 2 - x$ .

a)  $-2$

b)  $\left(\frac{9}{2}, \frac{17}{4}\right)$

c)  $x = 0, f'(x) = -4$   
 $x = 3, f'(3) = 2$

## Worked example

Find the coordinates of the point(s) where the gradient is 10:

$$y = x^3 + 6x^2 - 11x + 7$$

## Your turn

Find the coordinates of the point(s) where the gradient is 2:

$$y = x^3 - 3x^2 - 7x + 8$$

$(-1, 11)$  and  $(3, -13)$

## Worked example

A curve has gradient function

$$\frac{dy}{dx} = 3x^2 + 7$$

Work out the values of  $x$  for which the rate of change of  $y$  with respect to  $x$  is 55

## Your turn

A curve has gradient function

$$\frac{dy}{dx} = 5x^2 - 7$$

Work out the values of  $x$  for which the rate of change of  $y$  with respect to  $x$  is 38

$$x = \pm 3$$

## Worked example

$$y = 3x^2 + bx$$

The rate of change of  $y$  with respect to  $x$  when  $x = 4$  is triple the rate of change of  $y$  with respect to  $x$  when  $x = -2$ .

Work out the value of  $b$

## Your turn

$$y = 2x^3 + ax$$

The rate of change of  $y$  with respect to  $x$  when  $x = 2$  is twice the rate of change of  $y$  with respect to  $x$  when  $x = -1$ .

Work out the value of  $a$

$$a = 12$$