

Probability

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Worked example

Describe using impossible, unlikely, even chance, likely or certain the probability that:

You will walk to Mars.

The day after Monday is Tuesday.

You roll a three on a fair die.

You flip a tails on a fair coin.

Your turn

Describe using impossible, unlikely, even chance, likely or certain the probability that:

You roll an even number on a fair die
Even chance

The day after Monday is Wednesday. **Impossible**

You roll a number between 1 and 6 on a fair die. **Certain**

You will go to space in your life.
Unlikely

Worked example

The probability of it raining today is 0.2.
What is the probability of it not raining?

The probability of there not being an earthquake today is 0.99.
What is the probability of there being an earthquake?

Your turn

The probability of it snowing today is 0.07.
What is the probability of it not snowing?

0.93

Worked example

Calculate the probability of:

Flipping a tails from one toss of a fair coin

Rolling a 5 on a fair 6-sided die

Being born on a Monday

Your turn

Calculate the probability of:

Flipping a heads from one toss of a fair coin

$$\frac{1}{2}$$

Rolling a 5 on a fair 6-sided die

$$\frac{1}{6}$$

Being born on a Monday

$$\frac{1}{7}$$

Worked example

Calculate the probability of:

Flipping a tails or a heads from one toss of a fair coin

Rolling an odd number on a fair 6-sided die

Being born on a day beginning with S

Your turn

Calculate the probability of:

Rolling a number from 1 to 6 on a fair 6-sided die

1

Rolling a 7 on a fair 6-sided die

0

Being born on a day beginning with W

$\frac{1}{7}$

Worked example

If you flip a coin 300 times, how many times would you expect to flip a tails?

If you roll a die 600 times, how many times would you expect to roll a 1 or a 2?

Your turn

If you flip a coin 600 times, how many times would you expect to flip a heads?

300

If you roll a die 300 times, how many times would you expect to roll a 5?

50

Worked example

There are counters in a bag.

Colour	Red	Blue	Yellow
Number	5	10	15

One counter is taken out the bag.
Find the probability that it is:

Red

Blue

Yellow

Your turn

There are counters in a bag.

Colour	Purple	Orange	Green
Number	10	45	5

One counter is taken out the bag.
Find the probability that it is:

Purple $\frac{1}{6}$

Orange $\frac{3}{4}$

Green $\frac{1}{12}$

Worked example

There are counters in a bag.

Colour	Red	Blue	Yellow
Number	5	10	15

One counter is taken out the bag.
Find the probability that it is:

Red or blue

Blue or yellow

Red or yellow

Your turn

There are counters in a bag.

Colour	Purple	Orange	Green
Number	10	45	5

One counter is taken out the bag.
Find the probability that it is:

Purple or orange $\frac{11}{12}$

Orange or green $\frac{5}{6}$

Purple or green $\frac{1}{4}$

Worked example

There are counters in a bag.

Colour	Red	Blue	Yellow
Number	5	10	15

One counter is taken out the bag.
Find the probability that it is:

Red and blue

Blue and yellow

Red and yellow

Your turn

There are counters in a bag.

Colour	Purple	Orange	Green
Number	10	45	5

One counter is taken out the bag.
Find the probability that it is:

Purple and orange 0

Orange or green $\frac{5}{6}$

Purple or green $\frac{1}{4}$

Worked example

There are counters in a bag.

Colour	Red	Blue	Yellow
Number	5	10	15

One counter is taken out the bag.
It is not replaced.

Then another counter is taken out the bag.

Find the probability that:

Both counters are red

Both counters are blue

Both counters are yellow

Your turn

There are counters in a bag.

Colour	Purple	Orange	Green
Number	10	45	5

One counter is taken out the bag.
It is not replaced.

Then another counter is taken out the bag.

Find the probability that:

Both counters are purple

$$\frac{3}{118}$$

Both counters are orange

$$\frac{33}{59}$$

Both counters are green

$$\frac{1}{177}$$

Worked example

There are counters in a bag.

Colour	Red	Blue	Yellow
Number	5	10	15

One counter is taken out the bag.
It is not replaced.

Then another counter is taken out
the bag.

Find the probability that:

Neither counter is red

Neither counter is blue

Neither counter is yellow

Your turn

There are counters in a bag.

Colour	Purple	Orange	Green
Number	10	45	5

One counter is taken out the bag.
It is not replaced.

Then another counter is taken out
the bag.

Find the probability that:

Neither counter is purple

$$\frac{245}{354}$$

Neither counter is orange

$$\frac{7}{118}$$

Neither counter is green

$$\frac{99}{118}$$

Worked example

There are counters in a bag.

Colour	Red	Blue	Yellow
Number	5	10	15

One counter is taken out the bag.
It is not replaced.

Then another counter is taken out
the bag.

Find the probability that:

The counters are different colours

Your turn

There are counters in a bag.

Colour	Purple	Orange	Green
Number	10	45	5

One counter is taken out the bag.
It is not replaced.

Then another counter is taken out
the bag.

Find the probability that:

The counters are different colours

$$\frac{145}{354}$$

Worked example

There are counters in a bag.

Colour	Red	Blue	Yellow
Number	5	10	15

One counter is taken out the bag.
The colour is recorded and it is
put back into the bag, then
another counter is taken out.
Find the probability that:

Both counters are red

Both counters are blue

Both counters are yellow

Your turn

There are counters in a bag.

Colour	Purple	Orange	Green
Number	10	45	5

One counter is taken out the bag.
The colour is recorded and it is
put back into the bag, then
another counter is taken out.
Find the probability that:

Both counters are purple

$$\frac{1}{36}$$

Both counters are orange

$$\frac{9}{16}$$

Both counters are green

$$\frac{1}{144}$$

Worked example

There are counters in a bag.

Colour	Red	Blue	Yellow
Number	5	10	15

One counter is taken out the bag.
The colour is recorded and it is
put back into the bag, then
another counter is taken out.
Find the probability that:

Neither counter is red

Neither counter is blue

Neither counter is yellow

Your turn

There are counters in a bag.

Colour	Purple	Orange	Green
Number	10	45	5

One counter is taken out the bag.
The colour is recorded and it is
put back into the bag, then
another counter is taken out.
Find the probability that:

Neither counter is purple

$$\frac{25}{36}$$

Neither counter is orange

$$\frac{1}{16}$$

Neither counter is green

$$\frac{121}{144}$$

Worked example

There are counters in a bag.

Colour	Red	Blue	Yellow
Number	5	10	15

A counter is taken out at random.
The colour is recorded and it is
put back into the bag.
This is repeated two more times.
Find the probability that:

The counters are the same colour

Your turn

There are counters in a bag.

Colour	Purple	Orange	Green
Number	10	45	5

A counter is taken out at random.
The colour is recorded and it is
put back into the bag.
This is repeated two more times.
Find the probability that:

The counters are the same colour

$$\frac{41}{96}$$

Worked example

There are counters in a bag.

Colour	Red	Blue	Yellow
Number	5	10	15

A counter is taken out at random.
The colour is recorded and it is
put back into the bag.
This is repeated two more times.
Find the probability that:

The counters are different colours

Your turn

There are counters in a bag.

Colour	Purple	Orange	Green
Number	10	45	5

A counter is taken out at random.
The colour is recorded and it is
put back into the bag.
This is repeated two more times.
Find the probability that:

The counters are different colours

$$\frac{29}{72}$$

Worked example

There are counters in a bag.

Colour	Red	Blue	Yellow
Number	5	10	x

The probability of taking a yellow counter is $\frac{1}{2}$.

Find x

Your turn

There are counters in a bag.

Colour	Purple	Orange	Green
Number	10	45	x

The probability of taking a green counter is $\frac{1}{3}$.

Find x

$$x = 30$$

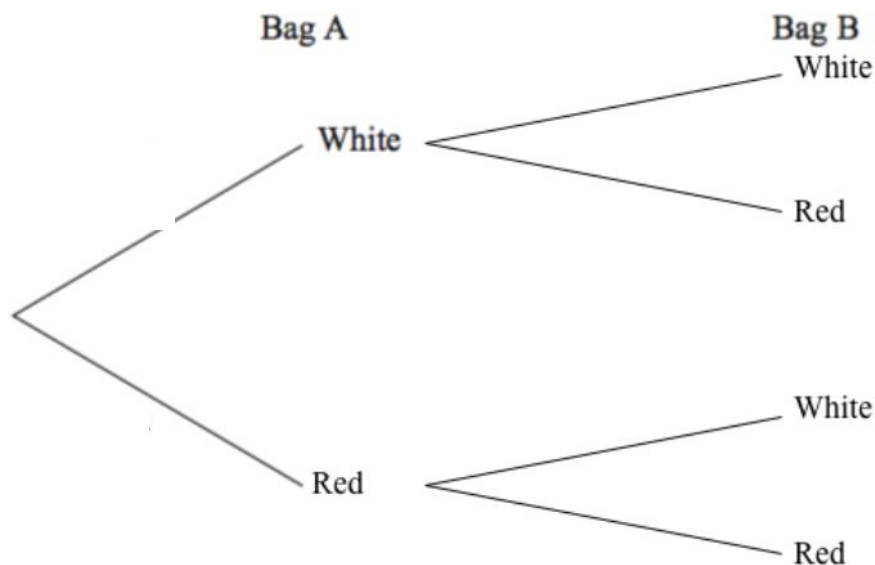
Worked example

In bag A there are 2 white and 5 red counters.

In bag B there are 7 white counters and 3 red counters.

A person takes at random one counter from A and one counter from B.

Draw a probability tree diagram to represent the situation:



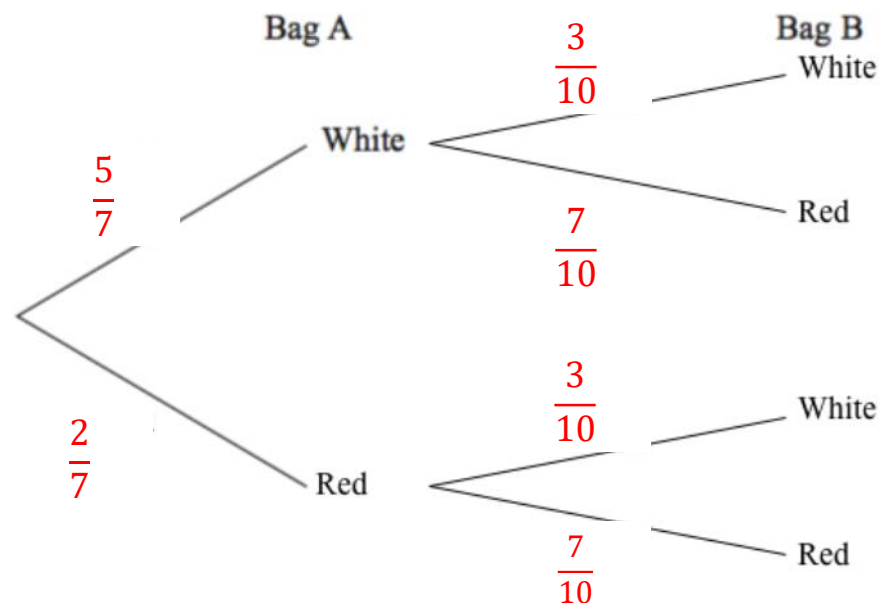
Your turn

In bag A there are 5 white and 2 red counters.

In bag B there are 3 white counters and 7 red counters.

A person takes at random one counter from A and one counter from B.

Draw a probability tree diagram to represent the situation:



Worked example

In bag A there are 2 white and 5 red counters.

In bag B there are 7 white counters and 3 red counters.

A person takes at random one counter from A and one counter from B.

Find the probability that the counters are the same colour

Your turn

In bag A there are 5 white and 2 red counters.

In bag B there are 3 white counters and 7 red counters.

A person takes at random one counter from A and one counter from B.

Find the probability that the counters are the same colour

$$\frac{29}{70}$$

Worked example

In bag A there are 2 white and 5 red counters.

In bag B there are 7 white counters and 3 red counters.

A person takes at random one counter from A and one counter from B.

Find the probability that the counters are different colours

Your turn

In bag A there are 5 white and 2 red counters.

In bag B there are 3 white counters and 7 red counters.

A person takes at random one counter from A and one counter from B.

Find the probability that the counters are different colours

$$\frac{41}{70}$$

Worked example

A person throws a biased coin two times.

The probability of flipping a heads on any throw is p

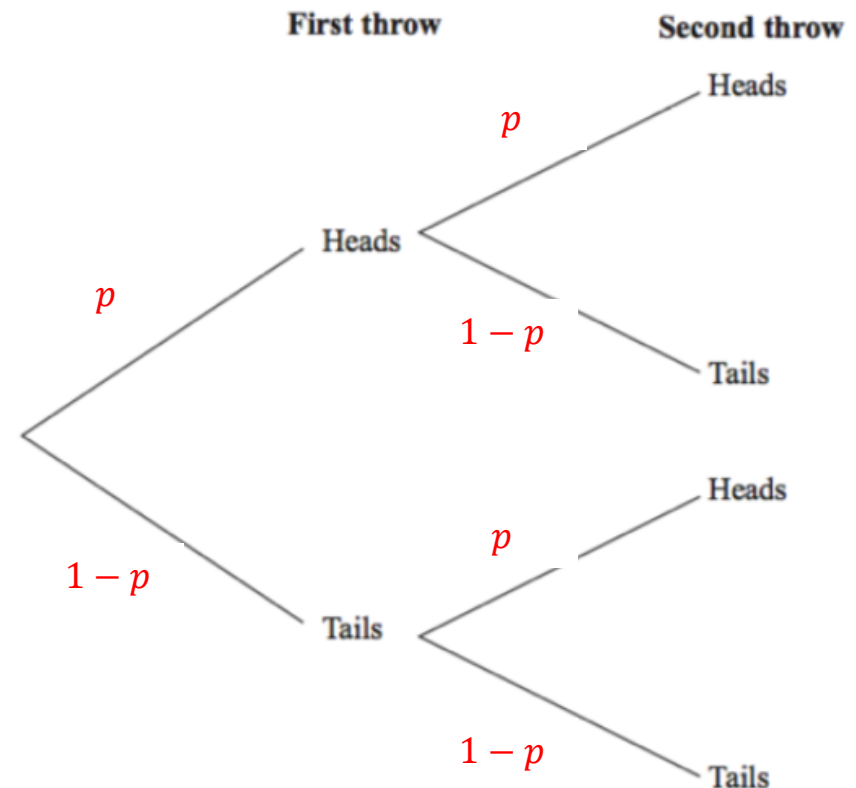
Draw a probability tree diagram to represent the situation.

Your turn

A person throws a biased coin two times.

The probability of flipping a heads on any throw is p

Draw a probability tree diagram to represent the situation.



Worked example

A person throws a biased coin two times.

The probability of flipping a heads on any throw is p

Find an expression for the probability of flipping:

Two heads

Two tails

One heads and one tails

Your turn

A person throws a biased coin two times.

The probability of flipping a heads on any throw is q

Find an expression for the probability of flipping:

Two heads q^2

Two tails $(1 - p)^2 = p^2 - 2p + 1$

One heads and one tails $p(1 - p)$

Worked example

Four fair 6-sided dice are thrown.
Work out the probability that a 5
is scored on at least two of the
four dice.

Your turn

Four fair 6-sided dice are thrown.
Work out the probability that a 5
is scored on at least one of the
four dice.

$$\frac{671}{1296}$$

Worked example

A biased 5-sided spinner can land on 1, 2, 3, 4 or 5

Number	1	2	3	4	5
Probability	0.2	0.3	0.1	0.05	

The spinner is spun 200 times.
Calculate the expected number of times the spinner will land on:

4

5

1 or 2

Your turn

A biased 4-sided spinner can land on 1, 2, 3, 4

Number	1	2	3	4
Probability	0.5	0.1	0.35	

The spinner is spun 200 times.
Calculate the expected number of times the spinner will land on:

3

70

4

10

1 or 2

120

Worked example

There are counters in a bag.

Colour	Red	Blue	Yellow
Number	5	10	15

More red counters are put into the bag.

The probability of taking at random a red counter is now $\frac{2}{7}$

How many red counters were put in the bag?

Your turn

There are counters in a bag.

Colour	Purple	Orange	Green
Number	10	45	5

More purple counters are put into the bag.

The probability of taking at random a purple counter is now $\frac{3}{8}$

How many purple counters were put in the bag?

20

Worked example

A person plays a game 40 times.

Prize	£1	£5	£10
Probability	0.25	0.075	0.025

The game is either win or lose.

Estimate the total value of the prizes they win.

Your turn

A person plays a game 40 times.

Prize	£1	£5	£10
Probability	0.5	0.15	0.05

The game is either win or lose.

Estimate the total value of the prizes they win.

£70

Worked example

A person plays a game 40 times.

Prize	£1	£5	£10
Probability	0.25	0.075	0.025

The game is either win or lose.
Each game costs 90p to play.
Estimate if the person playing the game will make a profit.

Your turn

A person plays a game 40 times.

Prize	£1	£5	£10
Probability	0.5	0.15	0.05

The game is either win or lose.
Each game costs £1 to play.
Estimate if the person playing the game will make a profit.

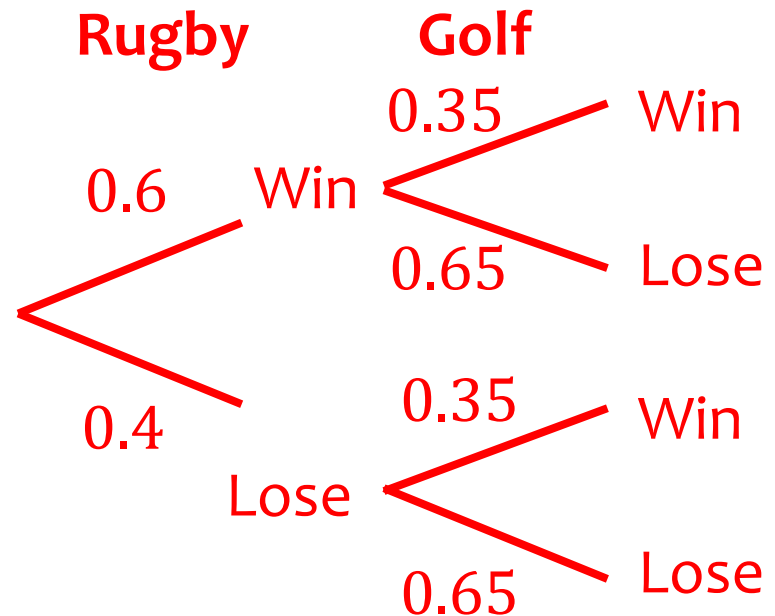
Yes: £30

Worked example

A person plays a game of tennis and then a game of golf.
They can only win or lose each game.
The probability of winning tennis is 0.3
The probability of winning golf is 0.7
The results of each game are independent of each other.
Draw a probability tree to represent this information

Your turn

A person plays a game of tennis and then a game of golf.
They can only win or lose each game.
The probability of winning tennis is 0.6
The probability of winning golf is 0.35
The results of each game are independent of each other.
Draw a probability tree to represent this information



Worked example

A person plays a game of tennis and then a game of golf.
They can only win or lose each game.
The probability of winning tennis is 0.3
The probability of winning golf is 0.7
The results of each game are independent of each other.
Calculate the probability that the person win both games

Your turn

A person plays a game of tennis and then a game of golf.
They can only win or lose each game.
The probability of winning tennis is 0.6
The probability of winning golf is 0.35
The results of each game are independent of each other.
Calculate the probability that the person win both games

$$\frac{21}{100} = 0.21$$

Worked example

A person plays a game of tennis and then a game of golf.
They can only win or lose each game.
The probability of winning tennis is 0.3
The probability of winning golf is 0.7
The results of each game are independent of each other.
Calculate the probability that the person loses both games

Your turn

A person plays a game of tennis and then a game of golf.
They can only win or lose each game.
The probability of winning tennis is 0.6
The probability of winning golf is 0.35
The results of each game are independent of each other.
Calculate the probability that the person loses both games

$$\frac{13}{50} = 0.26$$

Worked example

A person plays a game of tennis and then a game of golf.
They can only win or lose each game.
The probability of winning tennis is 0.3
The probability of winning golf is 0.7
The results of each game are independent of each other.
Calculate the probability that the person wins one and loses one

Your turn

A person plays a game of tennis and then a game of golf.
They can only win or lose each game.
The probability of winning tennis is 0.6
The probability of winning golf is 0.35
The results of each game are independent of each other.
Calculate the probability that the person wins one and loses one

$$\frac{53}{100} = 0.53$$

Worked example

A person plays a game of tennis and then a game of golf.
They can only win or lose each game.
The probability of winning tennis is 0.3
The probability of winning golf is 0.7
The results of each game are independent of each other.
Calculate the probability that the person wins at least one game

Your turn

A person plays a game of tennis and then a game of golf.
They can only win or lose each game.
The probability of winning tennis is 0.6
The probability of winning golf is 0.35
The results of each game are independent of each other.
Calculate the probability that the person wins at least one game

$$\frac{37}{50} = 0.74$$

Worked example

A person plays a game of tennis and then a game of golf.
They can only win or lose each game.
The probability of winning tennis is 0.3
The probability of winning golf is 0.7
The results of each game are independent of each other.
Calculate the probability that the person loses at least one game

Your turn

A person plays a game of tennis and then a game of golf.
They can only win or lose each game.
The probability of winning tennis is 0.6
The probability of winning golf is 0.35
The results of each game are independent of each other.
Calculate the probability that the person loses at least one game

$$\frac{79}{100} = 0.79$$

Worked example

The table shows the number of students in a group who study each subject.

	Maths	English	Science
Male	7	6	3
Female	8	4	12

A student is chosen at random.
Write down the probability that the student is:

Male and studies English

Female and studies Science

Female and studies Music

Your turn

The table shows the number of students in a group who study each subject.

	Maths	English	Science
Male	14	3	6
Female	16	2	24

A student is chosen at random.
Write down the probability that the student is:

Female and studies English $\frac{2}{65}$

Male and studies Maths $\frac{14}{65}$

Male and studies Geography 0

Worked example

The table shows the number of students in a group who study each subject.

	Maths	English	Science
Male	7	6	3
Female	8	4	12

A student is chosen at random.
Write down the probability that the student is:

Male and studies Maths or English

Female and studies Maths or Science

Female and studies English or French

Your turn

The table shows the number of students in a group who study each subject.

	Maths	English	Science
Male	14	3	6
Female	16	2	24

A student is chosen at random.
Write down the probability that the student is:

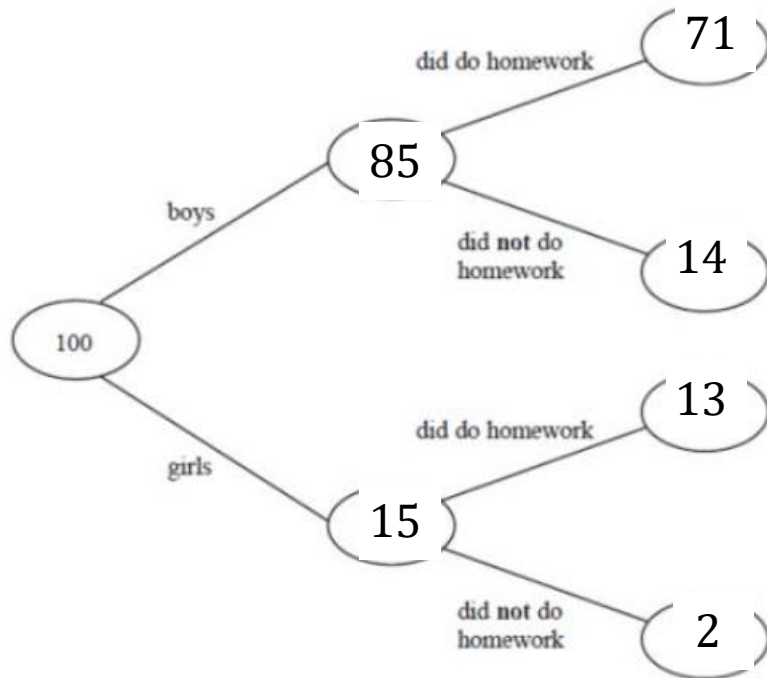
Female and studies English or Science $\frac{2}{5}$

Male and studies Maths or Science $\frac{4}{13}$

Male and studies Science or German $\frac{6}{65}$

Worked example

100 students had some homework.
The frequency tree shows the number of students who did the homework.



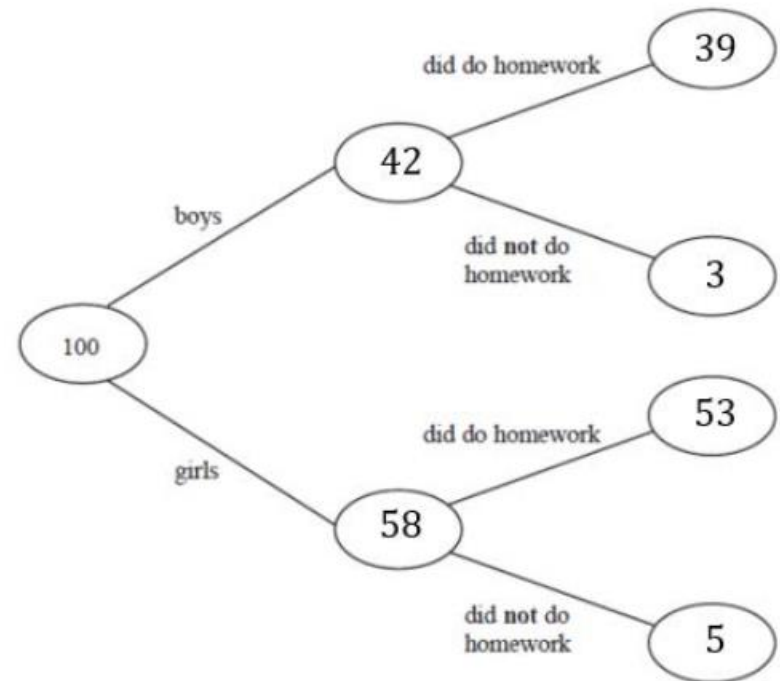
A student is chosen at random.
Write down the probability that this student:

Did their homework

Did not do their homework

Your turn

100 students had some homework.
The frequency tree shows the number of students who did the homework.

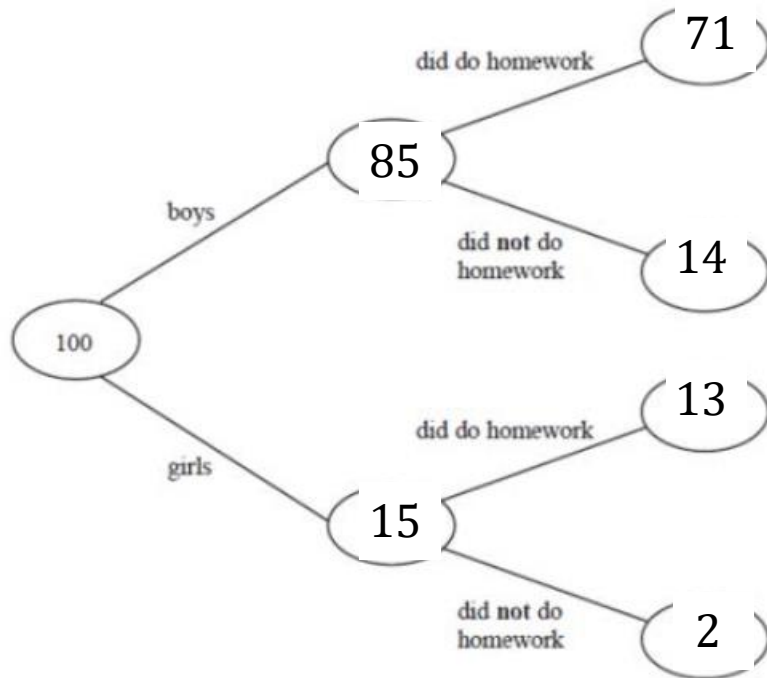


A student is chosen at random.
Write down the probability that this student did their homework

$$\frac{92}{100} = 0.92$$

Worked example

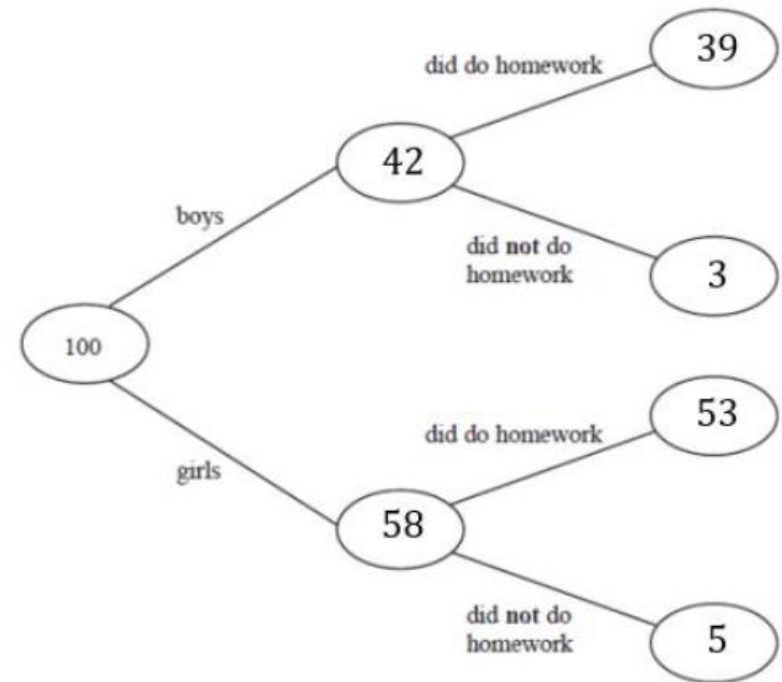
100 students had some homework.
The frequency tree shows the number of students who did the homework.



A boy is chosen at random.
Write down the probability that they did not do their homework.

Your turn

100 students had some homework.
The frequency tree shows the number of students who did the homework.



A girl is chosen at random.
Write down the probability that they did do their homework.

$$\frac{53}{58}$$

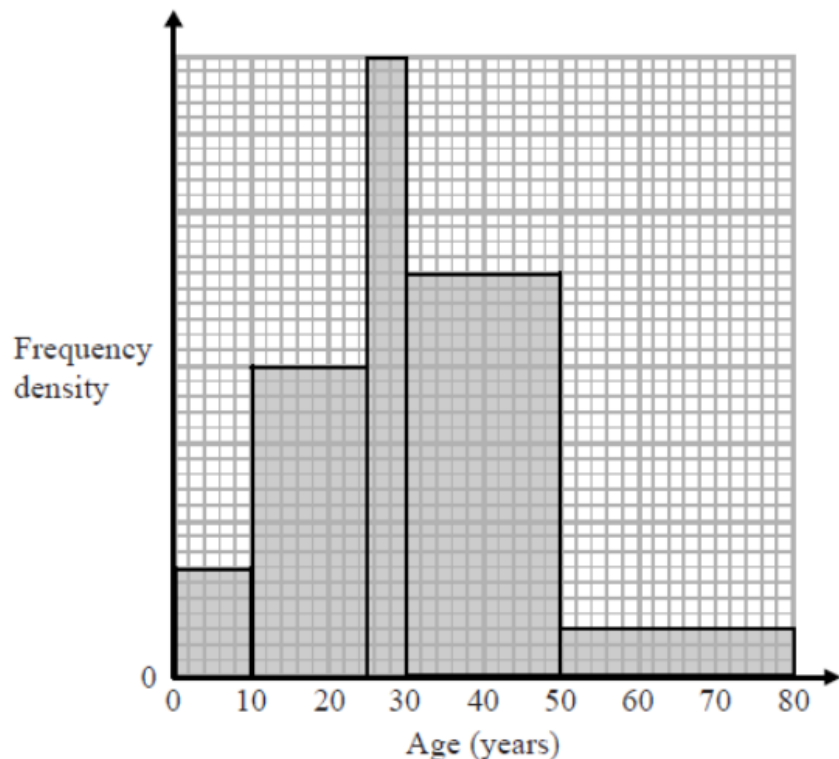
Worked example

The histogram shows the ages of members of a club.

There are 30 members aged between 10 and 25.

A member of the club is chosen at random.

What is the probability they are more than 25 years old?



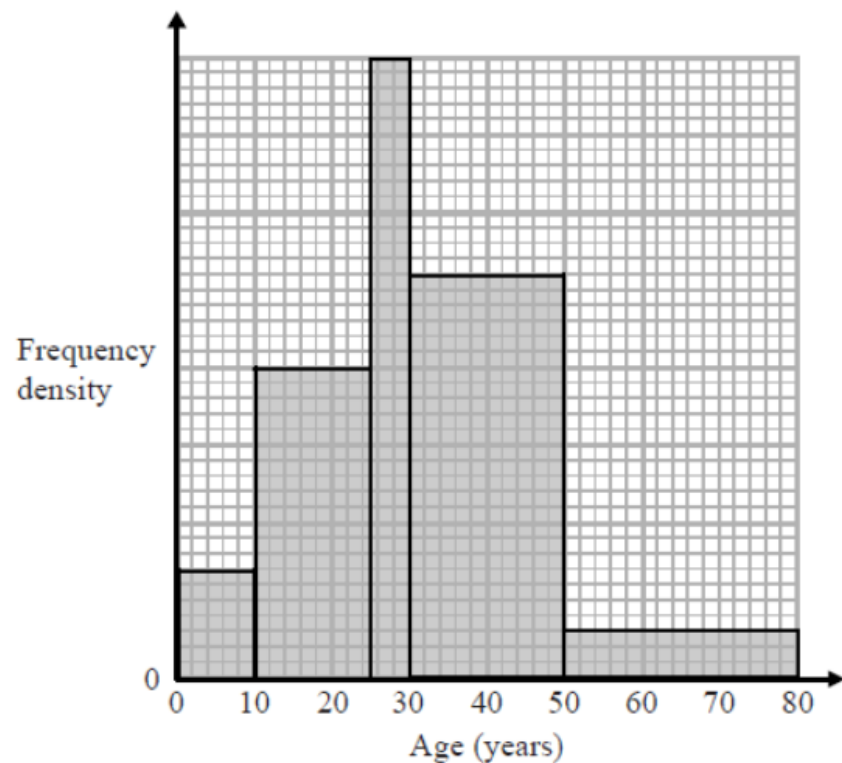
Your turn

The histogram shows the ages of members of a club.

There are 20 members aged between 25 and 30.

A member of the club is chosen at random.

What is the probability they are more than 30 years old? $\frac{61}{118}$



Worked example

There are counters in two bags

Bag A

Colour	Red	Blue	Yellow
Number	5	10	15

Bag B

Colour	Red	Blue	Yellow
Number	n	25	5

A person takes a counter from A then a counter from B.

The probability of taking two red discs is $\frac{1}{15}$. Find the value of n

Your turn

There are counters in two bags

Bag A

Colour	Red	Blue	Yellow
Number	5	6	1

Bag B

Colour	Red	Blue	Yellow
Number	n	17	0

A person takes a counter from A then a counter from B.

The probability of taking two red discs is $\frac{2}{15}$. Find the value of n

Worked example

The nine counters are in a bag.
A person takes at random a counter from the bag and does not replace the counter.

They then take at random a second counter from the bag.

Work out the probability that the number on the first counter added to the number on the second counter gives an even number.



Your turn

The nine counters are in a bag.
A person takes at random a counter from the bag and does not replace the counter.

They then take at random a second counter from the bag.

Work out the probability that the number on the first counter added to the number on the second counter gives an odd number.



$$\frac{5}{9}$$

Worked example

There are n sweets in a bag.
4 of the sweets are orange.
The rest are yellow.
Hannah takes at random a sweet from the bag. She eats the sweet.
Hannah then takes at random another sweet from the bag, and eats it.
The probability that Hannah eats two orange sweets is $\frac{2}{5}$.
Find n

Your turn

There are n sweets in a bag.
6 of the sweets are orange.
The rest are yellow.
Hannah takes at random a sweet from the bag. She eats the sweet.
Hannah then takes at random another sweet from the bag, and eats it.
The probability that Hannah eats two orange sweets is $\frac{1}{3}$.
Find n

$$n = 10$$

Worked example

There are counters in a bag.

At the start, 7 are red and the rest are blue.

A person takes at random a counter from the bag. They do not put it back in the bag. Then they take another counter at random.

The probability that the first counter is blue and the second counter is red is $\frac{21}{90}$

Work out the number of blue counters in the bag at the start.

Your turn

There are counters in a bag.

At the start, 7 are red and the rest are blue.

A person takes at random a counter from the bag. They do not put it back in the bag. Then they take another counter at random.

The probability that the first counter is blue and the second counter is red is $\frac{21}{80}$

Work out the number of blue counters in the bag at the start.

9

Worked example

The table shows 100 students, who each study one language. Two students are chosen at random.

	French	German
Female	26	30
Male	10	34

Calculate the probability that the two chosen students study the same language.

Your turn

The table shows 50 students, who each study one language. Two students are chosen at random.

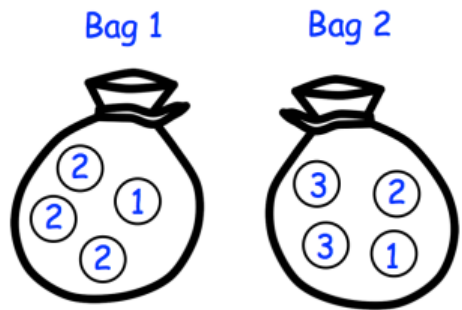
	Japanese	Spanish
Female	13	15
Male	5	17

Calculate the probability that the two chosen students study the same language.

$$\frac{64}{245}$$

Worked example

There are two bags with numbered discs as shown.



A person chooses a disc at random from bag 1.

If it is labelled 2, he puts the disc in bag 2.

If it is labelled 1, he does not put the disc in bag 2.

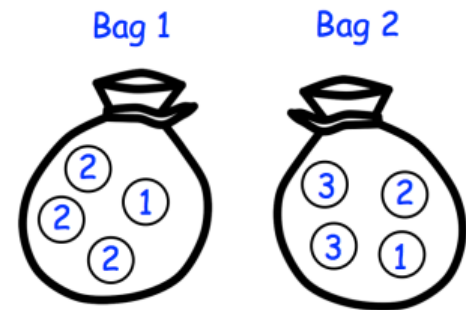
He then chooses a disc at random from bag 2.

He then adds the numbers of the two discs he selected to give his score.

Find the probability that his score is 5.

Your turn

There are two bags with numbered discs as shown.



A person chooses a disc at random from bag 1.

If it is labelled 1, he puts the disc in bag 2.

If it is labelled 2, he does not put the disc in bag 2.

He then chooses a disc at random from bag 2.

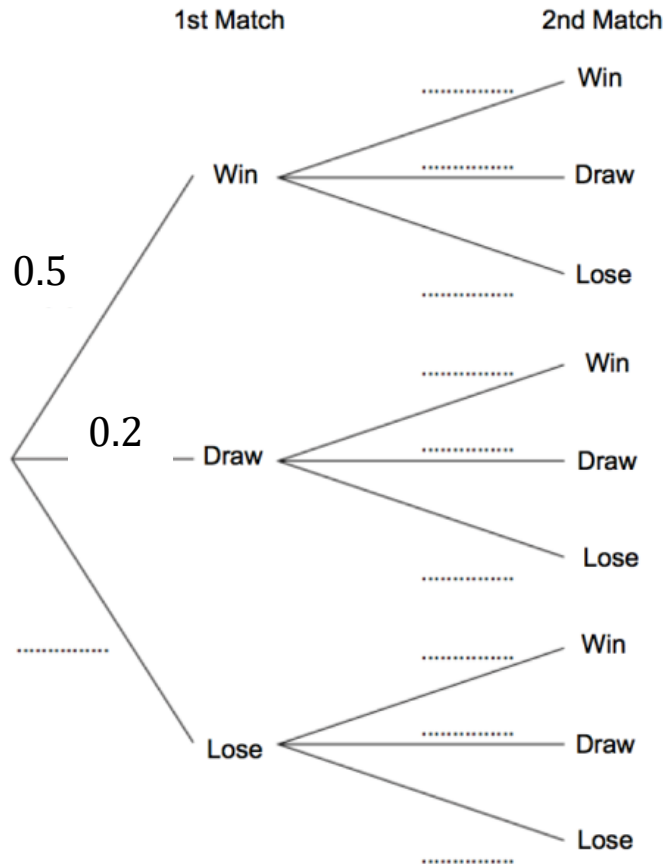
He then adds the numbers of the two discs he selected to give his score.

Find the probability that his score is 4.

$$\frac{23}{80}$$

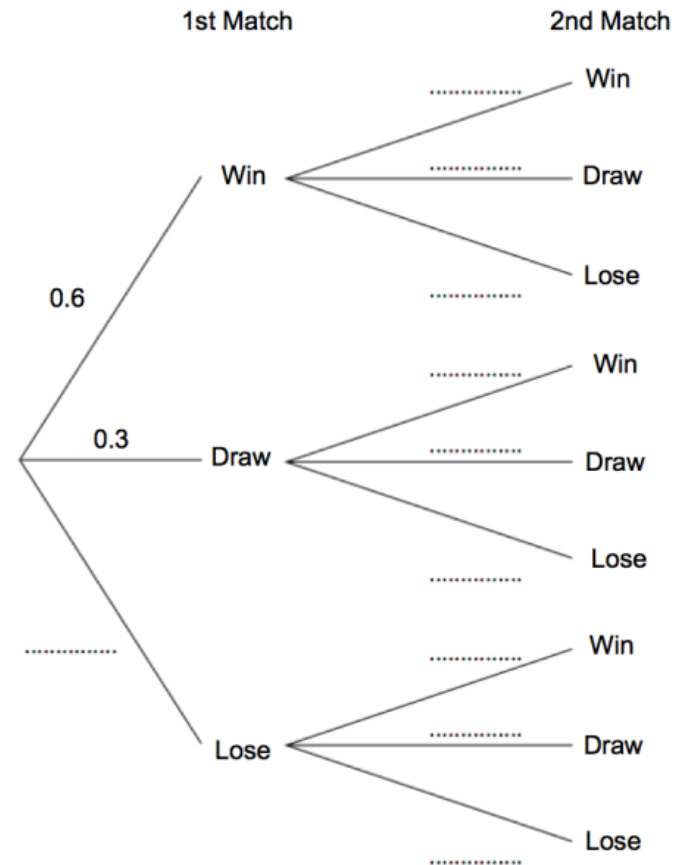
Worked example

A football team plays two matches.
Calculate the probability that they win at least one match.



Your turn

A football team plays two matches.
Calculate the probability that they win at least one match.



0.99

Worked example

A coin is flipped 30 times. The results are:

H T H H H H H H T T T H H T T
T T T H H H T T H H T H T H H

What are the relative frequencies for heads and tails?

The coin is flipped 300 more times. Estimate how many times the coin will land on tails.

Your turn

A coin is flipped 20 times. The results are:

T H T T T T T H H H
H H T H T H H H H H

What are the relative frequencies for heads and tails?

$$\text{Heads: } \frac{12}{20} = \frac{3}{5}$$
$$\text{Tails: } \frac{8}{20} = \frac{2}{5}$$

The coin is flipped 100 more times. Estimate how many times the coin will land on tails.

40

Worked example

The numbers 1 to 10 are in a bag.
A number is taken out, then replaced.
Another number is then taken out.
Calculate the probability that the two numbers taken out were different.

Your turn

The numbers 1 to 20 are in a bag.
A number is taken out, then replaced.
Another number is then taken out.
Calculate the probability that the two numbers taken out were different.

$$\frac{19}{20} = 0.95$$

Worked example

A and B are two events.

$$P(A) = 0.5$$

$$P(B) = 0.6$$

$$P(A \cup B) = 0.8$$

Calculate $P(A \cap B)$

Your turn

A and B are two events.

$$P(A) = 0.6$$

$$P(B) = 0.7$$

$$P(A \cup B) = 0.9$$

Calculate $P(A \cap B)$

0.4

Worked example

A and B are two events.

$$P(A) = 0.4$$

$$P(B) = 0.6$$

$$P(A \cap B) = 0.3$$

Calculate $P(A \cup B)$

Your turn

A and B are two events.

$$P(A) = 0.3$$

$$P(B) = 0.5$$

$$P(A \cap B) = 0.2$$

Calculate $P(A \cup B)$

0.6

Worked example

Two six-sided dice are rolled and the scores added together.

Find the probability that:

A 3 was rolled, given that the total was 6

The total was greater than 6, given that a 3 was rolled

Your turn

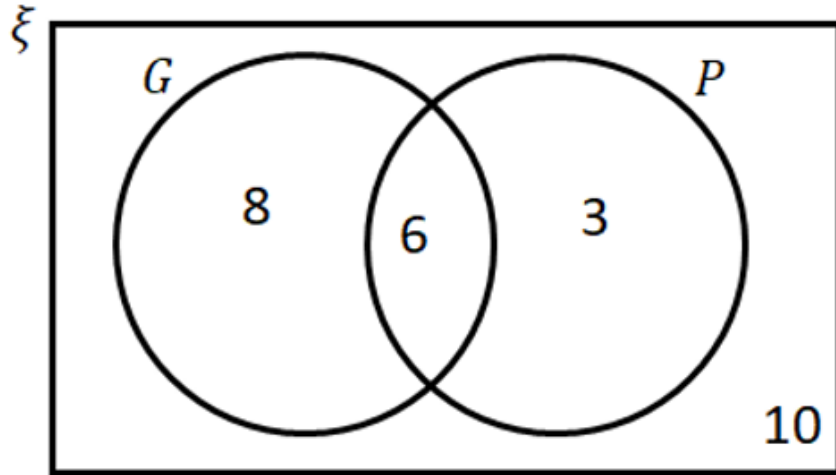
Two six-sided dice are rolled and the scores added together.

Find the probability that the total is greater than 3, given that a 2 was rolled

$$\frac{5}{7}$$

Worked example

The Venn diagram shows a sample of people who play the guitar (G) or piano (P).



Find the probability that a student plays the guitar, given that they play the piano

Your turn

Find the probability that a student plays the piano, given that they play the guitar

$$\frac{3}{7}$$

Worked example

Five coins are flipped. Find the number of ways of flipping four tails using Pascal's triangle.

Your turn

Seven coins are flipped. Find the number of ways of flipping four tails using Pascal's triangle.

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