

Surds

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Worked example

$$2\sqrt{3} + 5\sqrt{3}$$

$$3\sqrt{5} + 4\sqrt{5} + \sqrt{5}$$

Your turn

$$8\sqrt{2} + 6\sqrt{2} + \sqrt{2} + 3\sqrt{2}$$

$$18\sqrt{2}$$

Worked example

$$2\sqrt{3} + 5\sqrt{3} - \sqrt{3}$$

$$3\sqrt{5} + 4\sqrt{5} - 9\sqrt{5}$$

Your turn

$$8\sqrt{2} + 6\sqrt{2} - 17\sqrt{2}$$

$$-3\sqrt{2}$$

Worked example

$$2\sqrt{3} + 5 - \sqrt{3} + 4$$

$$3\sqrt{5} - 2 - 4\sqrt{5} + 7$$

Your turn

$$8\sqrt{2} - 9 - 6\sqrt{2} + 5$$

$$2\sqrt{2} - 4$$

Worked example

$$2\sqrt{3} + 5 - (\sqrt{3} + 4)$$

$$3\sqrt{5} - (2 - 4\sqrt{5}) + 7$$

Your turn

$$8\sqrt{2} - (9 - 6\sqrt{2}) + 5$$

$$14\sqrt{2} - 4$$

Worked example

$$\sqrt{3} \times \sqrt{3}$$

$$\sqrt{5} \times \sqrt{5}$$

Your turn

$$\sqrt{2} \times \sqrt{2}$$

2

Worked example

$$\sqrt{3} \times \sqrt{5}$$

$$\sqrt{7} \times \sqrt{11}$$

Your turn

$$\sqrt{2} \times \sqrt{13}$$

$$\sqrt{26}$$

Worked example

$$\sqrt{6} \div \sqrt{3}$$

$$\frac{\sqrt{21}}{\sqrt{7}}$$

Your turn

$$\frac{\sqrt{56}}{\sqrt{8}}$$
$$\sqrt{7}$$

Worked example

$$\frac{15\sqrt{10}}{5\sqrt{10}}$$

$$\frac{28\sqrt{11}}{4\sqrt{11}}$$

Your turn

$$\frac{48\sqrt{13}}{6\sqrt{13}}$$

8

Worked example

$$\sqrt{5} \times \sqrt{6} \div \sqrt{3}$$

$$\sqrt{2} \times \sqrt{15} \div \sqrt{6}$$

Your turn

$$\sqrt{3} \times \sqrt{10} \div \sqrt{2}$$

$$\sqrt{15}$$

Worked example

$$2\sqrt{7} \times 3\sqrt{7}$$

$$5\sqrt{11} \times -7\sqrt{11}$$

Your turn

$$7\sqrt{3} \times -4\sqrt{3}$$

$$-84$$

Worked example

$$(\sqrt{5})^2$$

$$(\sqrt{7})^2$$

Your turn

$$(\sqrt{11})^2$$

11

Worked example

$$(\sqrt[3]{5})^3$$

$$(\sqrt[3]{7})^3$$

Your turn

$$(\sqrt[3]{11})^3$$

11

Worked example

$$(\sqrt{5})^3$$

$$(\sqrt{7})^3$$

Your turn

$$(\sqrt{11})^3$$

$$11\sqrt{11}$$

Worked example

$$3\sqrt{5} \times \sqrt{7}$$

$$4\sqrt{3} \times 11\sqrt{2}$$

Your turn

$$5\sqrt{11} \times 2\sqrt{7}$$

$$10\sqrt{77}$$

Worked example

Simplify:

$$\sqrt{160}$$

$$\sqrt{180}$$

Your turn

Simplify:

$$\sqrt{320}$$

$$4\sqrt{2}$$

Worked example

Simplify:

$$\sqrt{72} - \sqrt{18}$$

$$\sqrt{125} - \sqrt{45}$$

Your turn

Simplify:

$$\sqrt{75} - \sqrt{12}$$

$$3\sqrt{3}$$

Worked example

Simplify $\sqrt{50}$ by writing it in the form $a\sqrt{b}$ where b is prime

Your turn

Simplify $\sqrt{32}$ by writing it in the form $a\sqrt{b}$ where b is prime

$$4\sqrt{2}$$

Worked example

Simplify $6\sqrt{45}$ by writing it in the form $a\sqrt{b}$ where b is prime

Your turn

Simplify $7\sqrt{99}$ by writing it in the form $a\sqrt{b}$ where b is prime

$$21\sqrt{11}$$

Worked example

Show that $\sqrt{27} + \sqrt{192} = a\sqrt{b}$

Your turn

Show that $\sqrt{28} + \sqrt{63} = a\sqrt{b}$

$$5\sqrt{7}$$

Worked example

Evaluate $\sqrt{2}(\sqrt{8} + \sqrt{50})$

Your turn

Evaluate $\sqrt{3}(\sqrt{12} + \sqrt{27})$

15

Worked example

Express in the form $a + b\sqrt{3}$:
 $(4 + \sqrt{3})(1 + 2\sqrt{3})$

Your turn

Express in the form $a + b\sqrt{5}$:
 $(2 + \sqrt{5})(3 + 4\sqrt{5})$

$$26 + 11\sqrt{5}$$

Worked example

Express in the form $a + b\sqrt{3}$:

$$(5 + 3\sqrt{3})^2$$

Your turn

Express in the form $a + b\sqrt{5}$:

$$(2 + 7\sqrt{5})^2$$

$$249 + 28\sqrt{5}$$

Worked example

Express in the form $a + b\sqrt{3}$:

$$(2\sqrt{3} - 5)^2$$

Your turn

Express in the form $a + b\sqrt{5}$:

$$(3\sqrt{5} - 2)^2$$

$$49 - 12\sqrt{5}$$

Worked example

Simplify

$$\sqrt{20} + \frac{55}{\sqrt{5}}$$

Your turn

Simplify

$$\sqrt{75} + \frac{24}{\sqrt{3}}$$

$$13\sqrt{3}$$

Worked example

Simplify:

$$(\sqrt{7} + 1)(\sqrt{7} - 1)$$

$$(\sqrt{5} - 2)(\sqrt{5} + 2)$$

Your turn

Simplify:

$$(\sqrt{11} - 3)(\sqrt{11} + 3)$$

2

Worked example

Simplify:

$$(3 + \sqrt{2})(3 - \sqrt{2})$$

$$(5 - \sqrt{3})(5 + \sqrt{3})$$

Your turn

Simplify:

$$(7 - \sqrt{5})(7 + \sqrt{5})$$

44

Worked example

Simplify:

$$(7 + 4\sqrt{2})(7 - 4\sqrt{2})$$

$$(5 - 2\sqrt{3})(5 + 2\sqrt{3})$$

Your turn

Simplify:

$$(9 - 3\sqrt{5})(9 + 3\sqrt{5})$$

36

Worked example

Simplify:

$$(7\sqrt{3} + 4\sqrt{2})(7\sqrt{3} - 4\sqrt{2})$$

$$(5\sqrt{7} - 2\sqrt{3})(5\sqrt{7} + 2\sqrt{3})$$

Your turn

Simplify:

$$(11\sqrt{5} - 3\sqrt{7})(11\sqrt{5} + 3\sqrt{7})$$

542

Worked example

Simplify $\frac{\sqrt{450}}{3}$ by writing it in the form $a\sqrt{b}$ where b is prime

Your turn

Simplify $\frac{\sqrt{288}}{4}$ by writing it in the form $a\sqrt{b}$ where b is prime

$$3\sqrt{2}$$

Worked example

Rationalise the denominator:

$$\frac{1}{\sqrt{2}}$$

$$\frac{2}{\sqrt{3}}$$

$$\frac{3}{\sqrt{5}}$$

Your turn

Rationalise the denominator:

$$\frac{4}{\sqrt{7}}$$

$$\frac{4\sqrt{7}}{7}$$

Worked example

Rationalise the denominator and simplify:

$$\frac{4}{\sqrt{2}}$$

$$\frac{6}{\sqrt{3}}$$

$$\frac{20}{\sqrt{5}}$$

Your turn

Rationalise the denominator and simplify:

$$\frac{35}{\sqrt{7}}$$

$$5\sqrt{7}$$

Worked example

Rationalise the denominator and simplify:

$$\frac{4}{3\sqrt{2}}$$

$$\frac{6}{5\sqrt{3}}$$

Your turn

Rationalise the denominator and simplify:

$$\frac{10}{2\sqrt{7}}$$

$$\frac{5\sqrt{7}}{7}$$

Worked example

Rationalise the denominator and simplify:

$$\frac{2 - 7\sqrt{3}}{\sqrt{3}}$$

$$\frac{3 - 5\sqrt{11}}{\sqrt{11}}$$

Your turn

Rationalise the denominator and simplify:

$$\frac{9 - 3\sqrt{5}}{\sqrt{5}}$$

$$\frac{9\sqrt{5} - 15}{5}$$

Worked example

Rationalise the denominator and simplify:

$$\frac{\sqrt{12}}{4\sqrt{3}}$$

$$\frac{\sqrt{20}}{3\sqrt{5}}$$

Your turn

Rationalise the denominator and simplify:

$$\frac{\sqrt{28}}{5\sqrt{7}}$$

$$\frac{2}{5}$$

Worked example

Rationalise the denominator and simplify:

$$\frac{3}{5 + \sqrt{2}}$$

$$\frac{4}{1 + \sqrt{3}}$$

Your turn

Rationalise the denominator and simplify:

$$\frac{18}{1 + \sqrt{7}}$$

$$-3 + 3\sqrt{7}$$

Worked example

Rationalise the denominator and simplify:

$$\frac{3}{5 - \sqrt{2}}$$

$$\frac{4}{1 - \sqrt{3}}$$

Your turn

Rationalise the denominator and simplify:

$$\frac{18}{1 - \sqrt{7}}$$

$$-3 - 3\sqrt{7}$$

Worked example

Rationalise the denominator and simplify:

$$\frac{3}{\sqrt{2} - 5}$$

$$\frac{4}{-1 + \sqrt{3}}$$

Your turn

Rationalise the denominator and simplify:

$$\frac{18}{\sqrt{7} - 1}$$

$$3 + 3\sqrt{7}$$

Worked example

Rationalise the denominator and simplify:

$$\frac{3 + \sqrt{2}}{\sqrt{2} - 5}$$

$$\frac{5 - \sqrt{3}}{-1 + \sqrt{3}}$$

Your turn

Rationalise the denominator and simplify:

$$\frac{2 - \sqrt{7}}{\sqrt{7} - 1}$$

$$\frac{-5 + \sqrt{7}}{6}$$

Worked example

Rationalise the denominator and simplify:

$$\frac{2\sqrt{3} + 5}{\sqrt{3} - 2}$$

Your turn

Rationalise the denominator and simplify:

$$\frac{2\sqrt{2} + 1}{\sqrt{2} - 3}$$

$$-1 - \sqrt{2}$$

Worked example

Rationalise the denominator and simplify:

$$\frac{3\sqrt{3} - 5}{2\sqrt{3} - 7}$$

Your turn

Rationalise the denominator and simplify:

$$\frac{5\sqrt{5} - 2}{2\sqrt{5} - 3}$$
$$4 + \sqrt{5}$$

Worked example

Rationalise the denominator and simplify:

$$\frac{11 + 3\sqrt{2}}{3\sqrt{7} + 5}$$

Your turn

Rationalise:

$$\frac{7 + 2\sqrt{3}}{11\sqrt{5} + 2}$$
$$\frac{77\sqrt{5} + 22\sqrt{15} - 4\sqrt{3} - 14}{601}$$

Worked example

Rationalise the denominator and simplify:

$$\frac{1}{12\sqrt{27}}$$

$$\frac{1}{16\sqrt{32}}$$

Your turn

Rationalise the denominator and simplify:

$$\frac{1}{8\sqrt{8}}$$

$$\frac{\sqrt{2}}{32}$$

Worked example

Rationalise the denominator and simplify:

$$\frac{(3 + \sqrt{2})(3 - \sqrt{2})}{\sqrt{7}}$$

$$\frac{(7 + \sqrt{5})(7 - \sqrt{5})}{\sqrt{11}}$$

Your turn

Rationalise the denominator and simplify:

$$\frac{(5 + \sqrt{3})(5 - \sqrt{3})}{\sqrt{22}}$$

$$\sqrt{22}$$

Worked example

Simplify, giving your answer in the form $a\sqrt{b}$:

$$\frac{\sqrt{54}}{3} + \frac{12}{\sqrt{6}}$$

Your turn

Simplify, giving your answer in the form $a\sqrt{b}$:

$$\frac{\sqrt{63}}{3} + \frac{21}{\sqrt{7}}$$

$$4\sqrt{7}$$

Worked example

Show that $\frac{1}{\frac{1}{\sqrt{3}}+1} = \frac{3}{2} - \frac{1}{2}\sqrt{3}$

Your turn

Show that $\frac{1}{1+\frac{1}{\sqrt{2}}} = 2 - \sqrt{2}$

$$= \frac{\sqrt{2}}{\sqrt{2} + 1}$$

$$= \frac{\sqrt{2}}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1}$$

$$= \frac{2 - \sqrt{2}}{2 + \sqrt{2} - \sqrt{2} - 1}$$

$$= \frac{2 - \sqrt{2}}{1}$$

$$= 2 - \sqrt{2}$$

Worked example

A rectangle has area 60 cm^2 and a width of $\sqrt{12} \text{ cm}$. Find the length of the rectangle in the form $a\sqrt{b}$

Your turn

A rectangle has area 64 cm^2 and a width of $\sqrt{32} \text{ cm}$. Find the length of the rectangle in the form $a\sqrt{b}$

$$8\sqrt{2} \text{ cm}$$

Worked example

A rectangle has an area of $(15 - 6\sqrt{3}) \text{ cm}^2$ and a width of $(2\sqrt{3} - 3) \text{ cm}$. Find the length of the rectangle in the form $a + b\sqrt{3}$

Your turn

A rectangle has an area of $(2 + \sqrt{2}) \text{ cm}^2$ and a width of $(3\sqrt{2} - 4) \text{ cm}$. Find the length of the rectangle in the form $a + b\sqrt{2}$

$$7 + 5\sqrt{2} \text{ cm}$$

Worked example

Simplify

$$(\sqrt{8} - \sqrt{2})^2$$

$$(\sqrt{20} - \sqrt{5})^2$$

Your turn

Simplify

$$(\sqrt{12} - \sqrt{3})^2$$

3

Worked example

Simplify

$$(3\sqrt{5} - 1)(2\sqrt{5} + 6)$$

Your turn

Simplify

$$(2\sqrt{3} - 4)(4\sqrt{3} + 2)$$

$$16 - 12\sqrt{3}$$

Worked example

Simplify

$$(3 + \sqrt{2})^2 - (3 - \sqrt{2})^2$$

$$(4 + \sqrt{5})^2 - (4 - \sqrt{5})^2$$

Your turn

Simplify

$$(2 + \sqrt{3})^2 - (2 - \sqrt{3})^2$$

$$8\sqrt{3}$$

Worked example

Simplify

$$(\sqrt{3} + 5)(\sqrt{3} - 3)(\sqrt{3} + 2)$$

Your turn

Simplify

$$(\sqrt{5} + 3)(\sqrt{5} - 2)(\sqrt{5} + 1)$$

4

Worked example

Express b and c in terms of a

$$(a + \sqrt{12})^2 = b + c\sqrt{3}$$

$$(a + \sqrt{20})^2 = b + c\sqrt{5}$$

Your turn

Express b and c in terms of a

$$(a + \sqrt{8})^2 = b + c\sqrt{2}$$

$$b = a^2 + 8$$

$$c = 4a$$

Worked example

Find the value of a and b :

$$(a - 3\sqrt{5})^2 = b - 42\sqrt{5}$$

Your turn

Express b and c in terms of a

$$(a - 2\sqrt{3})^2 = b - 20\sqrt{3}$$

$$a = 5, b = 37$$

Worked example

Find in its simplest form $a:b$,
given:

$$a = \sqrt{5} + \sqrt{c}$$

$$b = \sqrt{80} + \sqrt{d}$$

c and d are positive integers

$$c:d = 1:16$$

Your turn

Find in its simplest form $a:b$,
given:

$$a = \sqrt{7} + \sqrt{c}$$

$$b = \sqrt{63} + \sqrt{d}$$

c and d are positive integers

$$c:d = 1:9$$

$$a:b = 1:3$$