

Vectors

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Worked example

$$\mathbf{a} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

Write down as a column vector:

$$\mathbf{a} + \mathbf{b}$$

$$\mathbf{a} - \mathbf{b}$$

$$3\mathbf{a}$$

$$2\mathbf{a} + 3\mathbf{b}$$

Your turn

$$\mathbf{a} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

Write down as a column vector:

$$\mathbf{a} + \mathbf{b} \quad \begin{pmatrix} 9 \\ 5 \end{pmatrix}$$

$$\mathbf{a} - \mathbf{b} \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$2\mathbf{a} \quad \begin{pmatrix} 10 \\ 6 \end{pmatrix}$$

$$3\mathbf{a} + 2\mathbf{b} \quad \begin{pmatrix} 23 \\ 13 \end{pmatrix}$$

Worked example

$$\mathbf{a} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

Write down as a column vector:

$$\mathbf{a} + \mathbf{b}$$

$$\mathbf{a} - \mathbf{b}$$

$$3\mathbf{a}$$

$$2\mathbf{a} + 3\mathbf{b}$$

Your turn

$$\mathbf{a} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

Write down as a column vector:

$$\mathbf{a} + \mathbf{b} \quad \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\mathbf{a} - \mathbf{b} \quad \begin{pmatrix} 9 \\ -5 \end{pmatrix}$$

$$2\mathbf{a} \quad \begin{pmatrix} 10 \\ -6 \end{pmatrix}$$

$$3\mathbf{a} + 2\mathbf{b} \quad \begin{pmatrix} 7 \\ -5 \end{pmatrix}$$

Worked example

$$\mathbf{a} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

Work out the magnitude of:

$\mathbf{a}$

$\mathbf{b}$

Your turn

$$\mathbf{a} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

Work out the magnitude of:

$\mathbf{a}$

$$\sqrt{34}$$

$\mathbf{b}$

$$\sqrt{20} = 2\sqrt{5}$$

Worked example

$$\mathbf{a} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

Work out the magnitude of:

$\mathbf{a}$

$\mathbf{b}$

Your turn

$$\mathbf{a} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

Work out the magnitude of:

$\mathbf{a}$

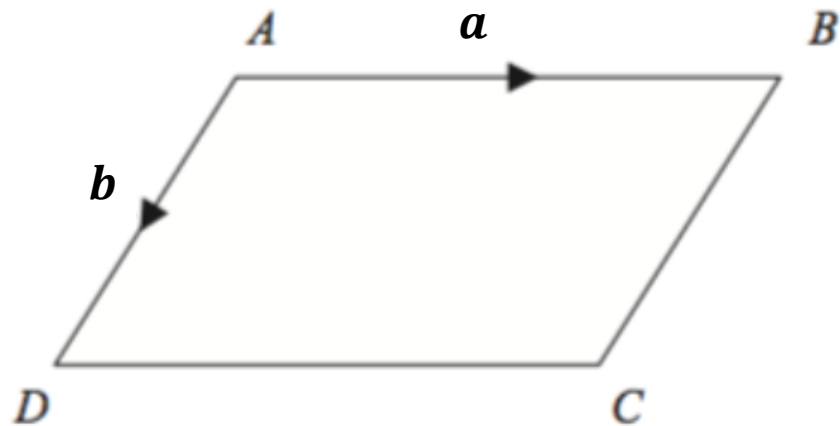
$$\sqrt{34}$$

$\mathbf{b}$

$$\sqrt{20} = 2\sqrt{5}$$

Worked example

ABCD is a parallelogram. Express the following in terms of ***a*** and ***b***:



$$\vec{AB} =$$

$$\vec{AD} =$$

$$\vec{BC} =$$

$$\vec{BA} =$$

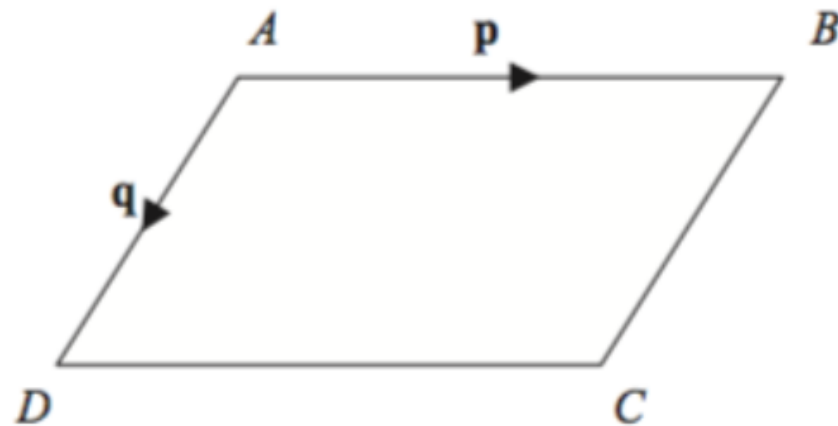
$$\vec{DA} =$$

$$\vec{AC} =$$

$$\vec{BD} =$$

Your turn

ABCD is a parallelogram. Express the following in terms of ***a*** and ***b***:



$$\vec{AD} = \vec{q}$$

$$\vec{AB} = \vec{p}$$

$$\vec{DC} = \vec{p}$$

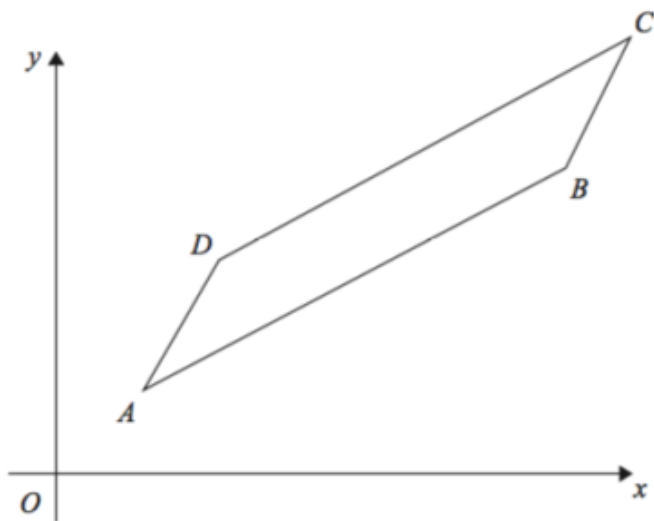
$$\vec{DA} = -\vec{q}$$

$$\vec{BA} = -\vec{p}$$

$$\vec{DB} = \vec{p} - \vec{q}$$

$$\vec{CA} = -\vec{p} - \vec{q}$$

Worked example



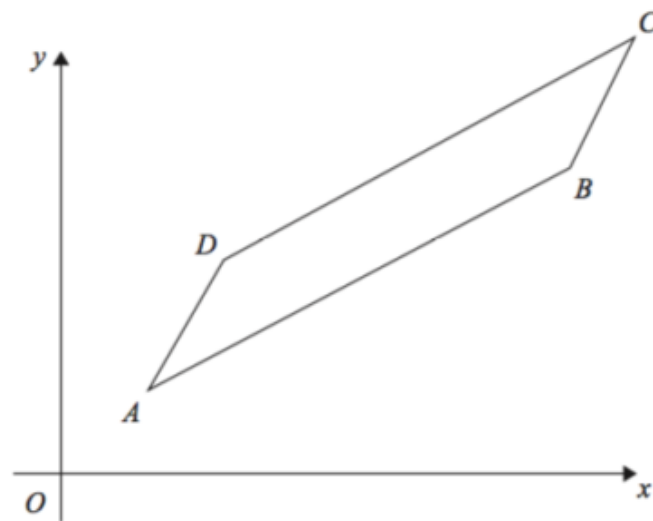
$$\overrightarrow{AD} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \overrightarrow{AB} = \begin{pmatrix} 10 \\ 6 \end{pmatrix}$$

ABCD is a parallelogram.

A has coordinates (5, 7).

Work out the coordinates of C

Your turn



$$\overrightarrow{AD} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \overrightarrow{AB} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

ABCD is a parallelogram.

A has coordinates (5, 7).

Work out the coordinates of C

(11, 12)

Worked example

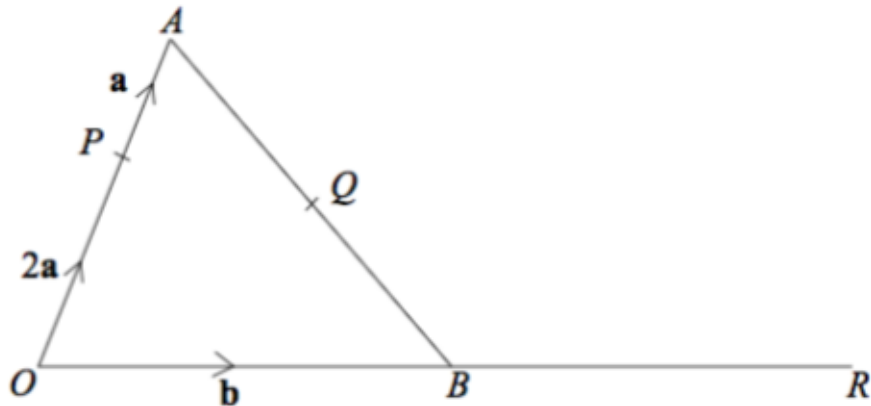
B is the midpoint of OR.

Q is the midpoint of AB.

$$\overrightarrow{PR} = 3\overrightarrow{PQ}$$

The length of PQ is 4 *cm*

Find the length of PR



Your turn

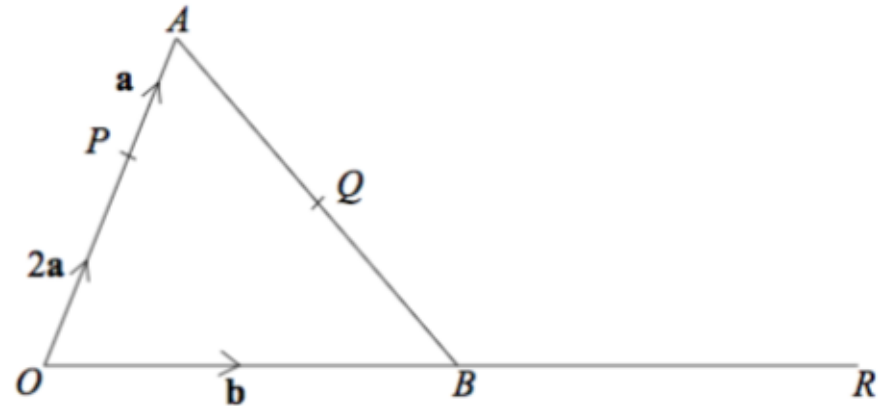
B is the midpoint of OR.

Q is the midpoint of AB.

$$\overrightarrow{PR} = 3\overrightarrow{PQ}$$

The length of PQ is 4 *cm*

Find the length of PR



12 *cm*

Worked example

$$\overrightarrow{OA} = \mathbf{a} + 2\mathbf{b}$$

$$\overrightarrow{OB} = 3\mathbf{a} - 5\mathbf{b}$$

Express the vectors in terms of $\mathbf{a}$ and $\mathbf{b}$:

$$\overrightarrow{AO} =$$

$$\overrightarrow{BO} =$$

$$\overrightarrow{AB} =$$

$$\overrightarrow{BA} =$$

Your turn

$$\overrightarrow{OA} = 2\mathbf{a} + \mathbf{b}$$

$$\overrightarrow{OB} = 5\mathbf{a} - 3\mathbf{b}$$

Express the vectors in terms of $\mathbf{a}$ and $\mathbf{b}$:

$$\overrightarrow{AO} = -2\mathbf{a} - \mathbf{b}$$

$$\overrightarrow{BO} = -5\mathbf{a} + 3\mathbf{b}$$

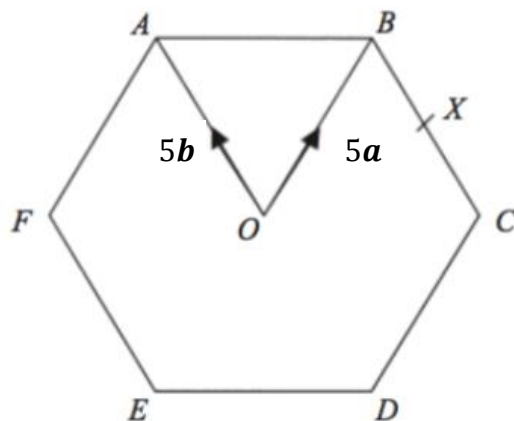
$$\overrightarrow{AB} = 3\mathbf{a} - 4\mathbf{b}$$

$$\overrightarrow{BA} = -4\mathbf{a} - 4\mathbf{b}$$

Worked example

ABCDEF is a regular hexagon with centre O.

Express the following in terms of **a** and/or **b**:



$$\overrightarrow{EF} =$$

$$\overrightarrow{DC} =$$

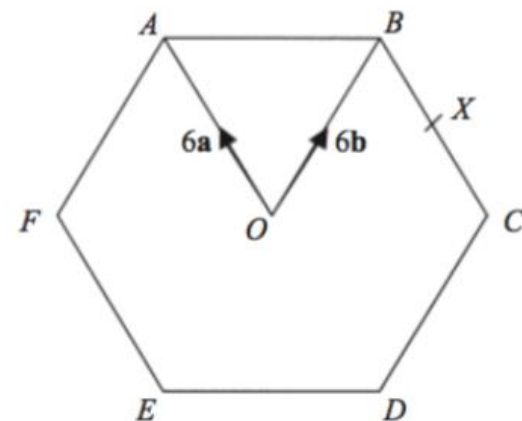
$$\overrightarrow{BC} =$$

$$\overrightarrow{AF} =$$

Your turn

ABCDEF is a regular hexagon with centre O.

Express the following in terms of **a** and/or **b**:



$$\overrightarrow{FA} = 6b$$

$$\overrightarrow{CB} = 6a$$

$$\overrightarrow{FE} = -6a$$

$$\overrightarrow{CD} = -6b$$

Worked example

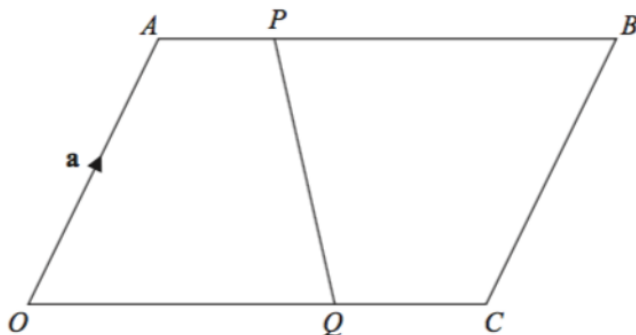
OABC is a parallelogram.

$$\overrightarrow{OA} = \mathbf{a} \text{ and } \overrightarrow{OC} = \mathbf{c}$$

$$\overrightarrow{AP} = \frac{1}{3} \overrightarrow{AB}$$

$$OQ = \frac{4}{5} \overrightarrow{OC}$$

Find, in terms of $\mathbf{a}$ and $\mathbf{c}$, $\overrightarrow{PQ}$



Your turn

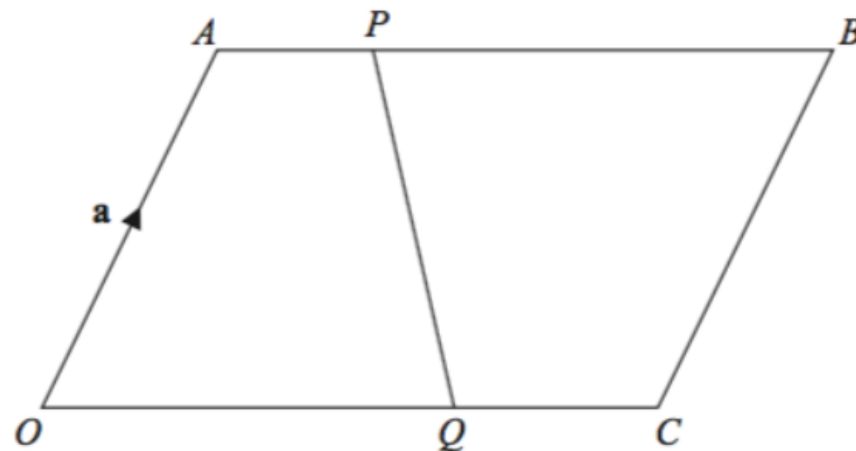
OABC is a parallelogram.

$$\overrightarrow{OA} = \mathbf{a} \text{ and } \overrightarrow{OC} = \mathbf{c}$$

$$\overrightarrow{AP} = \frac{1}{4} \overrightarrow{AB}$$

$$OQ = \frac{2}{3} \overrightarrow{OC}$$

Find, in terms of $\mathbf{a}$ and $\mathbf{c}$, $\overrightarrow{PQ}$



$$\frac{5}{12} \mathbf{c} - \mathbf{a}$$

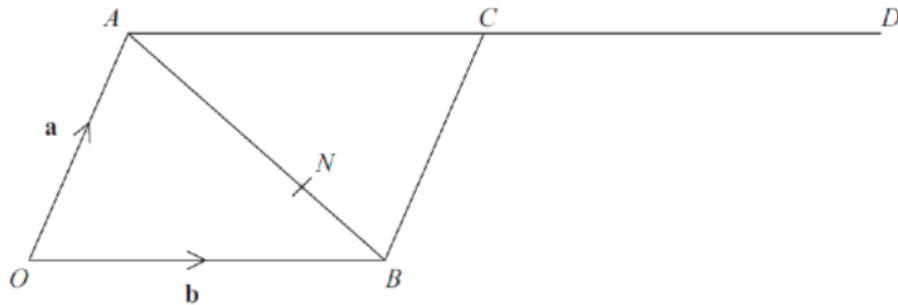
Worked example

OACB is a parallelogram.

D is the point such that $\overrightarrow{AC} = \overrightarrow{CD}$.

The point N divided AB in the ratio 4:1

Write an expression for $\overrightarrow{ON}$ in terms of ***a*** and ***b***



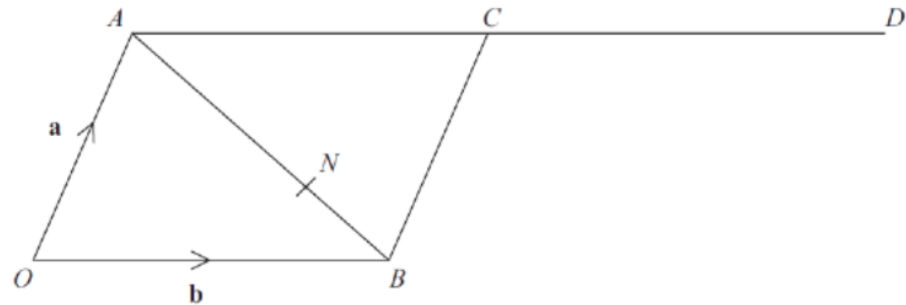
Your turn

OACB is a parallelogram.

D is the point such that $\overrightarrow{AC} = \overrightarrow{CD}$.

The point N divided AB in the ratio 2:1

Write an expression for $\overrightarrow{ON}$ in terms of ***a*** and ***b***



$$\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$$

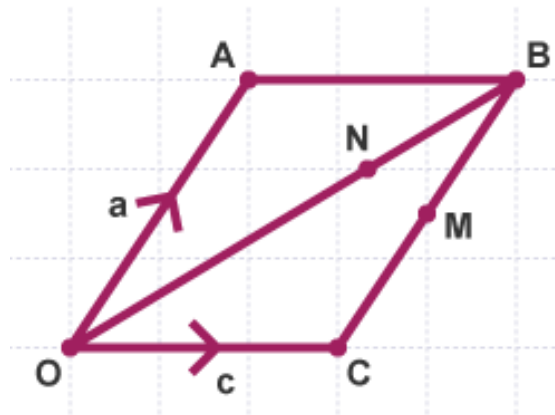
Worked example

OABC is a parallelogram

N is the point on OB such that $ON:NB = 2:1$

M is the midpoint of BC.

Use a vector method to show that ANM is a straight line.



Your turn

W is the midpoint of PQ.

X is the point on QR such that $QX:XR = 2:1$

PRY is a straight line.

R is the midpoint of the straight line PRY

$\overrightarrow{PW} = \mathbf{a}$ and $\overrightarrow{PR} = \mathbf{b}$

Use a vector method to show that WXY is a straight line.

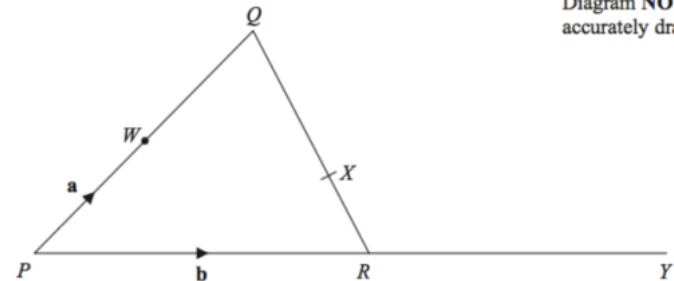


Diagram NOT
accurately drawn

$$\overrightarrow{WX} = \frac{1}{3}(-\mathbf{a} + 2\mathbf{b})$$

$$\overrightarrow{WX} = -\mathbf{a} + 2\mathbf{b}$$

$\overrightarrow{WX}$ is a multiple of $\overrightarrow{WY} \therefore$ parallel
W is a common point.

$\therefore$ WXY is a straight line.

Worked example

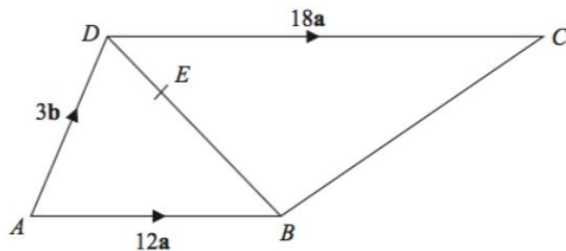
BACD is a trapezium.

AB is parallel to DC.

E is the point on diagonal DB such that

$$DE = \frac{1}{3} DB.$$

Show by a vector method that BC is parallel to AE.



Your turn

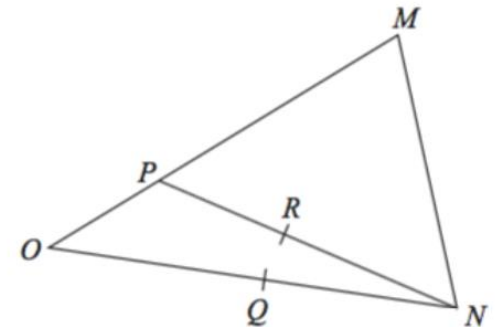
P is the point on OM such that $OP = \frac{1}{4} OM$.

Q is the midpoint of ON.

R is the midpoint of PN

$$\overrightarrow{OP} = \mathbf{p} \text{ and } \overrightarrow{OQ} = \mathbf{q}$$

Show by a vector method that QR is parallel to OP



$$\overrightarrow{QR} = \frac{1}{2} \mathbf{p}$$

$$\overrightarrow{OP} = \mathbf{p}$$

$$\overrightarrow{QR} = \frac{1}{2} \overrightarrow{OP}$$

$\overrightarrow{QR}$ is a multiple of $\overrightarrow{OP} \therefore$ parallel

Worked example

$$\overrightarrow{PQ} = 3\mathbf{a} + 4\mathbf{b}$$

$$\overrightarrow{PR} = -4\mathbf{a} + 2\mathbf{b}$$

The point M lies on PR such that
 $PM = \frac{2}{3}PR$

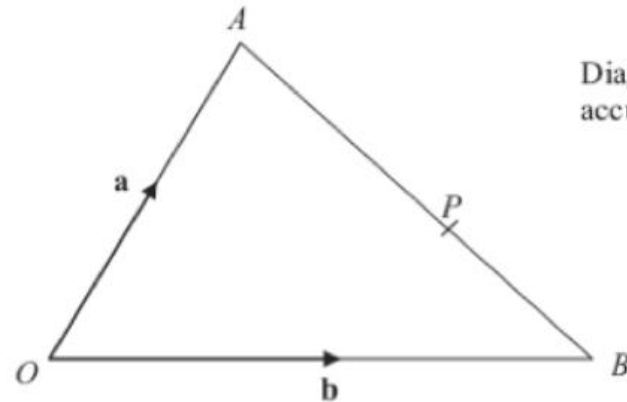
The point N lies on PQ such that
 $PN = \frac{1}{3}PQ$

Show that $\overrightarrow{MN} = k\mathbf{a}$ where k is a constant to be found.

Your turn

P is the point on AB such that
 $AP:PB = 3:2$

Show that $\overrightarrow{OP} = x(2\mathbf{a} + 3\mathbf{b})$,
 where x is a constant to be found.



$$x = \frac{1}{5}$$

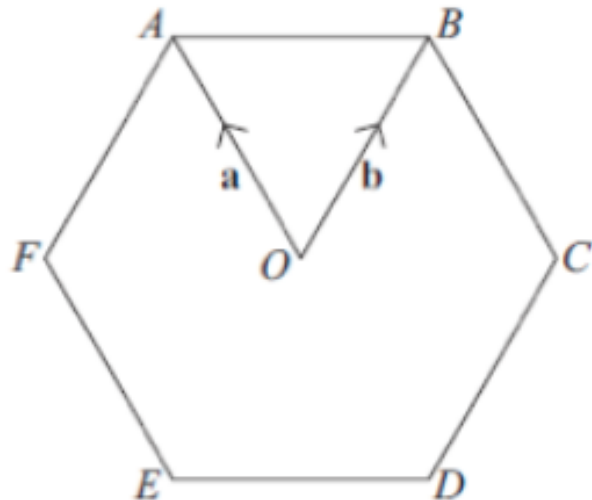
Worked example

ABCDEF is a regular hexagon with centre O.

$$\overrightarrow{OA} = \mathbf{a} \text{ and } \overrightarrow{OB} = \mathbf{b}$$

The line AB is extended to the point K so that $AB:BK = 1:3$

Write the vector $\overrightarrow{CK}$ in terms of $\mathbf{a}$ and $\mathbf{b}$



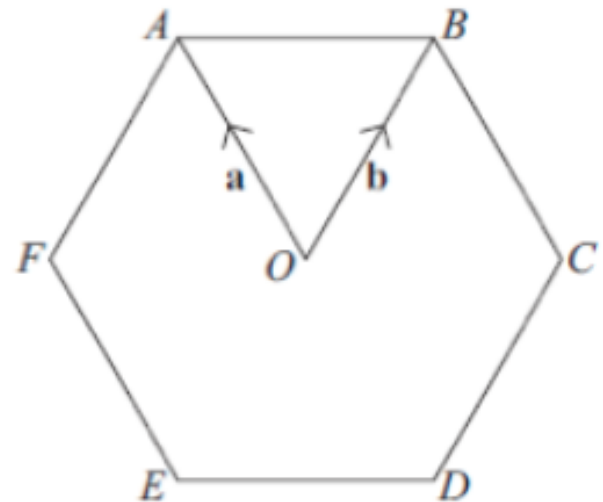
Your turn

ABCDEF is a regular hexagon with centre O.

$$\overrightarrow{OA} = \mathbf{a} \text{ and } \overrightarrow{OB} = \mathbf{b}$$

The line AB is extended to the point K so that $AB:BK = 1:2$

Write the vector $\overrightarrow{CK}$ in terms of $\mathbf{a}$ and $\mathbf{b}$



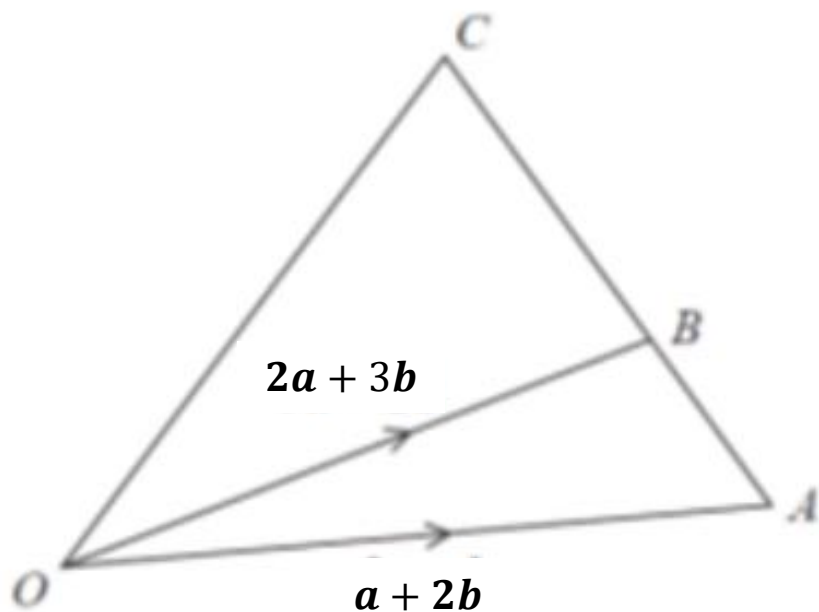
$$\overrightarrow{CK} = 2\mathbf{b} - \mathbf{a}$$

Worked example

ABC is a straight line.

$$AB:BC = 2:7$$

Express $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

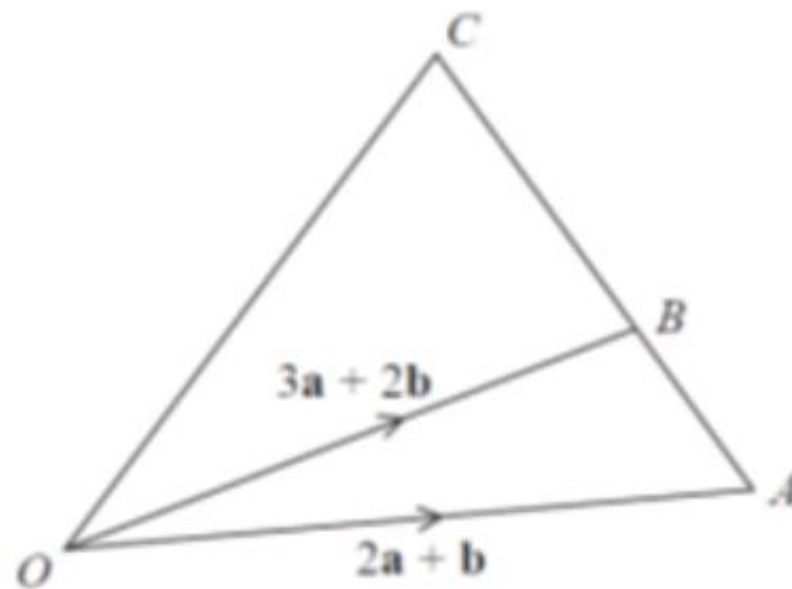


Your turn

ABC is a straight line.

$$AB:BC = 2:5$$

Express $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$



$$\overrightarrow{OC} = \frac{11}{2}\mathbf{a} + \frac{9}{2}\mathbf{b}$$