DIFFERENTIATION

CONTENTS

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Differentiate with respect to x:	Differentiate with respect to x:
x^2	x^5
	$5x^4$
x^3	
x^4	

Worked example	Your turn
Differentiate with respect to x:	Differentiate with respect to x:
$3x^2$	$-3x^5$
$-2x^3$	$-15x^4$
$5x^4$	

Worked example	Your turn
Differentiate with respect to x:	Differentiate with respect to x:
$\sqrt{x}$	$\sqrt[5]{x}$
	$\frac{1}{5}x^{-\frac{4}{5}}$
$\sqrt[3]{x}$	
$\sqrt[4]{x}$	

Worked example	Your turn
Differentiate with respect to x:	Differentiate with respect to x:
$\frac{1}{x}$	$\frac{1}{x^4}$
	$-4x^{-5} = -\frac{4}{x^5}$
$\frac{1}{x^2}$	
$\frac{1}{x^3}$	

Worked example	Your turn
Differentiate with respect to x:	Differentiate with respect to x:
$\frac{2}{x}$	$\frac{7}{8x^4}$
	$-\frac{7}{2}x^{-5} = -\frac{7}{2x^5}$
$\frac{3}{4x^2}$	
$\frac{6}{5x^3}$	

Worked example	Your turn
Differentiate with respect to x:	Differentiate with respect to x:
$\frac{2}{3}\sqrt{x}$	$\frac{3}{5}\sqrt{x}$
	$\frac{3}{10}x^{-\frac{1}{2}} = \frac{3}{10\sqrt{x}}$
$\frac{4}{7}\sqrt[3]{x}$	
$\frac{5}{6}\sqrt[4]{x}$	

Worked example	Your turn
Differentiate with respect to x:	Differentiate with respect to x:
$\frac{2}{3\sqrt{x}}$	$\frac{3}{5\sqrt{x}}$
	$-\frac{3}{10}x^{-\frac{3}{2}} = -\frac{3}{10x\sqrt{x}}$
$\frac{4}{7\sqrt[3]{x}}$	
$\frac{5}{6\sqrt[4]{x}}$	

Worked example	Your turn
Differentiate with respect to x:	Differentiate with respect to x:
$y = 4x^3 + 3x^2 + 2x + 1$	$y = 5x^4 - 2x^7 + 12345 - x^5$
	$\frac{dy}{dx} = 20x^3 - 14x^6 - 5x^4$
$f(x) = x^3 - 2x^5 - 3x^{-2} - 2$	

Worked example	Your turn
Differentiate with respect to x:	Differentiate with respect to x:
$y = 2\sqrt{x} + 3x^{\frac{4}{3}} - \frac{1}{x} + \frac{5}{x^2}$	$y = 3\sqrt{x} + 4x^{\frac{5}{3}} - \frac{5}{x} + \frac{1}{\sqrt[3]{x}}$
	$\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} + \frac{20}{3}x^{\frac{2}{3}} + 5x^{-2} - \frac{1}{3}x^{-\frac{4}{3}}$
$f(x) = 4\sqrt[3]{x} + 2x^{\frac{1}{4}} - \frac{5}{x^3} + \frac{3}{\sqrt{x}} + 6x^{-2}$	

Worked example	Your turn
Differentiate with respect to x: $y = \frac{(x+3)^2}{x}$	Differentiate with respect to x: $f(x) = \frac{(2x+3)^2}{5x}$
$f(x) = \frac{(3x-2)^2}{5x}$	$f'(x) = \frac{4}{5} - \frac{9}{5}x^{-2}$ $= \frac{4}{5} - \frac{9}{5x^2}$

Worked example	Your turn
Differentiate with respect to x: $$y = \frac{(x+4)^3}{5x^2}$$	Differentiate with respect to x: $$y = \frac{(x+2)^3}{3x^2}$$ $$\frac{dy}{dx} = \frac{1}{3} - 4x^{-2} - \frac{16}{3}x^{-3}$$ $$= \frac{1}{3} - \frac{4}{x^2} - \frac{16}{3x^3}$$

Worked example	Your turn
Find the gradient of the curve: $y = x^2 + 3x + 2$ at $(4, 30)$	Find the gradient of the curve: $y = 3x^2 - 2x + 1$ at $(-2, 17)$
	-14
$y = 2x^3 - x + 5$ at $(-1, 4)$	

Worked example	Your turn
Find the gradient of the curve: $y = 8\sqrt{x} + \frac{48}{x}$ at $(4, 28)$	Find the gradient of the curve: $y = 5\sqrt{x} - \frac{3}{x}$ at $(16, \frac{317}{16})$ $\frac{163}{256}$
$y = \frac{3}{x^2} - \frac{18}{\sqrt{x}}$ at $(9, -\frac{161}{27})$	

Worked example	Your turn
Find the coordinates of the point(s) where the gradient is 4: $y = x^2 - 8x + 3$ $y = 5x^2 - x + 7$	Find the coordinates of the point(s) where the gradient is 3: $y = 3x^2 - 9x + 7$ $(2, 1)$

Worked example	Your turn
Find the coordinates of the point(s) where the gradient is 10: $y = x^3 + 6x^2 - 11x + 7$	Find the coordinates of the point(s) where the gradient is 2: $y = x^3 - 3x^2 - 7x + 8$ $(-1, 11)$ and $(3, -13)$

Worked example	Your turn
Find the coordinates of the turning/stationary point(s) of the curves by differentiation: $y = x^2 + 6x - 2$ $y = 2x^3 + 6x^2 - 4$	Find the coordinates of the turning/stationary point(s) of the curves by differentiation: $y = x^3 + 3x^2 - 4$ $(-2, 0)$ and $(0, -4)$

Worked example	Your turn
Find the coordinates of the turning/stationary point(s) of the curves by differentiation: $y = \frac{2}{3}x^3 - 3.5x^2 + 3x + 5$	Find the coordinates of the turning/stationary point(s) of the curves by differentiation: $y = x^3 + \frac{1}{2}x^2 - 2x + 4$ $(-1, \frac{11}{2})$ and $(\frac{2}{3}, \frac{86}{27})$

Worked example	Your turn
Find the stationary points on the curve $y = \frac{5}{3}x^3 - 80x$	Find the stationary points on the curve $y = x^3 - 12x$ $(-2, 16)$ and $(2, -16)$

Worked example	Your turn
For the curve $y = f(x)$, $\frac{dy}{dx} = 723 + kx^5 + 2k$, where k is a constant. When $x = -3$, the gradient of the curve is 241. Find k.	For the curve $y = f(x)$, $\frac{dy}{dx} = \frac{3}{2} - kx^4 + k$, where k is a constant. When $x = -2$, the gradient of the curve is -6. Find k. $k = \frac{1}{2}$

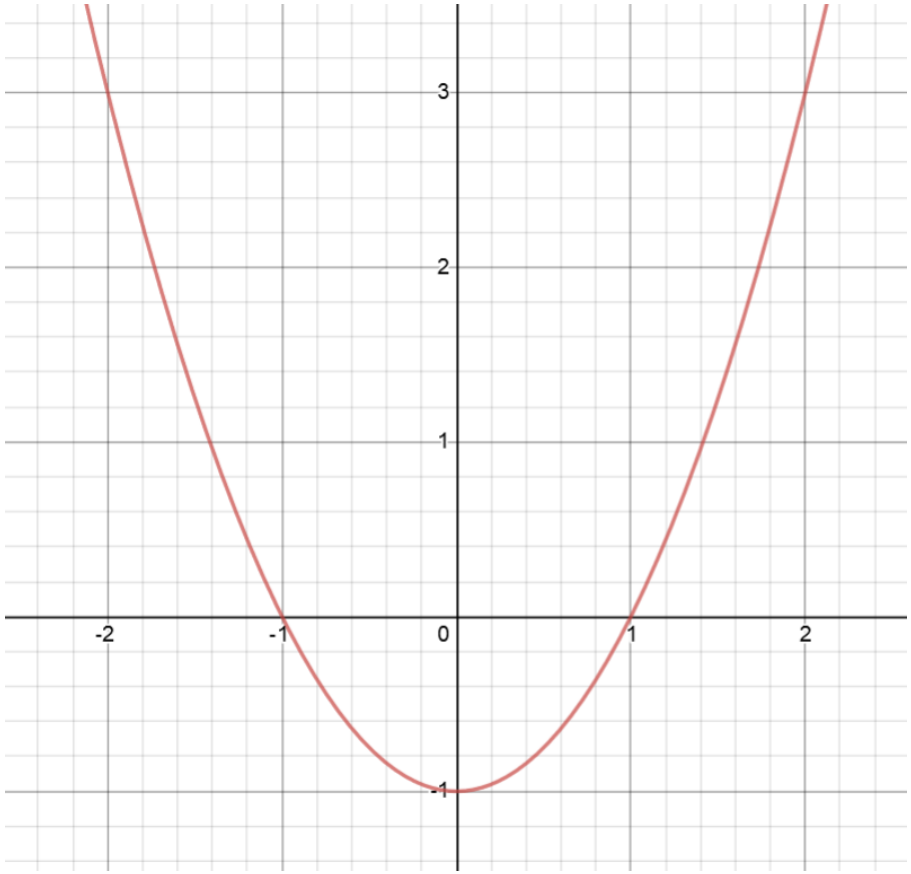
Worked example

Find an estimate for the gradient of each curve at the point where:

$x = 2$

$x = 0$

$x = -1$



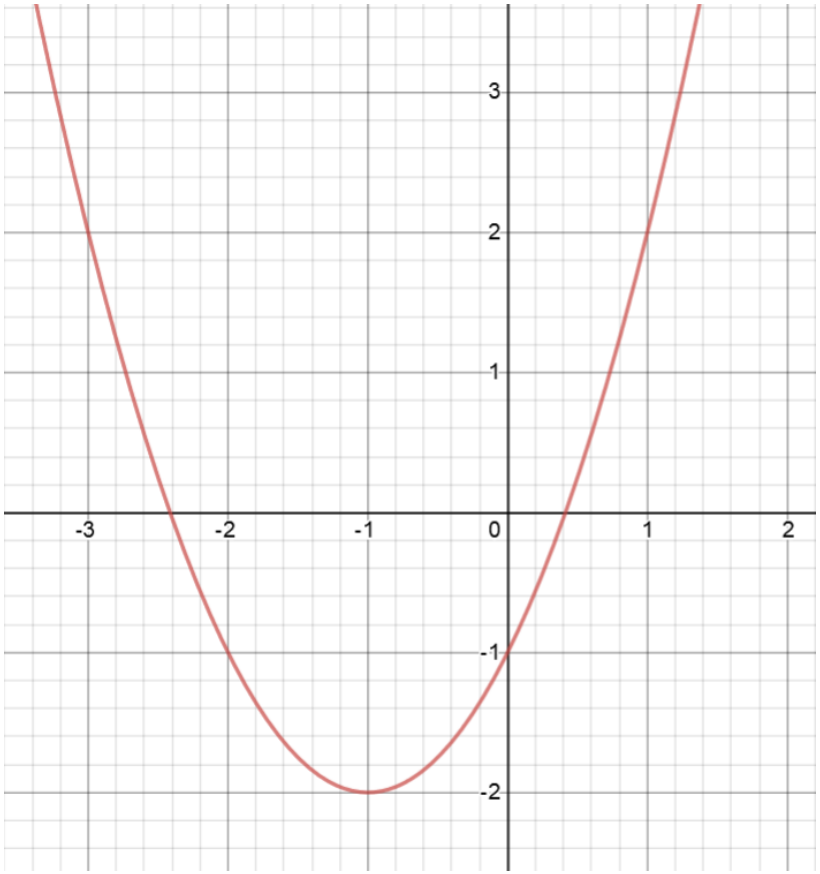
Your turn

Find an estimate for the gradient of each curve at the point where:

$x = 1$ 4

$x = -1$ 0

$x = -2$ -0.4



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Worked example	Your turn
The displacement, x metres, of a particle from a point O after t seconds is given by the formula: $x = 2t^3 - 3t + 5$ Find expressions for the velocity and acceleration of the particle.	The displacement, x metres, of a particle from a point O after t seconds is given by the formula: $x = 4t^3 + t - 3$ Find expressions for the velocity and acceleration of the particle. Velocity: $12t^2 + 1\ ms^{-1}$ Acceleration: $24t\ ms^{-2}$

Worked example	Your turn
The displacement, x metres, of a particle from a point O after t seconds is given by the formula: $x = t^2 + 5t$	The displacement, x metres, of a particle from a point O after t seconds is given by the formula: $x = t^2 + 3t$
a) Calculate the distance between the particle and O after 8 seconds	a) Calculate the distance between the particle and O after 5 seconds $40m$
b) Find the velocity of the particle after 8 seconds	b) Find the velocity of the particle after 6 seconds $15\ ms^{-1}$
c) After how many seconds is the velocity of the particle $10ms^{-1}$?	c) After how many seconds is the velocity of the particle $5ms^{-1}$? 1

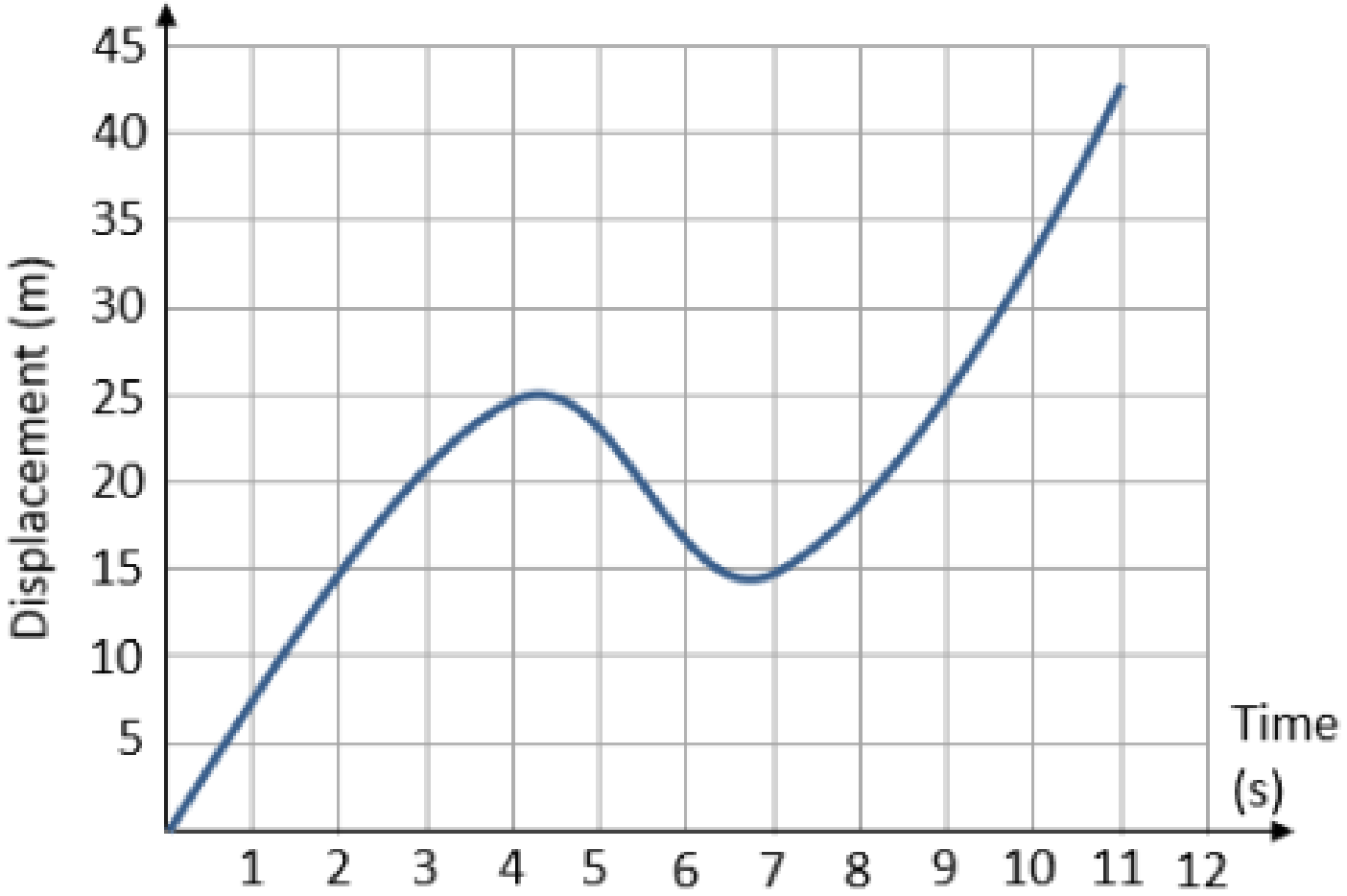
Worked example	Your turn
The velocity, $v\ ms^{-1}$, of a particle is given by the formula $v = t^2 - 6t + 8$, where t is the time in seconds since the particle started moving.	The velocity, $v\ ms^{-1}$, of a particle is given by the formula $v = t^2 - 5t + 6$, where t is the time in seconds since the particle started moving.
Find the time(s) when the particle is at rest.	Find the time(s) when the particle is at rest. $t = 2, \mathrm{t} = 3$
Find an expression for the acceleration, a	Find an expression for the acceleration, a $a = 2t - 5$
When is the acceleration 0?	When is the acceleration 0? $t = \frac{5}{2}$

Worked example	Your turn
The velocity, $v\ ms^{-1}$, of a particle is given by the formula $v = t^2 - 3t - 4$, where t is the time in seconds since the particle started moving.	The velocity, $v\ ms^{-1}$, of a particle is given by the formula $v = t^2 - 5t - 6$, where t is the time in seconds since the particle started moving.
Find the time(s) when the particle is at rest.	Find the time(s) when the particle is at rest. $t = 6$
Find an expression for the acceleration, a	Find an expression for the acceleration, a $a = 2t - 5$
When is the acceleration 0?	When is the acceleration 0? $t = \frac{5}{2}$

Worked example	Your turn
A particle is moving on the x-axis. At time t seconds, displacement x metres from O is given by $x = t^4 - 32t + 14$. Find:	A particle is moving on the x-axis. At time t seconds, displacement x metres from O is given by $x = t^3 - \frac{3}{2}t^2 - 36t$. Find:
The value of t when P is instantaneously at rest	The value of t when P is instantaneously at rest $t = 4$
The acceleration when $t = 1.5$	The acceleration when $t = 5$ $26\ ms^{-2}$

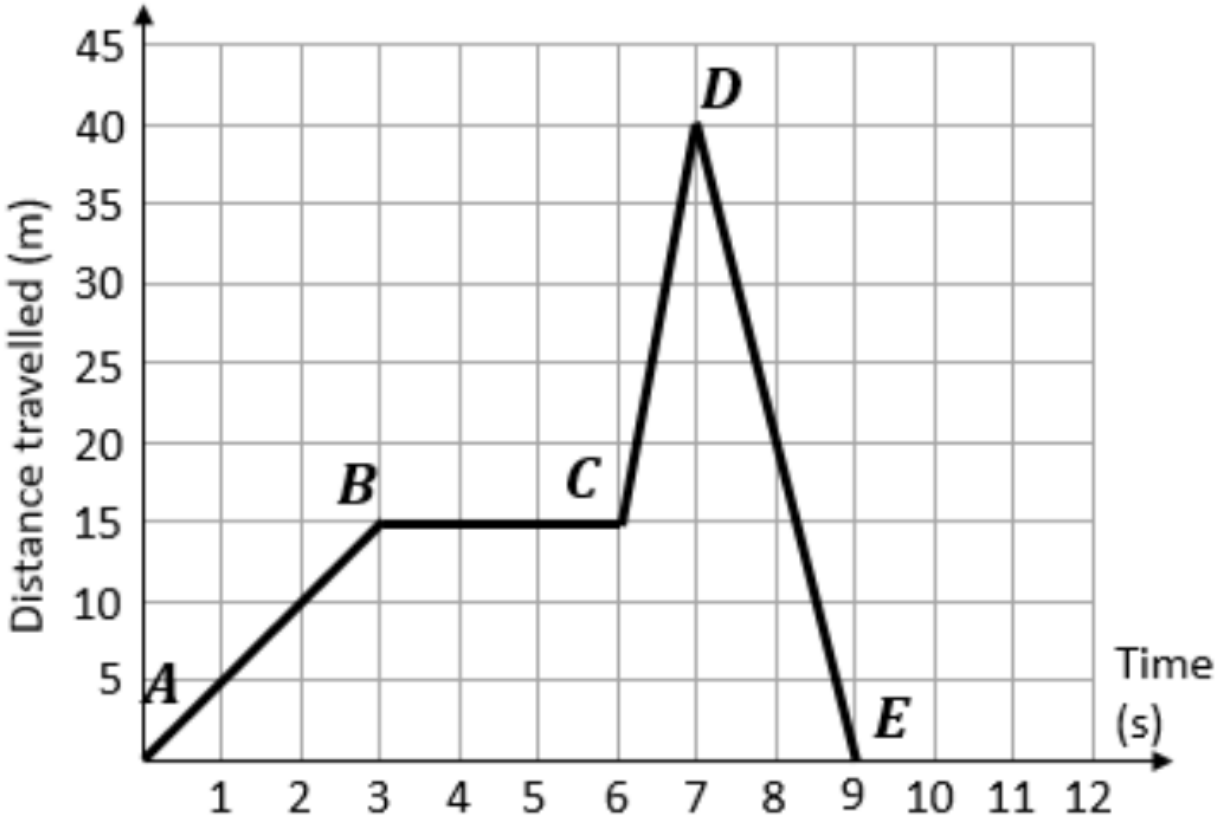
Worked example	Your turn
The graph shows the displacement of a particle. Estimate the velocity when $t = 3$	The graph shows the displacement of a particle. Estimate the velocity when $t = 10$

$8\ ms^{-1}$



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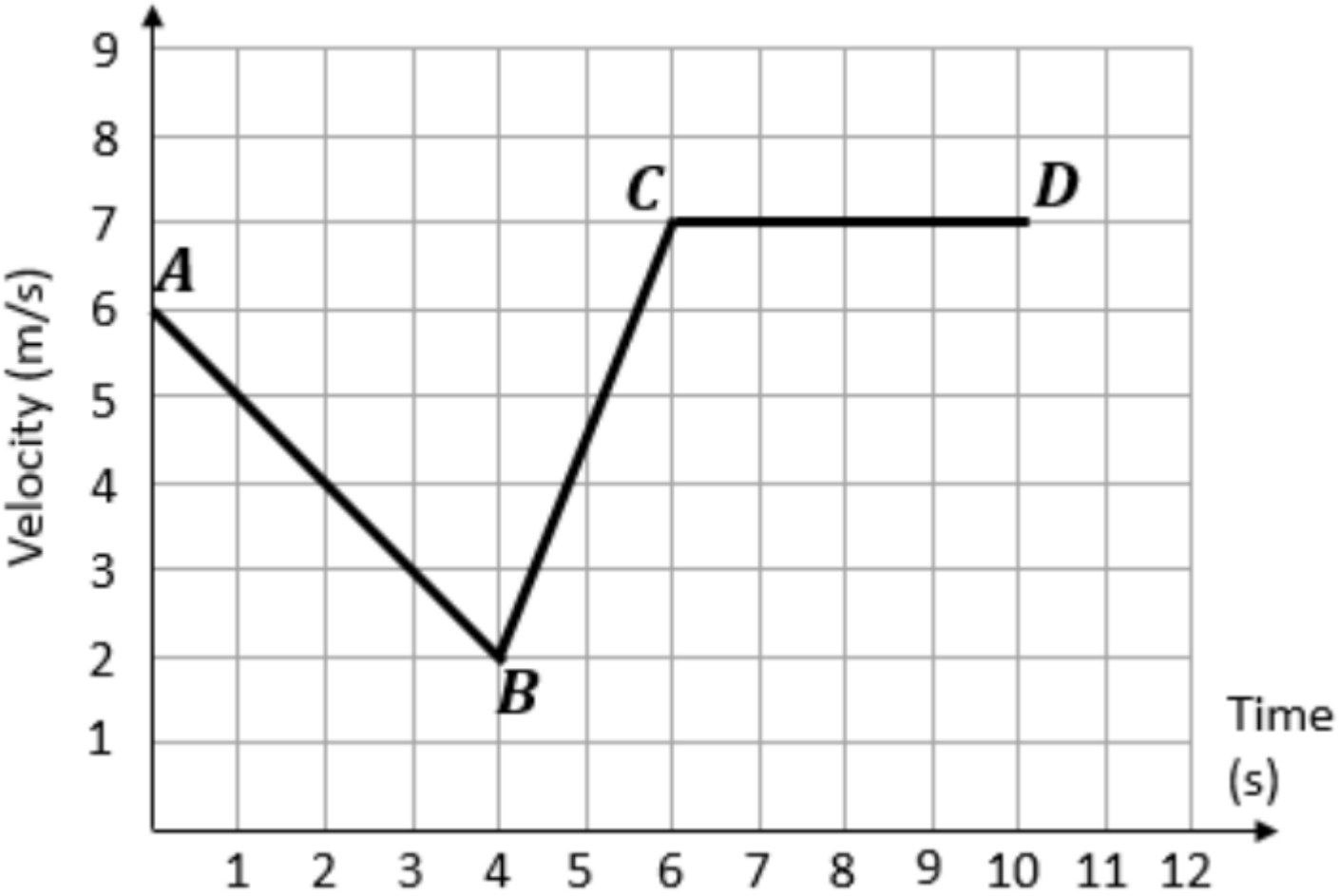
Worked example	Your turn
Determine the speed from: A to B B to C	Determine the speed from: C to D $25\ ms^{-1}$ D to E $20\ ms^{-1}$



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Worked example	Your turn
Determine the acceleration from:	Determine the acceleration from:
A to B	B to C $2.5\ ms^{-2}$

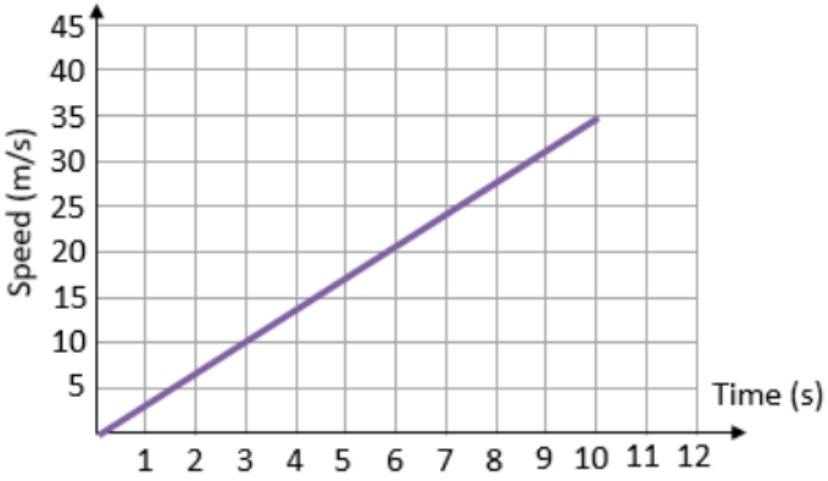
C to D

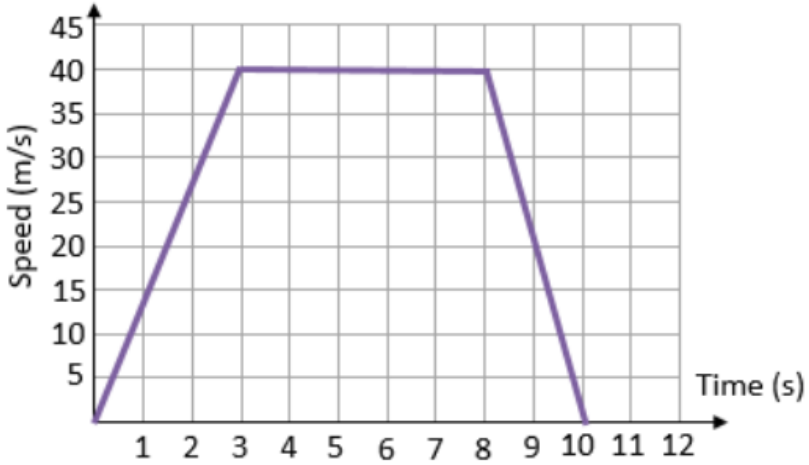


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Worked example

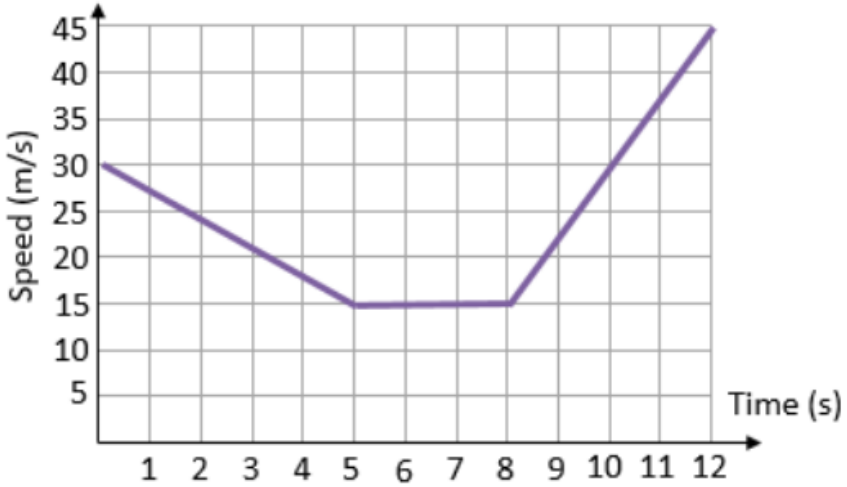
Calculate the distance travelled in the first 10 seconds:





Your turn

Calculate the distance travelled in the first 12 seconds:

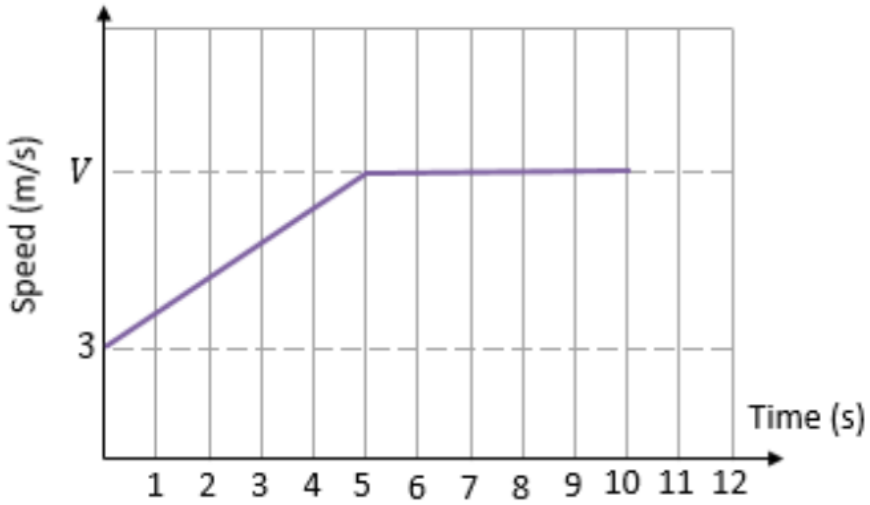


$277.5\ m$

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Worked example

The object travelled $155m$ in 10 seconds. Determine V



Your turn

The object travelled $150m$ in 10 seconds. Determine V



$V = 30$

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