

# COMPLETING THE SQUARE

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## Worked example

Complete the square for:

$$x^2 + 6x + 5$$

$$x^2 + 8x + 3$$

## Your turn

Complete the square for:

$$x^2 + 10x + 1$$

$$(x + 5)^2 - 24$$

## Worked example

Complete the square for:

$$x^2 - 6x + 5$$

$$x^2 - 8x - 3$$

## Your turn

Complete the square for:

$$x^2 - 10x + 1$$

$$(x - 5)^2 - 24$$

## Worked example

Complete the square for:

$$x^2 + 9x + 5$$

$$x^2 + 7x + 3$$

## Your turn

Complete the square for:

$$x^2 + 5x + 1$$

$$\left(x + \frac{5}{2}\right)^2 - \frac{21}{4}$$

## Worked example

Complete the square for:

$$x^2 - 5x - 3$$

$$x^2 - x + 2$$

## Your turn

Complete the square for:

$$x^2 - 3x - 2$$

$$\left(x - \frac{3}{2}\right)^2 - \frac{17}{4}$$

## Worked example

Complete the square for:

$$2x^2 + 12x + 1$$

$$3x^2 + 12x + 2$$

## Your turn

Complete the square for:

$$5x^2 + 40x + 3$$

$$5(x + 4)^2 - 77$$

## Worked example

Complete the square for:

$$2x^2 + 5x + 1$$

$$3x^2 + 7x + 2$$

## Your turn

Complete the square for:

$$5x^2 + 9x + 3$$

$$5\left(x + \frac{9}{10}\right)^2 - \frac{21}{20}$$

## Worked example

Complete the square for:

$$2x^2 - 5x + 3$$

$$5x^2 - 3x + 1$$

## Your turn

Complete the square for:

$$3x^2 - 7x + 2$$

$$3\left(x - \frac{7}{6}\right)^2 - \frac{25}{12}$$

## Worked example

Express in the form  $a(x + b)^2 + c$ :  
 $2x^2 - 5x + 3$

## Your turn

Express in the form  $a(x + b)^2 + c$ :  
 $3x^2 - 7x + 2$

$$3\left(x - \frac{7}{6}\right)^2 - \frac{25}{12}$$

## Worked example

Complete the square for:

$$3 + 5x - x^2$$

$$2 - 3x - x^2$$

## Your turn

Complete the square for:

$$5 - 7x - x^2$$

$$-\left(x + \frac{7}{2}\right)^2 + \frac{69}{4}$$

## Worked example

Complete the square for:

$$3 + 5x - 2x^2$$

$$2 - 3x - 5x^2$$

## Your turn

Complete the square for:

$$5 - 7x - 3x^2$$

$$-3 \left( x + \frac{7}{6} \right)^2 + \frac{109}{12}$$

## Worked example

Solve by completing the square:

$$x^2 + 8x + 3 = 0$$

$$x^2 + 10x - 4 = 0$$

## Your turn

Solve by completing the square:

$$x^2 + 6x + 4 = 0$$

$$x = -3 + \sqrt{5}, x = -3 - \sqrt{5}$$

## Worked example

Solve by completing the square:

$$2x^2 - 8x + 3 = 0$$

$$3x^2 - 10x - 4 = 0$$

## Your turn

Solve by completing the square:

$$5x^2 - 6x - 2 = 0$$

$$x = \frac{3 + \sqrt{19}}{5}, x = \frac{3 - \sqrt{19}}{5}$$

## Worked example

Solve using three methods:

$$x^2 + 6x + 8 = 0$$

$$x^2 + 6x + 8 = 0$$

$$x^2 + 6x + 8 = 0$$

## Your turn

Solve using three methods:

$$x^2 + 6x + 5 = 0$$

$$x = -5, x = -1$$

## Worked example

By completing the square, explain why the curve  $y = 2x^2 - 8x + 9$  does not intersect the  $x$ -axis

## Your turn

By completing the square, explain why the curve  $y = 2x^2 - 20x + 51$  does not intersect the  $x$ -axis

$$y = 2(x - 5)^2 + 1$$

Turning point at (5, 1)

## Worked example

A sequence has the  $n^{\text{th}}$  term  $n^2 - 6n + 10$ . By completing the square, show that every term is positive.

## Your turn

A sequence has the  $n^{\text{th}}$  term  $n^2 - 10n + 27$ . By completing the square, show that every term is positive.

$$n^2 - 10n + 27 = (n - 5)^2 + 2$$

$$k^2 \geq 0$$

$$(n - 5)^2 \geq 0$$

$$(n - 5)^2 + 2 \geq 2$$