QQQ – Pure Year 2 – Chapter 8 – Parametric Equations (Version 2)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1.

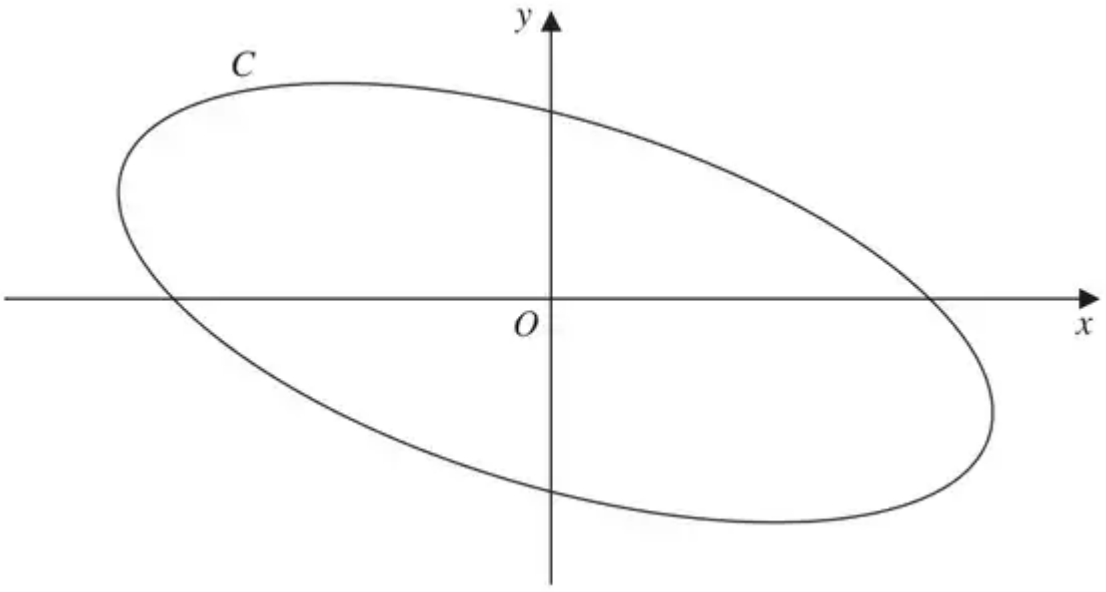


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t, \qquad 0 \leqslant t < 2\pi$$

(a) Show that

$$x + y = 2\sqrt{3}\cos t$$

(3)

(b) Show that a cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be determined. **(2)**

(Total 5 marks)

Edexcel C4 June 2014 Q5

2. Fig. 12 shows a curve C with parametric equations $x = 4t^2,\ y = 8t$. The point P, with parameter t, is a general point on the curve. Q is the point on the line $x + 4 = 0$ such that PQ is parallel to the x-axis. R is the point $(4, 0)$.

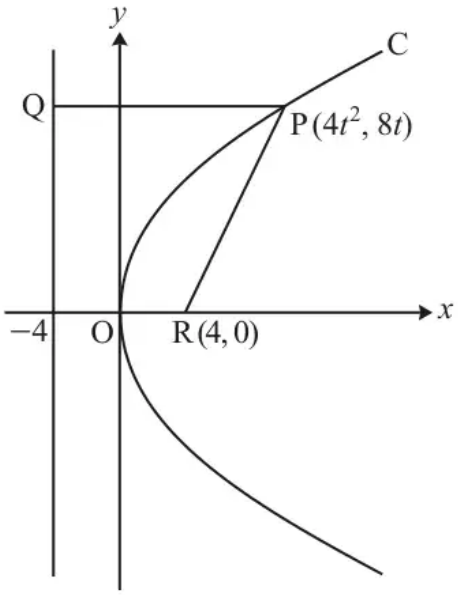


Fig. 12

(a) Show algebraically that P is equidistant from Q and R. **[4]**

(b) Find a cartesian equation of C. **[2]**

(Total 6 marks)

OCR MEI June 2019 Paper 1 Q12

3. The curve C is defined, for $0 \leqslant t < 2\pi$, by the parametric equations

$x = 4k + k\sin t, \quad y = 2 + 4\cos t,$

where k is a constant.

(a) Find a cartesian equation for C. You do **not** need to simplify your answer. **[2]**

You are given that C is a circle.

(b) **(i)** Determine the radius of C. **[2]**

(ii) Find the possible coordinates for the centre of C. **[2]**

(Total 6 marks)

OCR A June 2024 Paper 3 Q6

4.

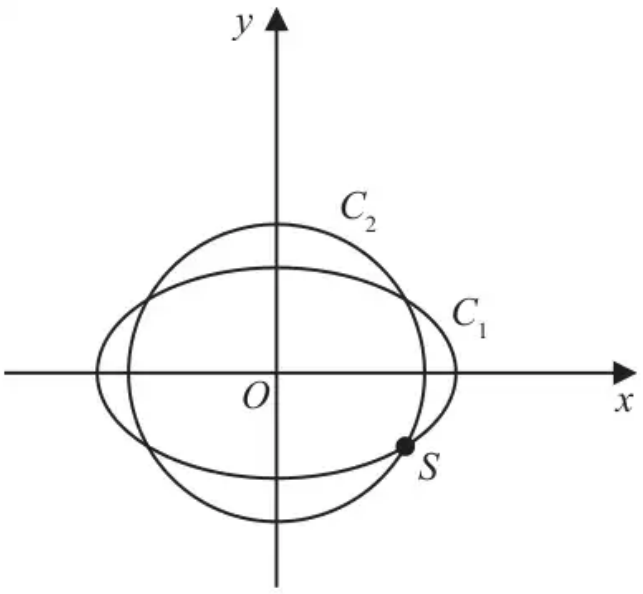


Figure 2

The curve C_1 with parametric equations

$$x = 10\cos t, \quad y = 4\sqrt{2}\sin t, \quad 0 \leqslant t < 2\pi$$

meets the circle C_2 with equation

$$x^2 + y^2 = 66$$

at four distinct points as shown in Figure 2.

Given that one of these points, S, lies in the 4th quadrant, find the Cartesian coordinates of S.

(6)

(Total 6 marks)

Edexcel June 2019 Paper 2 Q4

Solutions

1.

Mark scheme (a)

Scheme	Marks
$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t$	
Main Scheme $x = 4\left(\cos t \cos\left(\frac{\pi}{6}\right) - \sin t \sin\left(\frac{\pi}{6}\right)\right)$ $\cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t \cos\left(\frac{\pi}{6}\right) \pm \sin t \sin\left(\frac{\pi}{6}\right)$	M1 oe
So, $\{x + y\} = 4\left(\cos t \cos\left(\frac{\pi}{6}\right) - \sin t \sin\left(\frac{\pi}{6}\right)\right) + 2\sin t$ *Adds their expanded x (which is in terms of t) to $2\sin t$*	dM1
$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) + 2\sin t$	
$= 2\sqrt{3}\cos t$ * *Correct proof*	A1 *
	(3)

Notes

Alternative Method 1

Scheme	Marks
$x = 4\left(\cos t \cos\left(\frac{\pi}{6}\right) - \sin t \sin\left(\frac{\pi}{6}\right)\right)$ $\cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t \cos\left(\frac{\pi}{6}\right) \pm \sin t \sin\left(\frac{\pi}{6}\right)$	M1 oe
$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) = 2\sqrt{3}\cos t - 2\sin t$	
So, $x = 2\sqrt{3}\cos t - y$ *Forms an equation in x, y and t.*	dM1
$x + y = 2\sqrt{3}\cos t$ * *Correct proof*	A1 *
	(3)

Question 5 Notes

M1: $\cos\left(t+\frac{\pi}{6}\right) \rightarrow \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$ or $\cos\left(t+\frac{\pi}{6}\right) \rightarrow \left(\frac{\sqrt{3}}{2}\right)\cos t \pm \left(\frac{1}{2}\right)\sin t$

Note: If a candidate states $\cos(A+B) = \cos A\cos B \pm \sin A\sin B$, **but there is an error *in its application*** then give M1.

Awarding the dM1 mark which is dependent on the first method mark

Main dM1: Adds their expanded x (which is in terms of t) to $2\sin t$

Note: Writing $x+y=\ldots$ is not needed in the **Main Scheme** method.

Alt 1 dM1: Forms an equation in x, y and t.

A1*: Evidence of $\cos\left(\frac{\pi}{6}\right)$ and $\sin\left(\frac{\pi}{6}\right)$ evaluated and the proof is correct with no errors.

Note: $\{x+y\} = 4\cos\left(t+\frac{\pi}{6}\right)+2\sin t$, by itself is M0M0A0.

Mark scheme (b)

Scheme	Marks
Main Scheme $\left(\frac{x+y}{2\sqrt{3}}\right)^2+\left(\frac{y}{2}\right)^2=1$ *Applies* $\cos^2 t+\sin^2 t=1$ *to achieve an equation containing* ***only*** x*'s and* y*'s.*	M1
$\Rightarrow \frac{(x+y)^2}{12}+\frac{y^2}{4}=1$	
$\Rightarrow (x+y)^2+3y^2=12$ $(x+y)^2+3y^2=12\ \ \{a=3,\ b=12\}$	A1
	(2)
	(5 marks)

Notes

Alternative Method 1

Scheme	Marks
$(x+y)^2 = 12\cos^2 t = 12(1-\sin^2 t) = 12-12\sin^2 t$	
So, $(x+y)^2 = 12-3y^2$ *Applies* $\cos^2 t+\sin^2 t=1$ *to achieve an equation containing* ***only*** x*'s and* y*'s.*	M1

$\Rightarrow (x+y)^2+3y^2=12$ $(x+y)^2+3y^2=12$	A1
	(2)

[Alternative Method 2]

Scheme	Marks
$(x+y)^2=12\cos^2 t$ As $12\cos^2 t+12\sin^2 t=12$ then $(x+y)^2+3y^2=12$	M1, A1
	(2)

M1: Applies $\cos^2 t+\sin^2 t=1$ to achieve an equation containing **only** x's and y's.

A1: leading $(x+y)^2+3y^2=12$

SC: Award **Special Case B1B0** for a candidate who writes down **either**

- $(x+y)^2+3y^2=12$ from no working
- $a=3,\ b=12$, but **does not provide a correct proof**.

Note: Alternative method 2 is fine for M1 A1

Note: Writing $(x+y)^2=12\cos^2 t$ followed by $12\cos^2 t+a(4\sin^2 t)=b\Rightarrow a=3,\ b=12$ is SC: B1B0

Note: Writing $(x+y)^2=12\cos^2 t$ followed by $12\cos^2 t+a(4\sin^2 t)=b$

- states $a=3,\ b=12$
- and refers to either $\cos^2 t+\sin^2 t=1$ or $12\cos^2 t+12\sin^2 t=12$
- and there is no incorrect working

would get M1A1

2.

Mark scheme (a)

Scheme	Marks	AO
$x=4t^2,\quad y=8t$ $\mathrm{PQ}=4+4t^2$	B1	2.1
$\mathrm{PR}^2=(4t^2-4)^2+(8t)^2$	M1	2.1

$= 16t^4 + 32t^2 + 16$	M1	2.1
$= (4 + 4t^2)^2 = \text{PQ}^2$ So equidistant	A1	2.1
	[4]	

Notes

Due to the error on the paper, **all** candidates will get 4 marks for question 12(a) and all candidates will get 2 marks for 12 (b). The MS for the correct question is given for completeness.

(The question paper as sat printed $y = 4t$; OCR's post-exam correction changes it to $y = 8t$, as shown here.)

M1: Use of distance formula

M1: Must show some algebraic manipulation

Correct expression for PR or PR^2

A1: Conclusion comparing PQ and PR or PQ^2 and PR^2

Mark scheme (b)

Scheme	Marks	AO
Substitute $t = \frac{y}{8}$ to obtain $x = 4\left(\frac{y}{8}\right)^2$	M1	1.1a
$y^2 = 16x$	A1	1.1b
	[2]	

Notes

M1: Rearranged equation must be used

allow use of $t = \frac{\sqrt{x}}{2}$ oe used for M1A0.

A1: Allow any equivalent form including $y = \pm 4\sqrt{x}$

3.

Mark scheme (a)

Scheme	Marks	AO

$x = 4k + k\sin t,\ y = 2 + 4\cos t$ $\sin t = \dfrac{x-4k}{k},\ \cos t = \dfrac{y-2}{4}$ and use of $\sin^2 t + \cos^2 t = 1$	M1	3.1a
$\dfrac{(x-4k)^2}{k^2} + \dfrac{(y-2)^2}{16} = 1$	A1	1.1
	[2]	

Notes

M1: Re-arranges to obtain both $\sin t = \dfrac{x \pm 4k}{k}$ **and** $\cos t = \dfrac{y \pm 2}{4}$ **and** use $\sin^2 t + \cos^2 t = 1$ to eliminate t

A1: Allow any correct un-simplified cartesian form not involving trigonometric terms
ISW once a correct answer seen

SC for part **(a)**: If **M0** awarded then **SC B1** for $y = 2 + 4\cos\left(\arcsin\left(\dfrac{x-4k}{k}\right)\right)$ **or** $x = 4k + k\sin\left(\arccos\left(\dfrac{y-2}{4}\right)\right)$ **or** $\arcsin\left(\dfrac{x-4k}{k}\right) = \arccos\left(\dfrac{y-2}{4}\right)$ (or any correct form in terms of trig. functions – allow $\cos^{-1}$ for arccos etc.)

If considering one branch only e.g. $y\left(= 2 + 4\sqrt{1-\sin^2 t}\right) = 2 + 4\sqrt{1-\left(\dfrac{x-4k}{k}\right)^2}$ this scores **M1** only (or equivalent expression for x)

If considering both branches e.g. $y\left(= 2 \pm 4\sqrt{1-\sin^2 t}\right) = 2 \pm 4\sqrt{1-\left(\dfrac{x-4k}{k}\right)^2}$ then this scores **M1 A1** (or equivalent expression for x)

Note a (common) correct answer for **M1 A1** is:
$x^2 + y^2 = 16k^2 + 8k(x-4k) + (x-4k)^2 + 4 + 4(y-2) + (y-2)^2$

Mark scheme (b)

Scheme	Marks	AO
(i) C is a circle $\Rightarrow k^2 = 16$	M1	3.1a
$r = 4$	A1	1.1
	[2]	
(ii) $(16, 2), (-16, 2)$	B2	2.2a 2.2a
	[2]	

Notes

(b)(i)

M1: Setting c and d equal in their $\frac{(x \pm a)^2}{c} + \frac{(y \pm b)^2}{d} = 1$ **or** stating that $y_{\min} = -2$ **and** $y_{\max} = 6$

Possibly implied by correct value for r www

A1: $r = \pm 4$ is **A0** unless replaced with positive r only

(b)(ii) B2: B1 for either one correct centre **or** both x-coordinates correct

These marks are dependent on $r = 4$ from correct working or www

4.

Scheme	Marks	AO
$C_1: \ x = 10\cos t, \ y = 4\sqrt{2}\sin t, \ 0 \leqslant t < 2\pi; \ \ C_2: x^2 + y^2 = 66$		
Way 1 $(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$	M1	3.1a
$100(1 - \sin^2 t) + 32\sin^2 t = 66$ or $100\cos^2 t + 32(1 - \cos^2 t) = 66$	M1 A1	2.1 1.1b
$100 - 68\sin^2 t = 66 \Rightarrow \sin^2 t = \frac{1}{2} \Rightarrow \sin t = \ldots$ or $68\cos^2 t + 32 = 66 \Rightarrow \cos^2 t = \frac{1}{2} \Rightarrow \cos t = \ldots$	dM1	1.1b
Substitutes their solution back into the relevant original equation(s) to get the value of the x-coordinate and value of the corresponding y-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b
$S = (5\sqrt{2}, \ -4)$ or $x = 5\sqrt{2}, \ y = -4$ or $S = (\text{awrt } 7.07, \ -4)$	A1	3.2a
Note: Give final A0 for writing $x = 5\sqrt{2}, \ y = -4$ followed by $S = (-4, \ 5\sqrt{2})$		
	(6)	
	(6 marks)	

Notes

Way 2

Scheme	Marks	AO
$\{\cos^2 t + \sin^2 t = 1 \Rightarrow\}\ \left(\frac{x}{10}\right)^2 + \left(\frac{y}{4\sqrt{2}}\right)^2 = 1\ \{\Rightarrow 32x^2 + 100y^2 = 3200\}$	M1	3.1a
$\frac{x^2}{100} + \frac{66 - x^2}{32} = 1$ or $\frac{66 - y^2}{100} + \frac{y^2}{32} = 1$	M1 A1	2.1 1.1b
$32x^2 + 6600 - 100x^2 = 3200, x^2 = 50 \Rightarrow x = \ldots$ or $2112 - 32y^2 + 100y^2 = 3200, y^2 = 16 \Rightarrow y = \ldots$	dM1	1.1b
Substitutes their solution back into the relevant original equation(s) to get the value of the corresponding x-coordinate or y-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b
$S = (5\sqrt{2},\ -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07,\ -4)$	A1	3.2a
	(6)	

Way 3

Scheme	Marks	AO
$\{C_2 : x^2 + y^2 = 66 \Rightarrow\}\ x = \sqrt{66}\cos\alpha,\ y = \sqrt{66}\sin\alpha$ $\{C_1 = C_2 \Rightarrow\}\ 10\cos t = \sqrt{66}\cos\alpha,\ 4\sqrt{2}\sin t = \sqrt{66}\sin\alpha$ $\{\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow\}\ \left(\frac{10\cos t}{\sqrt{66}}\right)^2 + \left(\frac{4\sqrt{2}\sin t}{\sqrt{66}}\right)^2 = 1$	M1	3.1a
then continue with applying the mark scheme for Way 1		

Way 4

Scheme	Marks	AO
$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$	M1	3.1a
$100\left(\frac{1 + \cos 2t}{2}\right) + 32\left(\frac{1 - \cos 2t}{2}\right) = 66$	M1 A1	2.1 1.1b
$50 + 50\cos 2t + 16 - 16\cos 2t = 66 \Rightarrow 34\cos 2t + 66 = 66$ $\Rightarrow \cos 2t = \ldots$	dM1	1.1b
Substitutes their solution back into the original equation(s) to get the value of the x-coordinate and value of the y-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b

$S = (5\sqrt{2},\ -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07,\ -4)$	A1	3.2a
	(6)	

Way 5

Scheme	Marks	AO
$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$	M1	3.1a
$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66(\sin^2 t + \cos^2 t)$	M1 A1	2.1 1.1b
$100\cos^2 t + 32\sin^2 t = 66\sin^2 t + 66\cos^2 t \Rightarrow 34\cos^2 t = 34\sin^2 t$ $\Rightarrow \tan t = \ldots$	dM1	1.1b
Substitutes their solution back into the relevant original equation(s) to get the value of the x-coordinate and value of the corresponding y-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b
$S = (5\sqrt{2},\ -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07,\ -4)$	A1	3.2a
	(6)	

Notes for Question 4

Way 1

M1: Begins to solve the problem by applying an appropriate strategy.
E.g. Way 1: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.

M1: Uses the identity $\sin^2 t + \cos^2 t \equiv 1$ to achieve an equation in $\sin^2 t$ only or $\cos^2 t$ only

A1: A correct equation in $\sin^2 t$ only or $\cos^2 t$ only

dM1: dependent on both the previous M marks
Rearranges to make $\sin t = \ldots$ where $-1 \leqslant \sin t \leqslant 1$ or $\cos t = \ldots$ where $-1 \leqslant \cos t \leqslant 1$

Note: Condone 3rd M1 for $\sin^2 t = \dfrac{1}{2} \Rightarrow \sin t = \dfrac{1}{4}$

M1: See scheme

A1: Selects the correct coordinates for S
Allow either $S = (5\sqrt{2},\ -4)$ or $S = (\text{awrt } 7.07,\ -4)$

Way 2

M1: Begins to solve the problem by applying an appropriate strategy.
E.g. Way 2: A complete process of using $\cos^2 t + \sin^2 t \equiv 1$ to convert the parametric equation for C_1 into a Cartesian equation for C_1

M1: Complete valid attempt to write an equation in terms of x only or y only not involving trigonometry

A1: A correct equation in x only or y only not involving trigonometry

dM1: dependent on both the previous M marks
Rearranges to make $x = \ldots$ or $y = \ldots$

Note: their x^2 or their y^2 must be > 0 for this mark

M1: See scheme

Note: their x^2 and their y^2 must be > 0 for this mark

A1: Selects the correct coordinates for S

Allow either $S = (5\sqrt{2},\ -4)$ or $S = (\text{awrt } 7.07,\ -4)$ or $S = (\sqrt{50},\ -4)$ or $S = \left(\dfrac{10}{\sqrt{2}},\ -4\right)$

Way 3

M1: Begins to solve the problem by applying an appropriate strategy.
E.g. Way 3: A complete process of writing C_2 in parametric form, combining the parametric equations of C_1 and C_2 and applying $\cos^2\alpha + \sin^2\alpha \equiv 1$ to give an equation in one variable (i.e. t) only.

then continue with applying the mark scheme for Way 1

Way 4

M1: Begins to solve the problem by applying an appropriate strategy.
E.g. Way 4: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.

M1: Uses the identities $\cos 2t \equiv 2\cos^2 t - 1$ and $\cos 2t \equiv 1 - 2\sin^2 t$ to achieve an equation in $\cos 2t$ only

Note: At least one of $\cos 2t \equiv 2\cos^2 t - 1$ or $\cos 2t \equiv 1 - 2\sin^2 t$ must be correct for this mark.

A1: A correct equation in $\cos 2t$ only

dM1: dependent on both the previous M marks
Rearranges to make $\cos 2t = \ldots$ where $-1 \leqslant \cos 2t \leqslant 1$

M1: See scheme

A1: Selects the correct coordinates for S

Allow either $S = (5\sqrt{2},\ -4)$ or $S = (\text{awrt } 7.07,\ -4)$ or $S = (\sqrt{50},\ -4)$ or $S = \left(\dfrac{10}{\sqrt{2}},\ -4\right)$

Way 5

M1: Begins to solve the problem by applying an appropriate strategy.
E.g. Way 5: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.

M1: Uses the identity $\sin^2 t + \cos^2 t \equiv 1$ to achieve an equation in $\sin^2 t$ only and $\cos^2 t$ only **with no constant term**

A1: A correct equation in $\sin^2 t$ and $\cos^2 t$ containing no constant term

dM1: dependent on both the previous M marks
Rearranges to make $\tan t = \ldots$

M1: See scheme

A1: Selects the correct coordinates for S

Allow either $S = (5\sqrt{2},\ -4)$ or $S = (\text{awrt } 7.07,\ -4)$ or $S = (\sqrt{50},\ -4)$ or $S = \left(\dfrac{10}{\sqrt{2}},\ -4\right)$