QQQ – Pure Year 2 – Chapter 8 – Parametric Equations (Version 1)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

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1. A curve C has parametric equations

$$x = \frac{t^2 + 5}{t^2 + 1} \qquad y = \frac{4t}{t^2 + 1} \qquad t \in \mathbb{R}$$

Show that all points on C satisfy

$$(x - 3)^2 + y^2 = 4$$

(3)

(Total 3 marks)

Edexcel October 2021 Paper 1 Q13

2. A curve C has parametric equations

$$x = 3 + 2\sin t, \qquad y = 4 + 2\cos 2t, \qquad 0 \leqslant t < 2\pi$$

(a) Show that all points on C satisfy $y = 6 - (x - 3)^2$ **(2)**

(b) **(i)** Sketch the curve C.

(ii) Explain briefly why C does not include all points of $y = 6 - (x - 3)^2, \ \ x \in \mathbb{R}$ **(3)**

The line with equation $x + y = k$, where k is a constant, intersects C at two distinct points.

(c) State the range of values of k, writing your answer in set notation. **(5)**

(Total 10 marks)

Edexcel June 2018 Paper 1 Q14

3.

(a) At time t hours **after a high tide**, the height, h metres, of the tide and the velocity, v knots, of the tidal flow can be modelled using the parametric equations

$$v = 4 - \left(\frac{2t}{3} - 2\right)^2$$

$$h = 3 - 2\sqrt[3]{t - 3}$$

High tides and low tides occur alternately when the velocity of the tidal flow is zero.

A high tide occurs at 2 am.

(i) Use the model to find the height of this high tide. **[1 mark]**

(ii) Find the time of the first **low** tide after 2 am. **[3 marks]**

(iii) Find the height of this low tide. **[1 mark]**

(b) Use the model to find the height of the tide when it is flowing with maximum velocity. **[3 marks]**

(c) Comment on the validity of the model. **[2 marks]**

(Total 10 marks)

AQA June 2019 Paper 1 Q15

Solutions

1.

Scheme	Marks	AO
$(x-3)^2+y^2=\left(\dfrac{t^2+5}{t^2+1}-3\right)^2+\left(\dfrac{4t}{t^2+1}\right)^2$	M1	3.1a
$=\dfrac{\left(2-2t^2\right)^2+16t^2}{\left(t^2+1\right)^2}=\dfrac{4+8t^2+4t^4}{\left(t^2+1\right)^2}$	dM1	1.1b
$\dfrac{4\left(t^4+2t^2+1\right)}{\left(t^2+1\right)^2}=\dfrac{4\left(t^2+1\right)^2}{\left(t^2+1\right)^2}=4*$	A1*	2.1
	(3)	
	(3 marks)	

Notes

M1: Attempts to substitute the given parametric forms into the Cartesian equation or the lhs of the Cartesian equation. There may have been an (incorrect) attempt to multiply out the $(x-3)^2$ term.

dM1: Attempts to combine (at least the lhs) using correct processing into a single fraction, multiplies out and collects terms on the numerator.

A1*: Fully correct proof showing all key steps

Alt

Scheme	Marks	AO
$x=\dfrac{t^2+5}{t^2+1}\Rightarrow xt^2+x=t^2+5\Rightarrow t^2=\dfrac{5-x}{x-1}$ $y=\dfrac{4t}{t^2+1}\Rightarrow y^2=\dfrac{16t^2}{\left(t^2+1\right)^2}=\dfrac{16\left(\dfrac{5-x}{x-1}\right)}{\left(\dfrac{5-x}{x-1}+1\right)^2}$	M1	3.1a
$y^2=\dfrac{16\left(\dfrac{5-x}{x-1}\right)}{\left(\dfrac{5-x}{x-1}+1\right)^2}=16\left(\dfrac{5-x}{x-1}\right)\times\left(\dfrac{(x-1)}{5-x+x-1}\right)^2\Rightarrow$ $y^2=(5-x)(x-1)$	dM1	1.1b
$y^2=(5-x)(x-1)\Rightarrow y^2=6x-x^2-5$ $\Rightarrow y^2=4-(x-3)^2$ or other intermediate step	A1*	2.1

$\Rightarrow (x-3)^2+y^2=4$ *		
	(3)	

M1: Adopts a correct strategy for eliminating t to obtain an equation in terms of x and y only. See scheme. Other methods exist which also lead to an appropriate equation. E.g using $t=\dfrac{y}{x-1}$

dM1: Uses correct processing to eliminate the fractions and start to simplify

A1*: Fully correct proof showing all key steps

2.

Mark scheme (a)

Scheme	Marks	AO
Attempts to use $\cos 2t = 1-2\sin^2 t \Rightarrow \dfrac{y-4}{2} = 1-2\left(\dfrac{x-3}{2}\right)^2$	M1	2.1
$\Rightarrow y-4=2-4\times\dfrac{(x-3)^2}{4} \Rightarrow y=6-(x-3)^2$ *	A1*	1.1b
	(2)	

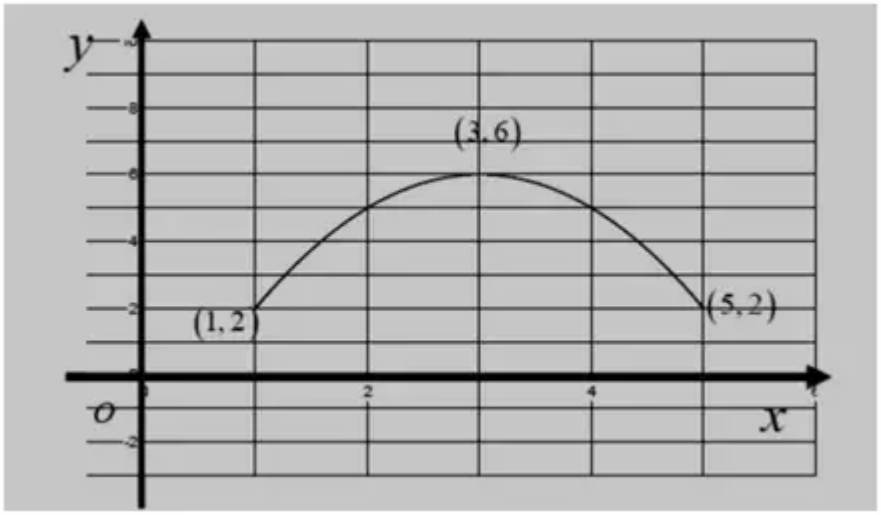
Notes

M1: Uses $\cos 2t = 1-2\sin^2 t$ in an attempt to eliminate t

A1*: Proceeds to $y=6-(x-3)^2$ without any errors
Allow a proof where they start with $y=6-(x-3)^2$ and substitute the parametric coordinates. M1 is scored for a correct $\cos 2t = 1-2\sin^2 t$ but A1 is only scored when both sides are seen to be the same AND a comment is made, hence proven, or similar .

Mark scheme (b)

Scheme	Marks	AO

 ∩ shaped parabola	M1	1.1b
Fully correct with 'ends' at $(1, 2)$ & $(5, 2)$	A1	1.1b
Suitable reason : Eg states as $x = 3 + 2\sin t,\ 1 \leqslant x \leqslant 5$	B1	2.4
	(3)	

Notes

(b) M1: For sketching a ∩ parabola with a maximum in quadrant one. It does not need to be symmetrical

A1: For sketching a ∩ parabola with a maximum in quadrant one and with end coordinates of $(1, 2)$ and $(5, 2)$

B1: Any suitable explanation as to why C does not include all points of $y = 6 - (x - 3)^2,\ \ x \in \mathbb{R}$

This should include a reference to **the limits on sin or cos** with a **link to a restriction on x or y.**

For example

'As $-1 \leqslant \sin t \leqslant 1$ then $1 \leqslant x \leqslant 5$' Condone in words '$x$ lies between 1 and 5' and strict inequalities

'As $\sin t \leqslant 1$ then $x \leqslant 5$' Condone in words 'x is less than 5'

'As $-1 \leqslant \cos(2t) \leqslant 1$ then $2 \leqslant y \leqslant 6$' Condone in words '$y$ lies between 2 and 6'

Withhold if the statement is incorrect Eg "because the domain is $2 \leqslant x \leqslant 5$"

Do not allow a statement on the top limit of y as this is the same for both curves

Mark scheme (c)

Scheme	Marks	AO
Either finds the lower value for $k = 7$ or deduces that $k < \frac{37}{4}$	B1	2.2a

Finds where $x+y=k$ meets $y=6-(x-3)^2$ $\Rightarrow k-x=6-(x-3)^2$ and proceeds to 3TQ in x or y	M1	3.1a
Correct 3TQ in x $\quad x^2-7x+(k+3)=0$ Or y $\quad y^2+(7-2k)y+\left(k^2-6k+3\right)=0$	A1	1.1b
Uses $b^2-4ac=0\Rightarrow 49-4\times1\times(k+3)=0\Rightarrow k=\left(\frac{37}{4}\right)$ or $(7-2k)^2-4\times1\times\left(k^2-6k+3\right)=0\Rightarrow k=\left(\frac{37}{4}\right)$	M1	2.1
Range of values for $k=\left\{k:7\leqslant k<\frac{37}{4}\right\}$	A1	2.5
	(5)	
	(10 marks)	

Notes

B1: Deduces either

- the correct that the lower value of $k=7$ This can be found by substituting into $(5,2)$
  $x+y=k\Rightarrow k=7$ or substituting $x=5$ into
  $x^2-7x+(k+3)=0\Rightarrow 25-35+k+3=0\Rightarrow k=7$
- or deduces that $k<\frac{37}{4}$ This may be awarded from later work

M1: For an attempt at the upper value for k.
Finds where $x+y=k$ meets $y=6-(x-3)^2$ once by using an appropriate method.
Eg. Sets $k-x=6-(x-3)^2$ and proceeds to a 3TQ

A1: Correct 3TQ $x^2-7x+(k+3)=0$ The = 0 may be implied by subsequent work

M1: Uses the "discriminant" condition. Accept use of $b^2=4ac$ oe or $b^2\ldots4ac$ where ... is any inequality leading to a critical value for k. Eg. one root $\Rightarrow 49-4\times1\times(k+3)=0\Rightarrow k=\frac{37}{4}$

A1: Range of values for $k=\left\{k:7\leqslant k<\frac{37}{4}\right\}$ Accept $k\in\left[7,\frac{37}{4}\right)$ or exact equivalent

Alternative (c)

Scheme	Marks	AO
As above	B1	2.2a

Finds where $x + y = k$ meets $y = 6 - (x - 3)^2$ once by using an appropriate method. Eg. Sets gradient of $y = 6 - (x - 3)^2$ equal to -1	M1	3.1a
$-2x + 6 = -1 \Rightarrow x = 3.5$	A1	1.1b
Finds point of intersection and uses this to find upper value of k. $y = 6 - (3.5 - 3)^2 = 5.75$ Hence using $k = 3.5 + 5.75 = 9.25$	M1	2.1
Range of values for $k = \{k : 7 \leqslant k < 9.25\}$	A1	2.5

3.

Mark scheme (a)

Scheme	Marks	AO
(i) Uses model with t=0 to find correct value of h *AWRT 5.9*	B1	AO3.4
(ii) Uses v=0 to set up quadratic equation for t	M1	AO3.4
Obtains t = 6 PI correct answer	A1	AO1.1b
Interprets their lowest positive solution correctly NMS can score 3	A1F	AO3.2a
(iii) Obtains correct h for their positive t provided h < 5.88 Accept 0.12 If given to more decimal places AWFW 0.115 to 0.116 FT their t allow negative h	B1F	AO3.4

Typical solution

(i)

5.88metres

(ii)

$$0 = 4 - \left(\frac{2t}{3} - 2\right)^2$$

$$t = 6$$

8 am

(iii)

0.12m

Mark scheme (b)

Scheme	Marks	AO
Identifies t=3 (for maximum velocity)	B1	AO3.1b
Substitutes their t into model for h PI their answer	M1	3.4
Finds correct height **with units**	A1	3.2a

Typical solution

$$t = 3$$

$$h = 3 - 2\sqrt[3]{3 - 3}$$

$h = 3$ metres

Mark scheme (c)

Scheme	Marks	AO
Explains that the validity of the model is limited by time.	B1	3.5b
Explains that the height continues to decrease after 6 hours (or after the first cycle or first low tide). Or explains there are no other times when v=0	B1	3.5a
	(10 marks)	

Typical solution

The model breaks down after one cycle of the tide.

After 6 hours the model shows the height continues to decrease.