

QQQ – Pure Year 2 – Chapter 12 – Vectors (Version 2)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. Two points A and B lie in a horizontal plane and have coordinates $(-2, 7)$ and $(3, 19)$ respectively.

A particle moves in a straight line from A to B under the action of a constant resultant force of magnitude 6.5 N

Express the resultant force in vector form.

[3 marks]

(Total 3 marks)

AQA June 2022 Paper 1 Q13

2.

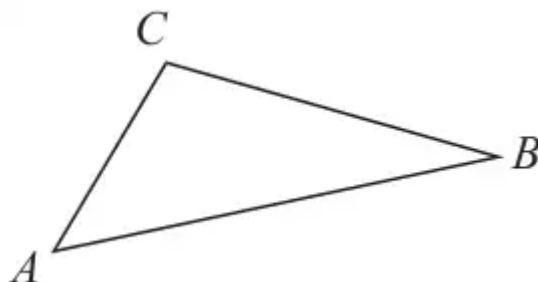


Figure 1

Figure 1 shows a sketch of triangle ABC .

Given that

- $\overrightarrow{AB} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$
- $\overrightarrow{BC} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$

(a) find $\overrightarrow{AC}$ **(2)**

(b) show that $\cos ABC = \frac{9}{10}$ **(3)**

(Total 5 marks)

Edexcel October 2021 Paper 1 Q6

3.

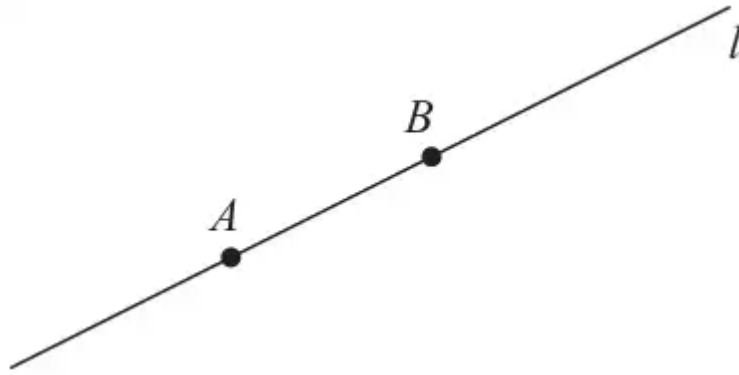


Figure 2

Figure 2 shows a sketch of the straight line l .

Line l passes through the points A and B .

Relative to a fixed origin O

- the point A has position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k}$

(a) Find $\overrightarrow{AB}$ (1)

Given that a point P lies on l such that

$$|\overrightarrow{AP}| = 2|\overrightarrow{BP}|$$

(b) find the possible position vectors of P . (4)

(Total 5 marks)

4. [In this question, $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in a horizontal plane.]

A particle P is moving on a smooth horizontal surface under the action of two forces.

Given that

- the mass of P is 2 kg
- the two forces are $(2\mathbf{i} + 4\mathbf{j})$ N and $(c\mathbf{i} - 2\mathbf{j})$ N, where c is a constant
- the magnitude of the acceleration of P is $\sqrt{5} \text{ m s}^{-2}$

find the two possible values of c .

(5)

(Total 5 marks)

Edexcel June 2024 Mechanics Q3

5. The points A and B have position vectors $\begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix}$ respectively.

(i) Find the exact length of AB .

[2]

(ii) Find the position vector of the midpoint of AB .

[1]

The points P and Q have position vectors $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$ respectively.

(iii) Show that $ABPQ$ is a parallelogram.

[3]

(Total 6 marks)

OCR A June 2018 Paper 2 Q2

Solutions

1.

Scheme	Marks	AO
Finds the vector $\overrightarrow{AB}$ with at least one component correct PI by the modulus calculation ACF	M1	1.1a
Uses Pythagoras to determine their $ \overrightarrow{AB} $ Condone notation errors for modulus.	M1	1.1a
Finds the correct force vector. Condone missing units. ACF	A1	1.1b
	(3 marks)	

Typical solution

$$\overrightarrow{AB} = 5\mathbf{i} + 12\mathbf{j}$$

$$|\overrightarrow{AB}| = \sqrt{5^2 + 12^2} = 13$$

$$\text{So } \mathbf{F} = \frac{6.5}{13}(5\mathbf{i} + 12\mathbf{j})$$

$$2.5\mathbf{i} + 6\mathbf{j} \text{ N}$$

2.

Mark scheme (a)

Scheme	Marks	AO
$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k} + \mathbf{i} + \mathbf{j} + 4\mathbf{k} = \dots$	M1	1.1b
$= -2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$	A1	1.1b
	(2)	

Notes

M1: Attempts $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

There must be attempt to add not subtract.

If no method shown it may be implied by **two** correct components

A1: Correct vector. Allow $-2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$ and $\begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix}$ but not $\begin{pmatrix} -2\mathbf{i} \\ -3\mathbf{j} \\ -1\mathbf{k} \end{pmatrix}$

Mark scheme (b)

Scheme	Marks	AO
At least 2 of $(AC^2) = "2^2 + 3^2 + 1^2", (AB^2) = 3^2 + 4^2 + 5^2, (BC^2) = 1^2 + 1^2 + 4^2$	M1	1.1b
$2^2 + 3^2 + 1^2 = 3^2 + 4^2 + 5^2 + 1^2 + 1^2 + 4^2 - 2\sqrt{3^2 + 4^2 + 5^2}\sqrt{1^2 + 1^2 + 4^2} \cos ABC$	M1	3.1a
$14 = 50 + 18 - 2\sqrt{50}\sqrt{18} \cos ABC$ $\Rightarrow \cos ABC = \frac{50 + 18 - 14}{2\sqrt{50}\sqrt{18}} = \frac{9}{10} *$	A1*	2.1
	(3)	
	(5 marks)	

Notes

M1: Attempts to "square and add" for at least 2 of the 3 sides. Follow through on their $\overrightarrow{AC}$

Look for an attempt at either $a^2 + b^2 + c^2$ or $\sqrt{a^2 + b^2 + c^2}$

M1: A correct attempt to apply a correct cosine rule to the given problem; Condone **slips** on the lengths of the sides but the sides must be in the correct position to find angle ABC

A1*: Correct completion with sufficient intermediate work to establish the printed result.

Condone different labelling, e.g. $ABC \leftrightarrow \theta$ as long as it is clear what is meant

It is OK to move from a correct cosine rule $14 = 50 + 18 - 2\sqrt{50}\sqrt{18} \cos ABC$

via $\cos ABC = \frac{54}{2\sqrt{50}\sqrt{18}}$ o.e. such as $\cos ABC = \frac{(5\sqrt{2})^2 + (3\sqrt{2})^2 - (\sqrt{14})^2}{2 \times 5\sqrt{2} \times 3\sqrt{2}}$ to

$\cos ABC = \frac{9}{10}$

(b) Alternative

Scheme	Marks	AO
$AB^2 = 3^2 + 4^2 + 5^2, BC^2 = 1^2 + 1^2 + 4^2$	M1	1.1b

$\overrightarrow{BA} \cdot \overrightarrow{BC} = (3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}) \cdot (\mathbf{i} + \mathbf{j} + 4\mathbf{k}) = 27 = \sqrt{3^2 + 4^2 + 5^2} \sqrt{1^2 + 1^2 + 4^2} \cos ABC$	M1	3.1a
$27 = \sqrt{50} \sqrt{18} \cos ABC \Rightarrow \cos ABC = \frac{27}{\sqrt{50} \sqrt{18}} = \frac{9}{10} *$	A1*	2.1

M1: Correct application of Pythagoras for sides AB and BC or their squares

M1: Recognises the requirement for and applies the scalar product

A1*: Correct completion with sufficient intermediate work to establish the printed result

3.

Mark scheme (a)

Scheme	Marks	AO
$(\overrightarrow{AB} =) 3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}$	B1	1.1b
	(1)	

Notes

B1: Correct vector. Allow $3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}$ or $\begin{pmatrix} 3 \\ 9 \\ 3 \end{pmatrix}$ but **not** $\begin{pmatrix} 3\mathbf{i} \\ 9\mathbf{j} \\ 3\mathbf{k} \end{pmatrix}$ and **not** $(3, 9, 3)$

Condone 9 for $\begin{pmatrix} 3 \\ 9 \\ 3 \end{pmatrix}$

Do **not** apply isw here but award for e.g. $3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k} = \begin{pmatrix} 3\mathbf{i} \\ 9\mathbf{j} \\ 3\mathbf{k} \end{pmatrix}$

E.g. if they obtain $\overrightarrow{AB} = 3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}$ and then say $\overrightarrow{AB} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$ then award B0

If part (a) is **not attempted** and the correct $\overrightarrow{AB}$ is seen in part (b) the B1 can be awarded there.

Mark scheme (b)

Scheme	Marks	AO
Examples: $\overrightarrow{OP} = \overrightarrow{OA} + 2\overrightarrow{AB} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + 2(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$ or $\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{AB} = 5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k} + (3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$ or	M1	3.1a

$\vec{OP} = \vec{OA} + \frac{2}{3}\vec{AB} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \frac{2}{3}(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) =$... or $\vec{OP} = \vec{OB} + \frac{1}{3}\vec{BA} = 5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k} - \frac{1}{3}(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) =$...		
$8\mathbf{i} + 15\mathbf{j} + 11\mathbf{k}$ or $4\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	A1	1.1b
Examples: $\vec{OP} = \vec{OA} + 2\vec{AB} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + 2(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$ or $\vec{OP} = \vec{OB} + \vec{AB} = 5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k} + (3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$ and $\vec{OP} = \vec{OA} + \frac{2}{3}\vec{AB} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \frac{2}{3}(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) =$... or $\vec{OP} = \vec{OB} + \frac{1}{3}\vec{BA} = 5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k} - \frac{1}{3}(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) =$...	dM1	3.1a
$8\mathbf{i} + 15\mathbf{j} + 11\mathbf{k}$ and $4\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	A1	2.2a
	(4)	
	(5 marks)	

Notes

General Guidance for part (b):

As with most vector questions we will see a variety of approaches (correct and incorrect).

In general, the marks are awarded as follows:

- M1 for a correct complete strategy to find at least one position for P (May be implied by at least 2 correct components)
- A1 for one correct position for P
- dM1 for a correct complete strategy to find both positions for P (May be implied by at least 2 correct components for both positions)
- A1 both correct positions for P and no others

Various examples are shown below.

Other methods will be seen but the above marking principles should be applied.

You can condone slips in their algebra/processing as long as the intention is clear.

The examples given below give the detail to look for depending on the approach.

If you see a response and you are not sure if it deserves credit use Review.

Note that adding vectors when they should be subtracting will generally score M0 but use review if necessary.

Note that sight of at least one correct position for P implies M1A1

M1: Attempts at least one correct strategy for finding P

A1: One correct position vector or allow coordinates for this mark e.g. (8, 15, 11) or (4, 3, 7) or

$x = \dots, y = \dots, z = \dots$

If given as a vector, allow e.g. $8\mathbf{i} + 15\mathbf{j} + 11\mathbf{k}$, $\begin{pmatrix} 8 \\ 15 \\ 11 \end{pmatrix}$ but not $\begin{pmatrix} 8\mathbf{i} \\ 15\mathbf{j} \\ 11\mathbf{k} \end{pmatrix}$

dM1: Attempts two correct strategies for finding P

A1: Both correct position **vectors**

Must both be vectors so e.g. $8\mathbf{i} + 15\mathbf{j} + 11\mathbf{k}$, $\begin{pmatrix} 8 \\ 15 \\ 11 \end{pmatrix}$ and $4\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$, $\begin{pmatrix} 4 \\ 3 \\ 7 \end{pmatrix}$ but not e.g. $\begin{pmatrix} 8\mathbf{i} \\ 15\mathbf{j} \\ 11\mathbf{k} \end{pmatrix}$

Condone e.g. $\begin{matrix} 8 \\ 15 \\ 11 \end{matrix}$ for $\begin{pmatrix} 8 \\ 15 \\ 11 \end{pmatrix}$

Alternative 1 using vector equation of l :

$$\mathbf{r} = \overrightarrow{OA} + \lambda \overrightarrow{AB} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 9 \\ 3 \end{pmatrix} \quad \left(\text{or e.g. } \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \right)$$

$$|\overrightarrow{AP}| = 2|\overrightarrow{BP}| \Rightarrow \left| \begin{pmatrix} 3\lambda \\ 9\lambda \\ 3\lambda \end{pmatrix} \right| = 2 \left| \begin{pmatrix} 3\lambda + 2 - 5 \\ 9\lambda - 3 - 6 \\ 3\lambda + 5 - 8 \end{pmatrix} \right| \Rightarrow 9\lambda^2 + 81\lambda^2 + 9\lambda^2 = 4[(3\lambda - 3)^2 + (9\lambda - 9)^2]$$

$$\Rightarrow \lambda = 2, \frac{2}{3} \Rightarrow \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ 9 \\ 3 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + \frac{2}{3} \begin{pmatrix} 3 \\ 9 \\ 3 \end{pmatrix}$$

M1: Forms the vector equation of line l using their $\overrightarrow{AB}$ from part (a) or by starting again, forms the vectors $\overrightarrow{AP}$ and $\overrightarrow{BP}$ then uses $|\overrightarrow{AP}| = 2|\overrightarrow{BP}|$ and Pythagoras to produce a quadratic equation in " λ " which they then solve to find " λ " and use correctly to find at least one position for P .

Note if they use $\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ for the direction, they should get $\lambda = 6, 2$

If all other work is correct, condone not squaring the "2" when applying Pythagoras

A1: See main scheme

dM1: As the first M and finds both positions for P

A1: See main scheme

Alternative 2 using P as $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and that $\overrightarrow{AP}$ and $\overrightarrow{BP}$ are parallel:

$$\overrightarrow{OP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \overrightarrow{AP} = \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix}, \overrightarrow{BP} = \begin{pmatrix} x-5 \\ y-6 \\ z-8 \end{pmatrix}$$

$$|\overrightarrow{AP}| = 2|\overrightarrow{BP}| \Rightarrow \left| \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix} \right| = 2 \left| \begin{pmatrix} x-5 \\ y-6 \\ z-8 \end{pmatrix} \right| \Rightarrow \begin{matrix} (x-2)^2 = 4(x-5)^2 \\ (y+3)^2 = 4(y-6)^2 \\ (z-5)^2 = 4(z-8)^2 \end{matrix}$$

$$\begin{matrix} (x-2)^2 = 4(x-5)^2 \Rightarrow x = 4, 8 \\ (y+3)^2 = 4(y-6)^2 \Rightarrow y = 3, 15 \\ (z-5)^2 = 4(z-8)^2 \Rightarrow z = 7, 11 \end{matrix} \Rightarrow \begin{pmatrix} 4 \\ 3 \\ 7 \end{pmatrix}, \begin{pmatrix} 8 \\ 15 \\ 11 \end{pmatrix}$$

M1: Sets P as a general point, forms $\overrightarrow{AP}$ and $\overrightarrow{BP}$ (either way round) then uses $|\overrightarrow{AP}| = 2|\overrightarrow{BP}|$ then squares components and equates to produce quadratic equations in x and y and z which they then solve to find at least one position for P . It is not just for finding values which are not then used to form a point (or vector).

If all other work is correct, condone not squaring the “2” when squaring.

A1: See main scheme

dM1: As the first M and finds both positions for P .

A1: See main scheme

Note that if the modulus is not used, this method can lead to one correct position for P e.g.

$$\overrightarrow{AP} = 2\overrightarrow{BP} \Rightarrow \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix} = 2 \begin{pmatrix} x-5 \\ y-6 \\ z-8 \end{pmatrix} \Rightarrow \begin{matrix} x = 8 \\ y = 15 \\ z = 11 \end{matrix} \text{ and scores M1 A1}$$

But it is possible to find the other position without squaring e.g.

$$|\overrightarrow{AP}| = 2|\overrightarrow{BP}| \Rightarrow \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix} = 2 \begin{pmatrix} 5-x \\ 6-y \\ 8-z \end{pmatrix} \Rightarrow \begin{matrix} x = 4 \\ y = 3 \\ z = 7 \end{matrix} \text{ and scores dM1 then A1 as main scheme.}$$

This requires at least 2 correct equations for x , y or z for the dM1

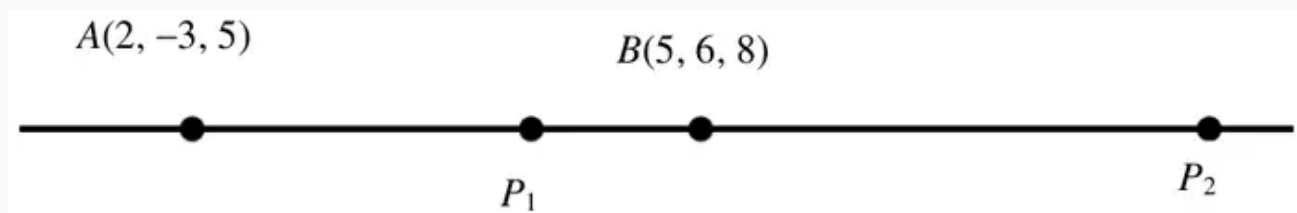
e.g. Alternative 3 using P as $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and eliminating 2 of the variables:

$$\overrightarrow{OP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \overrightarrow{AP} = \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix}, \overrightarrow{BP} = \begin{pmatrix} x-5 \\ y-6 \\ z-8 \end{pmatrix}$$

$$\overrightarrow{AP} = \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \Rightarrow \overrightarrow{AP} = \begin{pmatrix} x-2 \\ 3x-6 \\ x-2 \end{pmatrix}, \overrightarrow{BP} = \begin{pmatrix} x-5 \\ y-6 \\ z-8 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \Rightarrow \overrightarrow{BP} = \begin{pmatrix} x-5 \\ 3x-15 \\ x-5 \end{pmatrix}$$

$$|\overrightarrow{AP}| = 2|\overrightarrow{BP}| \Rightarrow (x-2)^2 + (3x-6)^2 + (x-2)^2 = 4[(x-5)^2 + (3x-15)^2 + (x-5)^2] \Rightarrow x$$

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} \text{ (or } \overrightarrow{OB} + \overrightarrow{BP}) = \begin{pmatrix} x \\ 3x-9 \\ x+3 \end{pmatrix} \Rightarrow \begin{pmatrix} 4 \\ 3 \\ 7 \end{pmatrix} \text{ or } \begin{pmatrix} 8 \\ 15 \\ 11 \end{pmatrix}$$



4.

Scheme	Marks	A0
Resultant force = $(2 + c)\mathbf{i} + (4 - 2)\mathbf{j}$	B1	1.1b
$\sqrt{(2 + c)^2 + 2^2}$ or $(2 + c)^2 + 2^2$	M1	3.1a
$(2 + c)^2 + 4 = 4 \times 5$ or $\sqrt{(2 + c)^2 + 2^2} = 2 \times \sqrt{5}$	M1	3.1a
OR		
$\mathbf{a} = \frac{1}{2}[(2 + c)\mathbf{i} + (4 - 2)\mathbf{j}]$ oe	M1	
$\frac{1}{4}[(2 + c)^2 + 4] = 5$ or $\frac{1}{2}\sqrt{(2 + c)^2 + 2^2} = \sqrt{5}$	M1	
$c = 2$ or $c = -6$	A1	1.1b
$c = 2$ and $c = -6$	A1	2.2a
	(5)	
	(5 marks)	

Notes

N.B. Allow the use of column vectors

B1 Seen or implied, with **i**'s and **j**'s collected

M1 Using Pythagoras, with or without the root, on their resultant force

N.B. This mark is available even if they've subtracted the two forces.

M1 Use of $F = ma$ to obtain an equation in c only.

OR

M1 Using $\mathbf{F} = m\mathbf{a}$ with their resultant force to obtain $\mathbf{a}$ with $\mathbf{i}$'s and $\mathbf{j}$'s collected

N.B. This mark is available even if they've subtracted the two forces.

M1 Using Pythagoras, with or without the root, on their $\mathbf{a}$ to obtain an equation in c only

A1 One correct value

A1 Two correct values

5.

Mark scheme (i)

Scheme	Marks	AO
$(1 - (-3))^2 + (-2 - (-1))^2 + (5 - 2)^2 (= 26)$	M1	1.1a
Length = $\sqrt{26}$ or 5.10 or 5.1 (2 sf)	A1	1.1
	[2]	

Notes

M1: Attempt. Allow with one sign error

$\sqrt{\quad}$ not nec'y

Mark scheme (ii)

Scheme	Marks	AO
$\begin{pmatrix} -1 \\ -1.5 \\ 3.5 \end{pmatrix}$	B1	1.1
	[1]	

Mark scheme (iii)

Scheme	Marks	AO
$\overrightarrow{BA} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$	M1	2.1

$\overrightarrow{PQ} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$	M1	1.1
$BA = PQ$ and $BA \parallel PQ$ and hence $ABPQ$ is a parallelogram (AG)	A1	2.2a
	[3]	

Notes

M1: or quote result for $\overrightarrow{BA}$ from (ii) or (i)(a)

M1: or similar methods with AQ & BP

or AB and QP etc

Allow find eg AB and PQ

or $\overrightarrow{BA} = \overrightarrow{PQ}$ with arrows

or $|BA| = |PQ|$ & $|BP| = |AQ|$ shown & stated

or $BA \parallel PQ$ & $BP \parallel AQ$ shown & stated

A1: Both statements needed, dep M1M1

SC Incorrect, but equal, vectors BA & PQ with correct conclusion SC B1

Allow without method

SC Lengths only seen: M1M0

Just $|BA| = |PQ|$ A0