

QQQ – Pure Year 2 – Chapter 12 – Vectors (Version 1)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. Relative to a fixed origin O

- the point A has position vector $5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$
- the point B has position vector $2\mathbf{i} + 4\mathbf{j} + a\mathbf{k}$

where a is a positive integer.

(a) Show that $|\overrightarrow{OA}| = \sqrt{38}$ (1)

(b) Find the smallest value of a for which

$$|\overrightarrow{OB}| > |\overrightarrow{OA}|$$

(2)

(Total 3 marks)

Edexcel June 2023 Paper 1 Q3

2. A particle P of mass 2.5 kg is in equilibrium under the action of three horizontal forces

$$\mathbf{F}_1 = \begin{pmatrix} 3 \\ -7 \end{pmatrix} \text{ N}, \mathbf{F}_2 = \begin{pmatrix} -5 \\ 10 \end{pmatrix} \text{ N and } \mathbf{F}_3.$$

(a) Find the force $\mathbf{F}_3$. [2]

The force $\mathbf{F}_3$ is changed to $\begin{pmatrix} 8 \\ 1 \end{pmatrix}$ N.

(b) Find the acceleration of P , giving your answer in column vector form. [2]

(Total 4 marks)

OCR A June 2024 Paper 2 Q9

3. Given that

- $\overrightarrow{PQ} = 2\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$
- $\overrightarrow{QR} = 6\mathbf{i} + 6\mathbf{k}$

(a) find $\overrightarrow{PR}$. (2)

(b) Hence show that triangle PQR is both right-angled and isosceles. (4)

(Total 6 marks)

Edexcel June 2025 Paper 1 Q10

4. Relative to a fixed origin O

- the point A has position vector $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $4\mathbf{j} + 6\mathbf{k}$
- the point C has position vector $-16\mathbf{i} + p\mathbf{j} + 10\mathbf{k}$

where p is a constant.

Given that A , B and C lie on a straight line,

(a) find the value of p . (3)

The line segment OB is extended to a point D so that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$

(b) Find $|\overrightarrow{OD}|$, writing your answer as a fully simplified surd. (3)

(Total 6 marks)

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5. Relative to a fixed origin O ,

- A is the point with position vector $12\mathbf{i}$
- B is the point with position vector $16\mathbf{j}$
- C is the point with position vector $(50\mathbf{i} + 136\mathbf{j})$
- D is the point with position vector $(22\mathbf{i} + 24\mathbf{j})$

(a) Show that AD is parallel to BC . (2)

Points A, B, C and D are used to model the vertices of a running track in the shape of a quadrilateral.

Runners complete one lap by running along all four sides of the track.

The lengths of the sides are measured in metres.

Given that a particular runner takes exactly 5 minutes to complete 2 laps,

(b) calculate the average speed of this runner, giving the answer in kilometres per hour. (4)

(Total 6 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
$(\vec{OA} =) \sqrt{5^2 + 3^2 + 2^2} = \sqrt{38}^*$	B1*	1.1b
	(1)	

Notes

B1*: Shows the magnitude of $|\vec{OA}|$ is $\sqrt{38}$. Must see $\sqrt{5^2 + 3^2 + 2^2}$ or e.g. $\sqrt{25 + 9 + 4}$. We need to see how the value 38 or $\sqrt{38}$ is formed using the three components. Withhold this mark for incorrect working such as $|\vec{OA}| = 5^2 + 3^2 + 2^2 = 38 \Rightarrow |\vec{OA}| = \sqrt{38}$ but do not penalise poor notation to denote vectors or the magnitude as long as the intention is clear as to what they are finding (OA instead of $|\vec{OA}|$ is fine). Do not penalise if their square root does not go fully over all three terms as long as the intention is clear. May find $|\vec{AO}|$ instead which is acceptable.

$(|\vec{OA}|^2 =) 25 + 9 + 4 = 38 \Rightarrow (|\vec{OA}| =) \sqrt{38}$ scores B1 (we see how 38 is found)

$(|\vec{OA}|^2 =) 25 + 9 + 4 \Rightarrow \sqrt{38}$ scores B1 (we see how 38 is found)

$25 + 9 + 4 = \sqrt{38}$ scores B0 (they are not equal)

$(|\vec{OA}|^2 =) 38 \Rightarrow (|\vec{OA}| =) \sqrt{38}$ scores B0 (no method seen to show how 38 is found)

Mark scheme (b)

Scheme	Marks	AO
$ \vec{OB} = \sqrt{2^2 + 4^2 + a^2} = \sqrt{20 + a^2}$ so when $a = 5$	M1	1.1b
$ \vec{OB} = \sqrt{20 + 25} = \sqrt{45}$		
$= 5$	Alcso	2.3
	(2)	
	(3 marks)	

Notes

M1: Attempts to find $|\vec{OB}|$ (or $|\vec{OB}|^2$) in terms of a and substitutes in a positive integer for a to find a value for $|\vec{OB}|$ (or $|\vec{OB}|^2$). e.g. $|\vec{OB}| = \sqrt{20 + a^2} \Rightarrow$ when $a = 2 \Rightarrow \sqrt{24}$. Also accept e.g. $|\vec{BO}|$ (or $|\vec{BO}|^2$).

Alternatively sets up an equation or an inequality e.g. $\sqrt{20 + a^2} > \sqrt{38}$ and proceeds to $a^2 > \dots$ (or $a^2 = \dots$). Condone sign slips in their rearrangement only.

Allow the use of $=, <$ or $>$ for this mark. “20” + $a^2 \dots 38 \Rightarrow a^2 \dots$ “18”

(may be implied by sight of $\sqrt{18} = 4.24 \dots$)

A1: 5 cso (answer on its own with no incorrect working seen scores M1A1). Withhold this mark if $|\vec{OB}|$ (or $|\vec{OB}|^2$) is incorrect (or $|\vec{BO}|$ or $|\vec{BO}|^2$). Do not be concerned with the notation as long as the intention is clear or implied as to what they are trying to calculate (the calculations must be correct). Withhold this mark if at any point they set $|\vec{OB}| < |\vec{OA}|$ but accept an argument leading to an answer of 5 from either $|\vec{OB}| > |\vec{OA}|$ or $|\vec{OB}| = |\vec{OA}|$

2.

Mark scheme (a)

Scheme	Marks	AO
$\begin{pmatrix} 3 \\ -7 \end{pmatrix} + \begin{pmatrix} -5 \\ 10 \end{pmatrix} + \mathbf{F}_3 = \mathbf{0}$	M1	3.3
$\mathbf{F}_3 = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \text{ (N)}$	A1	1.1
	[2]	

Notes

M1: Use $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$

Correct answer implies this mark

Allow sign errors. One correct component for $\mathbf{F}_3$ can imply this mark

A1: cao – isw if magnitude of the force is found after correct vector seen

oe e.g. $-\begin{pmatrix} -2 \\ 3 \end{pmatrix}, 2\mathbf{i} - 3\mathbf{j}$

Mark scheme (b)

Scheme	Marks	AO
$\begin{pmatrix} 3 \\ -7 \end{pmatrix} + \begin{pmatrix} -5 \\ 10 \end{pmatrix} + \begin{pmatrix} 8 \\ 1 \end{pmatrix} = 2.5\mathbf{a}$	M1	3.4
$\mathbf{a} = \begin{pmatrix} 2.4 \\ 1.6 \end{pmatrix} (\text{m s}^{-2})$	A1	1.1
	[2]	

Notes

M1: Use of $\mathbf{F} = m\mathbf{a}$ with $m = 2.5$ and at least one correct component for $\mathbf{F} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$ (either stated or implied by working)

A1: cao – isw if magnitude of the acceleration is found after correct vector seen

3.

Mark scheme (a)

Scheme	Marks	AO
Attempts to add $\overrightarrow{PQ} = 2\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$ and $\overrightarrow{QR} = 6\mathbf{i} + 6\mathbf{k}$	M1	1.1b
$(\overrightarrow{PR} =) 8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$	A1	1.1b
	(2)	

Notes

If part (a) is not attempted and the correct $\overrightarrow{PR}$ is seen in part (b) then M1A1 can be awarded

M1: Attempts to add $\overrightarrow{PQ} = 2\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$ and $\overrightarrow{QR} = 6\mathbf{i} + 6\mathbf{k}$ with at least one correct component of $8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$. A typical misread of $\overrightarrow{QR}$ as $6\mathbf{i} + 6\mathbf{j}$ can score for at least one correct component of $8\mathbf{i} + 14\mathbf{j} - 2\mathbf{k}$

A1: Correct vector. Allow $8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$ or $\begin{pmatrix} 8 \\ 8 \\ 4 \end{pmatrix}$ but **not** $\begin{pmatrix} 8\mathbf{i} \\ 8\mathbf{j} \\ 4\mathbf{k} \end{pmatrix}$ and **not** (8, 8, 4)

Condone 8 for $\begin{pmatrix} 8 \\ 8 \\ 4 \end{pmatrix}$ Do not apply isw here but award for e.g. $8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k} = \begin{pmatrix} 8\mathbf{i} \\ 8\mathbf{j} \\ 4\mathbf{k} \end{pmatrix}$

E.g. if they obtain $\overrightarrow{PR} = 8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$ and then say $\overrightarrow{PR} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ then award A0

Mark scheme (b)

Scheme	Marks	AO
Attempts to show the triangle is isosceles (or right-angled) e.g. Attempts $ \vec{PQ} = \sqrt{2^2 + 8^2 + (-2)^2}$ and $ \vec{QR} = \sqrt{6^2 + 6^2}$	M1	3.1a
Shows e.g. $ \vec{PQ} = \vec{QR} = \sqrt{72} (= 6\sqrt{2})$	A1	1.1b
Attempts to show the triangle is isosceles and right-angled e.g. attempts to find the lengths of all three sides AND e.g. attempts to compare lengths via use of " $a^2 + b^2 = c^2$ "	M1	1.1b
e.g. Shows that $ \vec{PQ} ^2 + \vec{QR} ^2 = \vec{PR} ^2$ as $72 + 72 = 144$ so PQR is a right-angled triangle	A1	2.1
	(4)	
	(6 marks)	

Notes

Note that M1A0M1A1 is not possible. If they have an incorrect vector in part (a) then the maximum score is M1A1M1A0. A misread of $\vec{QR}$ as $6\mathbf{i} + 6\mathbf{j}$ in (b) can only score a maximum M1A1M1A0

They may attempt to show the triangle is either isosceles or right-angled in either order. You will need to look through their solution and award the order which scores most marks. Usually it will be isosceles first. Condone slips to be recovered.

To show the triangle is isosceles they only need to show **two sides** (or **two angles**) are the same. They do not need to consider the other side to show it is isosceles.

M1: Attempts to show that the triangle is either isosceles (**or** right-angled). See table below.

A1: Fully shows that the triangle is isosceles (or right-angled). See table below. Allow slips in their method if recovered as long as they proceed to correct lengths or values. **A conclusion that the triangle is isosceles or right-angled is not required for this mark.**

Isosceles	Requirement for M1 examples	Requirement for A1 examples
Us in	Attempts to find the length or length ² of PQ and QR :	States or shows that $ \vec{PQ} = \vec{QR} (= \sqrt{72} (= 6\sqrt{2}))$,

g le ng th s	$\left(\left \overrightarrow{PQ}\right \right)=\sqrt{2^2+8^2+(-2)^2}\left(=\sqrt{72}=6\sqrt{2}\right)$ or seen as e.g. $2\sqrt{1^2+4^2+(-1)^2}\left(=2\sqrt{18}\right)$ $\left(\left \overrightarrow{QR}\right \right)=\sqrt{6^2+6^2}\left(=\sqrt{72}=6\sqrt{2}\right)$ or may be seen as e.g. $3\sqrt{2^2+2^2}\left(=3\sqrt{8}\right)$ May be implied by e.g. $6\sqrt{2}$ Condone missing brackets around $(-2)^2$ provided the intention is clear to square and add implied by e.g. $6\sqrt{2}$	or equivalent. Accept e.g. $PQ^2 = QR^2$ or “both are 72” Condone poor notation and/or labelling of lengths provided they are not clearly referring to the longest length. e.g. achieves $6\sqrt{2}$ for both PQ and QR and states they are the same scores M1A1 Only stating isosceles without a comparison of PQ and QR is A0 Uses the sine rule with the lengths and angles embedded in the correct places e.g. $\frac{\sin \angle QPR}{QR} = \frac{\sin \angle PRQ}{PQ}$ Deduces $\sin \angle QPR = \sin \angle PRQ$ so the angles are the same o.e.
Ri gh t- an gl ed	Requirement for M1 examples	Requirement for A1 examples
Us in g le ng th s	Attempts to find all three lengths or lengths² $\left(\left \overrightarrow{PR}\right \right)=\sqrt{“8”^2+“8”^2+“4”^2}\left(=12\right)$ or may be seen as e.g. $4\sqrt{“2”^2+“2”^2+“1”^2}$ or implied by their 12 AND ATTEMPTS $\left(PQ^2+QR^2=PR^2 \Rightarrow\right) “72”+“72”=“144”$ Attempts to find all three lengths or lengths² (see above for guidance) AND ATTEMPTS the cosine rule correctly to find $\cos PQR=\frac{“72”+“72”-“144”}{2 \times “\sqrt{72}” \times “\sqrt{72}”}$ o.e.	States or shows that $PQ ^2+ QR ^2= PR ^2$, or equivalent Condone poor notation and/or labelling of lengths. States or shows $\cos PQR = 0$

Scalar dot product (Further Maths)	Attempts e.g. $\begin{pmatrix} 2 \\ 8 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 0 \\ 6 \end{pmatrix} = (2 \times 6) + (8 \times 0) + (-2 \times 6)$ oe Must see the calculation for M1	States or shows that $(2 \times 6) + (8 \times 0) + (-2 \times 6) = 0$ oe (they do not need to write anything more for this mark)
	$(\cos PQR =) \frac{(2 \times 6) + (8 \times 0) + (-2 \times 6)}{“\sqrt{72}” \times “\sqrt{72}”}$ oe	$\cos PQR = \frac{(2 \times 6) + (8 \times 0) + (-2 \times 6)}{\sqrt{72} \times \sqrt{72}} = 0$

M1: Attempts to show that the triangle is both isosceles **and** right-angled. Usually they will have shown the triangle is isosceles and attempt to show that it is right-angled. **Here are some examples but there will be others:**

Right-angled	Method examples (required for second method mark)
Using lengths	Attempts to find all three lengths or lengths ² AND ATTEMPTS $(PQ^2 + QR^2 = PR^2 \Rightarrow) “72” + “72” = “144”$
	Attempts to find all three lengths or lengths ² (see above for guidance) AND ATTEMPTS the cosine rule in an attempt to find $\cos PQR = \frac{“72” + “72” - “144”}{2 \times “\sqrt{72}” \times “\sqrt{72}”}$
Scalar dot product	Attempts $\begin{pmatrix} 2 \\ 8 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 0 \\ 6 \end{pmatrix} = (2 \times 6) + (8 \times 0) + (-2 \times 6)$
	$(\cos PQR =) \frac{(2 \times 6) + (8 \times 0) + (-2 \times 6)}{“\sqrt{72}” \times “\sqrt{72}”}$ oe

Alternatively, having already shown the triangle is right-angled they may:

- use trigonometry to show that the two base angles are both 45°
- attempt to find the required lengths or lengths² of PQ and PR if not already found
(Note they may also find the length or length² of QR and may use the sine rule or cosine rule)

Note if they have shown either property then we will condone making an assumption of the other property to justify the size of angle PRQ and/or angle QPR and may use trigonometry to show either

$$\left(\sin \theta = \frac{6\sqrt{2}}{12} \Rightarrow \right) \theta = \arcsin \left(\frac{6\sqrt{2}}{12} \right) = 45^\circ \quad (\text{we must see arcsin or } \sin^{-1})$$

$$\sin \theta = \frac{6\sqrt{2}}{12} \Rightarrow \sin \theta = \frac{\sqrt{2}}{2} \left(\text{or } \frac{1}{\sqrt{2}} \right) \Rightarrow \theta = 45^\circ \quad (\text{since this is a known angle})$$

$$\sin 45^\circ = \frac{\sqrt{2}}{2} \left(\text{or } \frac{1}{\sqrt{2}} \right) = \frac{6\sqrt{2}}{12} \quad (\text{This requires all 3 equivalences (can be in any order)})$$

Using trigonometry (SOHCAHTOA) to show the second property they will e.g.

- Use triangle PQR and assume right angled
- Split triangle PQR into two right angled congruent triangles using the isosceles property

If they use triangle PQR then it requires

- For M1: Calculations showing the required property to score M1
- For A1: To either draw a labelled right angled triangle or state the assumption that it is right angled and conclude

If they split the triangle PQR in half then they have formed two right angled congruent triangles which (provided they had two lengths the same already) will not be an assumption of a right angle. **Then it requires**

- For M1: Calculations to find either at least one of the base angles of 45° OR to find the right angle. It should be clear whether they
 - found a base angle of PQR and doubled it or
 - found half of the right angle and doubled it.
- For A1: Fully correct calculations and conclusion

Note if they find a 45 degree angle and double it then it needs to be clear whether this is the right angle or if it is the sum of two base angles because there are four angles of size 45 degrees via this method.

They would have to e.g. mention about the angle sum of a triangle or show a clearly labelled diagram and calculations with labels that match the diagram.

Note that use of $\tan$ is unlikely to score because if they just use two equal lengths for the two shorter sides of their right angled triangle, then $\tan(A) = 1$, so the angles will always be 45° , 45° and 90° – it is the use of the length of 12 (or 6) which is going to lead to showing the triangle is right angled via these routes.

A1: Fully shows that the triangle is isosceles and right-angled **and concludes** that the triangle is **both isosceles and right-angled**. These conclusions may appear at separate stages of their solution. Condone poor notation and/or labelling of lengths provided the intention is clear. Condone slips if recovered. If they have a preamble then they must have a minimal conclusion e.g. proven, tick, QED

Note if they have an incorrect vector in part (a) then the maximum score is M1A1M1A0

4.

Mark scheme (a)

Scheme	Marks	AO
Attempts two of the relevant vectors $\pm \overrightarrow{AB} = \pm(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k})$ $\pm \overrightarrow{AC} = \pm(-20\mathbf{i} + (p + 3)\mathbf{j} + 5\mathbf{k})$ $\pm \overrightarrow{BC} = \pm(-16\mathbf{i} + (p - 4)\mathbf{j} + 4\mathbf{k})$	M1	3.1a
Uses two of the three vectors in such a way as to find the value of p . E.g. $p + 3 = 5 \times 7$	dM1	2.1
$p = 32$	A1	1.1b
	(3)	

(a) Alternative:

Scheme	Marks	AO
$\mathbf{r}_{AB} = 4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \lambda(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k})$	M1	3.1a
$4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \lambda(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k}) = -16\mathbf{i} + p\mathbf{j} + 10\mathbf{k} \Rightarrow \lambda = 5$ $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \lambda(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k}) = -16\mathbf{i} + p\mathbf{j} + 10\mathbf{k} \Rightarrow p = 35 - 3$	dM1	2.1
$p = 32$	A1	1.1b

Notes

M1: Attempts two of the three relevant vectors by **subtracting** either way around. See scheme.

Allow equivalent work e.g. $\pm \overrightarrow{AB} = \pm (\overrightarrow{OB} + \overrightarrow{AO})$

If no working is shown, method can be implied by 2 correct components.

dM1: For the key step in using the fact that if the vectors are parallel, they will be multiples of each other (where the multiple is something other than 1) to find p .

E.g. $p + 3 = 5 \times 7, p - 4 = \frac{4}{5}(p + 3), p - 4 = 4 \times 7$

A1: $p = 32$ (Condone 32j)

For reference, $\overrightarrow{BC} = 4\overrightarrow{AB}$, $\overrightarrow{AC} = 5\overrightarrow{AB}$, $\overrightarrow{BC} = \frac{4}{5}\overrightarrow{AC}$, $\overrightarrow{AC} = \frac{5}{4}\overrightarrow{BC}$

Note that candidates generally only need to use 2 components to find p and if the 3rd component has errors but is not used, full marks can be awarded.

Alternative:

M1: Forms the vector equation using A or B as position and $\pm\overrightarrow{AB}$ as the direction

dM1: For the key step in using the fact that C lies on the line to find p

A1: $p = 32$ (Condone 32j)

For reference, $\overrightarrow{BC} = 4\overrightarrow{AB}$, $\overrightarrow{AC} = 5\overrightarrow{AB}$, $\overrightarrow{BC} = \frac{4}{5}\overrightarrow{AC}$, $\overrightarrow{AC} = \frac{5}{4}\overrightarrow{BC}$

Note that candidates generally only need to use 2 components to find p and if the 3rd component has errors but is not used, full marks can be awarded.

There will be other approaches e.g. using “gradients” and “ratios” and the method marks can be implied – if you are unsure if such attempts deserve credit use Review

Mark scheme (b)

Scheme	Marks	AO
Deduces that $\overrightarrow{OD} = \lambda\overrightarrow{OB} = 4\lambda\mathbf{j} + 6\lambda\mathbf{k}$ and attempts $\overrightarrow{CD} = 16\mathbf{i} + (4\lambda - \text{“32”})\mathbf{j} + (6\lambda - 10)\mathbf{k}$	M1	3.1a
Correct attempt at λ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$ $\overrightarrow{CD} = 16\mathbf{i} + (4\lambda - \text{“32”})\mathbf{j} + (6\lambda - 10)\mathbf{k}$ $\overrightarrow{OA} = 4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ $4\lambda - 32 = -12 \Rightarrow \lambda = \dots$ OR $6\lambda - 10 = 20 \Rightarrow \lambda = \dots$	dM1	1.1b
$ \overrightarrow{OD} = 5 \times \sqrt{4^2 + 6^2} = 10\sqrt{13}$	A1	1.1b
	(3)	
	(6 marks)	

(b) Alternative:

Scheme	Marks	AO
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Deduces that $\overrightarrow{OD} = \lambda \overrightarrow{OB} = 4\lambda \mathbf{j} + 6\lambda \mathbf{k}$ and attempts $\overrightarrow{OD} = \overrightarrow{OC} + \mu \overrightarrow{OA} = -16\mathbf{i} + 32\mathbf{j} + 10\mathbf{k} + \mu(4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$	M1	3.1a
Correct attempt at λ or μ using the fact that $\lambda \overrightarrow{OB} = \overrightarrow{OC} + \mu \overrightarrow{OA}$ E.g. $-16 + 4\mu = 0 \Rightarrow \mu = 4$	dM1	1.1b
$ \overrightarrow{OD} = 5 \times \sqrt{4^2 + 6^2} = 10\sqrt{13}$	A1	1.1b
	(3)	

Notes

(b) Vector approach

M1: Deduces that $\overrightarrow{OD} = \lambda \overrightarrow{OB} = 4\lambda \mathbf{j} + 6\lambda \mathbf{k}$ and attempts $\overrightarrow{CD} = 16\mathbf{i} + (4\lambda - "32")\mathbf{j} + (6\lambda - 10)\mathbf{k}$

dM1: Correct attempt at finding λ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$

E.g.

$$16\mathbf{i} + (4\lambda - "32")\mathbf{j} + (6\lambda - 10)\mathbf{k} = 4\alpha\mathbf{i} - 3\alpha\mathbf{j} + 5\alpha\mathbf{k} \Rightarrow \alpha = 4 \Rightarrow 4\lambda - "32" = -3 \times "4" \Rightarrow \lambda = \dots$$

A1: $|\overrightarrow{OD}| = 10\sqrt{13}$

Alternative:

M1: Deduces that $\overrightarrow{OD} = \lambda \overrightarrow{OB} = 4\lambda \mathbf{j} + 6\lambda \mathbf{k}$ and attempts

$$\overrightarrow{OD} = \overrightarrow{OC} + \mu \overrightarrow{OA} = -16\mathbf{i} + 32\mathbf{j} + 10\mathbf{k} + \mu(4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$$

dM1: Correct attempt at finding λ or μ using the fact that $\lambda \overrightarrow{OB} = \overrightarrow{OC} + \mu \overrightarrow{OA}$

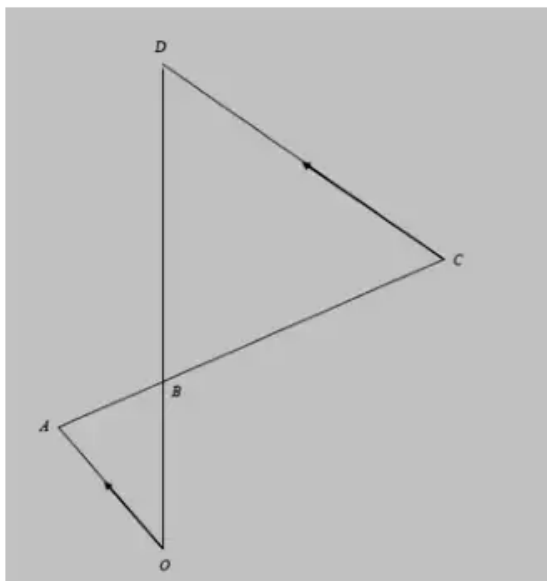
E.g. $(-16 + 4\mu)\mathbf{i} + ("32" - 3\mu)\mathbf{j} + (10 + 5\mu)\mathbf{k} = 4\lambda\mathbf{j} + 6\lambda\mathbf{k} \Rightarrow -16 + 4\mu = 0 \Rightarrow \mu = \dots$

May also solve simultaneously using y and z components to find λ or μ

A1: $|\overrightarrow{OD}| = 10\sqrt{13}$

Note that the correct vector is $20\mathbf{j} + 30\mathbf{k}$

(b) Similar triangle approach



M1: For the key step in recognising that triangle BCD and triangle BAO are similar with a ratio of lengths of 4:1

dM1: States or uses the fact that $|\overrightarrow{OD}| = 5 \times |\overrightarrow{OB}|$

Stating this will score M1 dM1 provided there is no evidence of incorrect work

Note that they may establish this result using the work from (a) but must be used here to score.

A1: $|\overrightarrow{OD}| = 10\sqrt{13}$

5.

Mark scheme (a)

Scheme	Marks	AO
$\overrightarrow{AD} = 10\mathbf{i} + 24\mathbf{j}$ and $\overrightarrow{BC} = 50\mathbf{i} + 120\mathbf{j}$	M1	1.1b
$\overrightarrow{AD} = \frac{1}{5}\overrightarrow{BC}$ therefore AD is parallel to BC *	A1*cs0	2.2a
	(2)	

Notes

M1: Attempts to find both $\overrightarrow{AD} = \pm(10\mathbf{i} + 24\mathbf{j})$ and $\overrightarrow{BC} = \pm(50\mathbf{i} + 120\mathbf{j})$.

May be seen as column vectors.

Condone poor notation with column vectors e.g. $\begin{pmatrix} 10\mathbf{i} \\ 24\mathbf{j} \end{pmatrix}$ or $\frac{10}{24}$ for $\begin{pmatrix} 10 \\ 24 \end{pmatrix}$

This mark can be scored for at least one correct component for each vector if no method is shown.

May be implied if they go straight for ratios (gradients) e.g. $\pm \frac{24 - 0}{22 - 12}$, $\pm \frac{16 - 136}{0 - 50}$, $\pm \frac{0 - 50}{16 - 136}$

Some candidates use distances in an attempt to prove part (a) e.g. finding $10^2 + 24^2$ and $50^2 + 120^2$ in which case the M1 can be implied. Adding vectors scores M0

A1* cso: This mark requires

- correct work showing AD is parallel to BC by showing that e.g. $\overrightarrow{AD} = \pm \frac{1}{5} \overrightarrow{BC}$ or equivalent e.g. $\overrightarrow{BC} = \pm 5 \overrightarrow{AD}$ or e.g. $\overrightarrow{AD} = 2(5\mathbf{i} + 12\mathbf{j})$ $\overrightarrow{BC} = 10(5\mathbf{i} + 12\mathbf{j})$ or e.g. $BC = \pm 5AD$ (i.e. the vector arrows are not required) or by showing the ratio/gradient of the lines through AD and BC are equal
e.g. $\frac{24}{10} = \frac{120}{50}$. Condone e.g. $\frac{50\mathbf{i} + 120\mathbf{j}}{10\mathbf{i} + 24\mathbf{j}} = 5$
- a (minimal) conclusion e.g. $\checkmark$, hence shown, etc. which may be in a preamble e.g. if they are parallel then $AD = kBC$...etc. If using ratios/gradients they need to say that they are parallel.
- vectors correctly calculated but allow e.g. $\overrightarrow{AD} = -10\mathbf{i} - 24\mathbf{j}$ and allow poor column vector notation as above

Using reciprocal gradients for both is acceptable for A1 even if they call them “gradients”.

Do not credit work in part (b) in part (a) unless used in part (a)

Mark scheme (b)

Scheme	Marks	AO
Attempt to find at least two lengths between AB , BC , CD and AD $ \overrightarrow{BC} = \sqrt{50^2 + 120^2} = 130$, $ \overrightarrow{DA} = \sqrt{10^2 + 24^2} = 26$	M1	1.1b
$ \overrightarrow{AB} = \sqrt{12^2 + 16^2} = 20$, $ \overrightarrow{CD} = \sqrt{28^2 + 112^2} = 28\sqrt{17}$ (awrt 115 m)	A1	1.1b
Average speed = $\frac{2(26 + 130 + 20 + 28\sqrt{17})}{5/60} / 1000$	dM1	3.1b
awrt = 6.99 (km/h)	A1	3.2a
	(4)	
	(6 marks)	

Notes

M1: Attempts to use Pythagoras to find **at least two** of the lengths of the quadrilateral.

May be implied by at least 2 correct lengths.

For reference

$$\pm \vec{AB} = \pm(-12\mathbf{i} + 16\mathbf{j}), \pm \vec{BC} = \pm(50\mathbf{i} + 120\mathbf{j}), \pm \vec{CD} = \pm(-28\mathbf{i} - 112\mathbf{j}), \pm \vec{DA} = \pm(10\mathbf{i} + 24\mathbf{j})$$

Allow ft if using their vectors from part (a) provided subtraction was used. Do not be concerned about the signs of individual components but must be using subtraction (but condone $\pm \vec{AB} = \pm(12\mathbf{i} + 16\mathbf{j})$) to find the vectors and then squaring and adding components and then taking the square root.

A1: At least 2 lengths correct: If units are given they must be correct.

$$|\vec{AB}| = \sqrt{12^2 + 16^2} = 20, \quad |\vec{CD}| = \sqrt{28^2 + 112^2} = 28\sqrt{17} \text{ (allow awrt 115 m)}$$

$$|\vec{BC}| = 10\sqrt{5^2 + 12^2} = 130, \quad |\vec{DA}| = 2\sqrt{5^2 + 12^2} = 26$$

Allow if they are working in km e.g. $|\vec{AB}| = 0.02$ etc.

M1A1 is implied by a total distance of awrt 291 (m) or possibly a multiple of this if they are doubling (awrt 583) or e.g. multiplying by 24 (awrt 6990) etc.

dM1: For an attempt at an average speed ignoring any attempt to get the correct units.

They must have attempted all 4 lengths for this mark.

There must be some indication that they have divided by 5 but this may be implied.

Allow this mark if they calculate the average speed for 2 laps or 1 lap e.g.

$$\frac{\text{"291"} \times 2}{5}, \frac{\text{"291"}}{5}, \text{"291"} \times 12, \text{"291"} \times 2 \times 12 \text{ or e.g. if they divide by 2.5 or multiply by 24.}$$

A1: awrt 6.99 (km/h). or anything which truncates to 6.99 e.g. 6.995. Units are **not** required but if they are given they must be correct. Isw once a correct answer is seen.

An exact answer is acceptable for the final A1 in (b): $4.224 + 0.672\sqrt{17}$

Special Case:

Some candidates are misinterpreting/misreading the position vector for B as $16\mathbf{i}$ rather than $16\mathbf{j}$

This is usually implied by their vectors/ratios e.g.

$$\vec{AD} = \pm(10\mathbf{i} + 24\mathbf{j}) \quad \text{and} \quad \vec{BC} = \pm(34\mathbf{i} + 136\mathbf{j})$$

or e.g.

$$\pm \frac{24 - 0}{22 - 12}, \pm \frac{50 - 16}{136 - 0}$$

For part (a), the maximum possible score is **M1A0** with the conditions for the M mark as described in the main scheme.

For part (b) the maximum possible score is **M1A1M1A0** as follows:

M1: Attempts to use Pythagoras to find **at least two** of the lengths of the quadrilateral as defined in the main scheme.

For reference

$$\pm \vec{AB} = \pm 4\mathbf{i}, \pm \vec{BC} = \pm(34\mathbf{i} + 136\mathbf{j}), \pm \vec{CD} = \pm(-28\mathbf{i} - 112\mathbf{j}), \pm \vec{DA} = \pm(10\mathbf{i} + 24\mathbf{j})$$

A1: Correct lengths for AD and CD . If units are given they must be correct.

This may **not** be scored for correct ft lengths for AB or BC

So requires both:

$$|\overrightarrow{CD}| = \sqrt{28^2 + 112^2} = 28\sqrt{17} \text{ (allow awrt 115 m)}$$

$$|\overrightarrow{DA}| = \sqrt{10^2 + 24^2} = 26$$

dM1: As above for an attempt at an average speed ignoring any attempt to get the correct units.

A0: Not available

If the position vector for B is not misinterpreted/misread in part (b) then full marks are available.