

QQQ – Pure Year 2 – Chapter 1 – Algebraic Fractions (Version 2)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. The expression

$$\frac{12x^2 + 3x + 7}{3x - 5}$$

can be written as

$$Ax + B + \frac{C}{3x - 5}$$

State the value of A

Circle your answer.

[1 mark]

- 3
- 4
- 7
- 9

(Total 1 marks)

AQA June 2024 Paper 1 Q3

2. Express $\frac{13 - x}{(x - 3)(x + 2)}$ in partial fractions.

[3]

(Total 3 marks)

OCR MEI June 2022 Paper 1 Q2

3. Simplify each of the following.

(a) $(2a^2)^3 \times \frac{3}{4}a^{-1}$

[2]

(b) $\frac{4x^2 - 9}{(2x^2 + 5x - 12)(2x + 3)}$

[2]

(Total 4 marks)

OCR A June 2024 Paper 3 Q1

4. Express

$$\frac{3x + 5}{x^2 + x - 12} - \frac{2}{x - 3}$$

as a single fraction in its simplest form.

(4)

(Total 4 marks)

Edexcel C3 June 2013 (R) Q1

5. Given that

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \quad x \neq \pm 2$$

find the values of the constants a, b, c, d and e .

(4)

(Total 4 marks)

Edexcel C3 June 2013 Q1

6. Chloe is attempting to write $\frac{2x^2 + x}{(x + 1)(x + 2)^2}$ as partial fractions, with constant numerators.

Her incorrect attempt is shown below.

Step 1	$\frac{2x^2 + x}{(x + 1)(x + 2)^2} \equiv \frac{A}{x + 1} + \frac{B}{(x + 2)^2}$
Step 2	$2x^2 + x \equiv A(x + 2)^2 + B(x + 1)$
Step 3	Let $x = -1 \Rightarrow A = 1$ Let $x = -2 \Rightarrow B = -6$
Answer	$\frac{2x^2 + x}{(x + 1)(x + 2)^2} \equiv \frac{1}{x + 1} - \frac{6}{(x + 2)^2}$

(a) (i) By using a counter example, show that the answer obtained by Chloe cannot be correct.

[2 marks]

(ii) Explain her mistake in Step 1.

[1 mark]

(b) Write $\frac{2x^2 + x}{(x + 1)(x + 2)^2}$ as partial fractions, with constant numerators.

[4 marks]

(Total 7 marks)

AQA June 2020 Paper 1 Q9

Solutions

1.

Scheme	Marks	AO
Circles the 2 nd answer	B1	1.1b
	(1 mark)	

Typical solution

4

2.

Scheme	Marks	AO
$\frac{13 - x}{(x - 3)(x + 2)} = \frac{A}{(x - 3)} + \frac{B}{(x + 2)}$ $13 - x = A(x + 2) + B(x - 3)$	M1	1.1a
$A = 2, B = -3$	A1	1.1b
So $\frac{2}{(x - 3)} - \frac{3}{(x + 2)}$	A1	1.1b
	[3]	

Notes

M1: Clearing the denominators oe

A1: For one correct coefficient

A1: Correct partial fractions seen

Accept just values for A and B if defined

3.

Mark scheme (a)

Scheme	Marks	AO
$6a^5$	B2	1.1
		1.1

	[2]	
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Notes

B2: B1 for 6 **or** a^5 in their final answer **or** for $8a^6 \times \frac{3}{4}a^{-1}$

$\frac{24}{4}a^5$ as a final answer is **B1 B0**

Mark scheme (b)

Scheme	Marks	AO
$\frac{(2x - 3)(2x + 3)}{(2x - 3)(x + 4)(2x + 3)}$	M1	1.1
$\frac{1}{x + 4}$	A1	1.1
	[2]	

Notes

M1: M1 for either $4x^2 - 9 = (2x - 3)(2x + 3)$ **or** $2x^2 + 5x - 12 = (2x - 3)(x + 4)$

A correct answer www implies **M1**

A1: oe, for example, $(x + 4)^{-1}$

Do not ISW if further incorrect simplification occurs e.g. $\frac{1}{x+4} = \frac{1}{x} + \frac{1}{4}$ is **A0**

4.

Scheme	Marks
$x^2 + x - 12 = (x + 4)(x - 3)$	B1
Attempt as a single fraction $\frac{(3x + 5)(x - 3) - 2(x^2 + x - 12)}{(x^2 + x - 12)(x - 3)}$ or $\frac{3x + 5 - 2(x + 4)}{(x + 4)(x - 3)}$	M1
$= \frac{x - 3}{(x + 4)(x - 3)}, = \frac{1}{(x + 4)} \text{ cao}$	A1, A1
	(4 marks)

Notes

B1 For correctly factorising $x^2 + x - 12 = (x + 4)(x - 3)$. It could appear anywhere in their solution

M1 For an attempt to combine two fractions. The denominator must be correct for 'their' fractions.

The terms could be separate but one term must have been modified.

Condone invisible brackets.

Examples of work scoring this mark are;

$$\frac{(3x+5)(x-3)}{(x^2+x-12)(x-3)} - \frac{2(x^2+x-12)}{(x^2+x-12)(x-3)} \quad \text{Two separate terms}$$

$$\frac{3x+5-2x+4}{(x+4)(x-3)} \quad \text{Single term, invisible bracket}$$

$$\frac{(3x+5)}{(x^2+x-12)(x-3)} - \frac{2(x^2+x-12)}{(x^2+x-12)(x-3)} \quad \text{Separate terms, only one numerator modified}$$

A1 Correct un simplified answer $\frac{x-3}{(x+4)(x-3)}$

If $\frac{x^2-6x-9}{(x^2+x-12)(x-3)}$ scored M1 the fraction must be subsequently be reduced to a correct

$$\frac{x-3}{x^2+x-12} \text{ or } \frac{(x-3)(x-3)}{(x+4)(x-3)(x-3)} \text{ to score this mark.}$$

A1 cao $\frac{1}{(x+4)}$

Do Not isw in this question.

The method of partial fractions is perfectly acceptable and can score full marks

$$\underbrace{\frac{3x+5}{(x+4)(x-3)}}_{B1} - \frac{2}{x-3} = \underbrace{\frac{1}{x+4} + \frac{2}{x-3}}_{M1A1} - \frac{2}{x-3} = \underbrace{\frac{1}{x+4}}_{A1}$$

5.

By Division

Scheme	Marks
$ \begin{array}{r} \overline{3x^2-2x+7} \\ x^2(+0x)-4 \overline{3x^4-2x^3-5x^2+(0x)-4} \\ \underline{3x^4+0x^3-12x^2} \\ -2x^3+7x^2+0x \\ \underline{-2x^3+0x^2+8x} \\ 7x^2-8x-4 \\ \underline{7x^2+0x-28} \\ -8x+24 \end{array} $	
$a = 3$	B1

Long division as far as	M1
$ \begin{array}{r} 3x^2 - 2x \dots\dots \\ x^2(+0x) - 4 \overline{) 3x^4 - 2x^3 - 5x^2 + (0x) - 4} \\ \underline{3x^4 + 0x^3 - 12x^2} \\ -2x^3 + \dots\dots\dots \\ \underline{-2x^3 + \dots\dots\dots} \end{array} $	
Two of $b = -2$ $c = 7$ $d = -8$ $e = 24$	A1
All four of $b = -2$ $c = 7$ $d = -8$ $e = 24$	A1
	(4 marks)

Notes

B1 Stating $a = 3$. This can also be scored by the coefficient of x^2 in $3x^2 - 2x + 7$

M1 Using long division by $x^2 - 4$ and getting as far as the 'x' term. The coefficients need not be correct. Award if you see the whole number part as $\dots x^2 + \dots x$ following some working. You may also see this in a table/ grid.

Long division by $(x + 2)$ will not score anything until $(x - 2)$ has been divided into the new quotient. It is very unlikely to score full marks and the mark scheme can be applied.

A1 Achieving two of $b = -2$ $c = 7$ $d = -8$ $e = 24$.

The answers may be embedded within the division sum and can be implied.

A1 Achieving all of $b = -2$ $c = 7$ $d = -8$ and $e = 24$

Accept a correct long division for 3 out of the 4 marks scoring B1M1A1A0

Need to see $a=...$, $b=...$, or the values embedded in the rhs for all 4 marks

Alt 1 By Multiplication

Scheme	Marks
* $3x^4 - 2x^3 - 5x^2 - 4 \equiv (ax^2 + bx + c)(x^2 - 4) + dx + e$	
Compares the x^4 terms $a = 3$	B1
Compares coefficients to obtain a numerical value of one further constant $-2 = b, \quad -5 = -4a + c \Rightarrow c = ..,$	M1
Two of $b = -2$ $c = 7$ $d = -8$ $e = 24$	A1
All four of $b = -2$ $c = 7$ $d = -8$ $e = 24$	A1
	(4 marks)

B1 Stating $a = 3$. This can also be scored for writing $3x^4 = ax^4$

M1 Multiply out expression given to get *. Condone slips only on signs of either expression.
Then compare the coefficient of any term (other than x^4) to obtain a numerical value of one further constant. In reality this means a valid attempt at either b or c

The method may be implied by a correct additional constant to a .

A1 Achieving two of $b = -2$ $c = 7$ $d = -8$ $e = 24$

A1 Achieving all of $b = -2$ $c = 7$ $d = -8$ and $e = 24$

6.

Mark scheme (a)

Scheme	Marks	A0
(i) Deduces an appropriate value for x and substitutes into at least one side of the given identity Any value of $x \neq -2, -1$	M1	2.2a
Shows that LHS $\neq$ RHS and concludes that Chloe's answer must be incorrect Accept $\frac{2x^2 + x}{(x+1)(x+2)^2} \neq \frac{1}{x+1} - \frac{6}{(x+2)^2}$	R1	2.1
	(2)	
(ii) Explains that Chloe should have included an additional term with $x + 2$ in the denominator or Explains that Chloe should have included $(Bx + C)$ as the numerator for $(x + 2)^2$	E1	2.3
	(1)	

Typical solution

(i)

$$\frac{2x^2 + x}{(x+1)(x+2)^2} \equiv \frac{1}{x+1} - \frac{6}{(x+2)^2}$$

Let $x = 0 \Rightarrow LHS = 0$

$$RHS = \frac{1}{1} - \frac{6}{4} = -\frac{1}{2} \neq 0$$

∴ Chloe's answer must be incorrect

(ii)

Chloe should have included $\frac{C}{x+2}$

Mark scheme (b)

Scheme	Marks	AO
Writes an identity of the correct form Condone use of equals signs	M1	1.1a
Uses a suitable method to obtain all three of 'their' constants. For example by substituting or comparing coefficients Only award the M1 if the identity used results from correctly removing fractions from 'their' chosen partial fraction form	M1	3.1a
Obtains any two correct constants If $Bx + C$ is used, then $B = 1$ and $C = -4$	A1	1.1b
Obtains all three correct values for the constant numerators	A1	1.1b
	(4)	
	(7 marks)	

Typical solution

$$\frac{2x^2 + x}{(x+1)(x+2)^2} \equiv \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$2x^2 + x \equiv A(x+2)^2 + B(x+1)(x+2) + C(x+1)$$

$$x = -1 \Rightarrow A = 1$$

$$x = -2 \Rightarrow C = -6$$

$$x^2 : A + B = 2 \Rightarrow B = 1$$

$$\frac{2x^2 + x}{(x+1)(x+2)^2} \equiv \frac{1}{x+1} + \frac{1}{x+2} - \frac{6}{(x+2)^2}$$