

QQQ – Pure Year 1 – Chapter 7 – Polynomials (Version 2)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. Given that $(x - 2)$ is a factor of $2x^3 + kx - 4$, find the value of the constant k . [2]

(Total 2 marks)

OCR A October 2021 Paper 2 Q1

2. Simplify fully $\frac{2x^2 + 9x - 18}{12x^2 - 8x - 15}$ [3 marks]

(Total 3 marks)

AQA June 2024 Paper 2 Q15

3. Kaya is investigating the function

$$f(x) = 2x^3 - 7x^2 - 12x + 45$$

Kaya makes two statements.

Statement 1: $f(3) = 0$

Statement 2: this shows that $(x + 3)$ must be a factor of $f(x)$.

- (a) State, with a reason, whether each of Kaya's statements is correct. [2 marks]

- (b) Fully factorise $f(x)$. [3 marks]

(Total 5 marks)

AQA June 2022 Paper 2 Q5

4. The polynomial $x^3 - 4x^2 + 10x - 21$ is denoted by $f(x)$.

- (a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$. [2]

- (b) The polynomial $f(x)$ can be written as $(x - 3)(x^2 + bx + c)$ where b and c are constants.
Find the values of b and c . [2]

- (c) Show that $x = 3$ is the only real root of the equation $f(x) = 0$. [2]

(Total 6 marks)

OCR MEI June 2024 Paper 1 Q6

5.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 2x^3 - 3ax^2 + bx + 8a$$

where a and b are constants.

Given that $(x - 4)$ is a factor of $f(x)$,

(a) use the factor theorem to show that

$$10a = 32 + b$$

(2)

Given also that $(x - 2)$ is a factor of $f(x)$,

(b) express $f(x)$ in the form

$$f(x) = (2x + k)(x - 4)(x - 2)$$

where k is a constant to be found.

(4)

(c) Hence,

(i) state the number of real roots of the equation $f(x) = 0$

(ii) write down the largest root of the equation $f\left(\frac{1}{3}x\right) = 0$

(2)

(Total 8 marks)

Solutions

1.

Scheme	Marks	AO
$2(2)^3 + 2k - 4 = 0$	M1	1.1
$k = -6$	A1	1.1
	[2]	

Notes

M1: Sets $f(2)$ equal to 0 Or any other **complete** method

2.

Answer	Mark	Comments
$(2x - 3)(x + 6)$ or $(6x + 5)(2x - 3)$	M1	oe factorisation using two brackets eg $2(x - 1.5)(x + 6)$
$(2x - 3)(x + 6)$ and $(6x + 5)(2x - 3)$	M1dep	oe factorisations using two brackets with a common algebraic factor eg $2(x - 1.5)(x + 6)$ and $(x - 1.5)(12x + 10)$
$\frac{x + 6}{6x + 5}$	A1	

Additional guidance

M1 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts

Condone one transcription error for A1 after fully correct answer seen

Accept use of multiplication signs throughout	
Condone missing final bracket in factorisations and ignore irrelevant brackets eg1 $(2x - 3)(x + 6$ eg2 $\frac{(6 + x)}{(6x + 5)}$	M1 M2A1
$(2x + 3)(x + 6)$ and $(6x + 5)(2x + 3)$ Answer $\frac{x + 6}{6x + 5}$	M0M0A0
$\frac{x + 6}{6x + 5}$ followed by further incorrect work eg $\frac{6x}{6x + 5}$	M2A0

3.

Mark scheme (a)

Scheme	Marks	AO
Shows that 3 has been substituted and $f(3) = 0$ and states that Kaya is correct OE	E1	2.4
States that Kaya is incorrect and gives the correct conclusion Kaya should have reached Or States that Kaya is incorrect and that $f(-3) \neq 0$ Or States that Kaya is incorrect and provides evidence that $(x + 3)$ is not a factor of $f(x)$	E1	2.3
	(2)	

Typical solution

$$f(3) = 54 - 63 - 36 + 45 = 0$$

Kaya is correct that $f(3) = 0$

But Kaya's conclusion is wrong

$(x - 3)$ is a factor of $f(x)$

Mark scheme (b)

Scheme	Marks	AO
Divides $f(x)$ by $x - 3$ or $x + 3$ or uses inspection. Must obtain $2x^2$ or 15 or finds another root of $f(x) = 0$	M1	1.1a
Finds correct quadratic factor or both remaining roots of $f(x) = 0$	A1	1.1b
Factorises $f(x)$ correctly ISW	A1	1.1b
	(3)	
	(5 marks)	

Typical solution

$$\begin{aligned}
 &(2x^3 - 7x^2 - 12x + 45) / (x - 3) \\
 &= 2x^2 - x - 15 \\
 &(x - 3)^2(2x + 5)
 \end{aligned}$$

4.

Mark scheme (a)

Scheme	Marks	AO
$f(3) = 3^3 - 4 \times 3^2 + 10 \times 3 - 21$ $= 27 - 36 + 30 - 21 = 0$	M1	2.1
[So by the factor theorem] $(x - 3)$ is a factor	A1	2.2a
	[2]	

Notes

M1: Substitutes $x = 3$ into expression for $f(x)$. Do not allow for algebraic division

A1: Argues from zero clearly seen

Mark scheme (b)

Scheme	Marks	AO
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Algebraic division gives $f(x) = (x - 3)(x^2 - x + 7)$	M1	3.1a
So $b = -1$ and $c = 7$	A1	1.1
	[2]	

Notes

M1: Attempts to divide by $(x - 3)$ as far as linear term. Allow arithmetic slips. Also allow for expanding and equating coefficients.

A1: Allow seen in correct product of factors or correct algebraic division or multiplication grid. Correct factorisation by inspection scores both marks

Mark scheme (c)

Scheme	Marks	AO
$[f(x) = 0 \text{ when } x = 3]$ or when $x^2 - x + 7 = 0$		
Discriminant $(-1)^2 - 4 \times 1 \times 7 = -27 < 0$	M1	2.1
So no additional real roots	A1	2.5
	[2]	

Notes

M1: Finds discriminant. Also allow for equivalent argument using quadratic formula or completing the square.

A1: www Clear argument from their negative discriminant. FT their b and c . Condone missing reference to the fact that $x = 3$ is a root.

(Corrected from the printed mark scheme: the discriminant is printed as $(-1)^2 - 4 \times 3 \times 7 = -27$; the coefficient of x^2 is 1, so it should read $(-1)^2 - 4 \times 1 \times 7 = -27$.)

5.

Mark scheme (a)

Scheme	Marks	AO
$(f(4) =) 2 \times 4^3 - 3a \times 4^2 + 4b + 8a = 0$	M1	1.1b

$128 + 4b = 40a \Rightarrow 32 + b = 10a *$	A1*	1.1b
	(2)	

Notes

M1: Attempts $f(4) = 0$ leading to an equation in a and b only. Condone slips. The $= 0$ may be implied by further work for this mark. Attempts using algebraic division score M0A0.

A1*: Simplifies and rearranges to the given answer with no errors. There must be at least one intermediate stage of working between their first expression or equation and the given answer and $= 0$ must be correctly seen at some point in their solution or at the start e.g. stating $f(4) = 0$. Isw if they achieve the given answer but then attempt to make a or b the subject.

Minimum acceptable is e.g. $128 - 48a + 4b + 8a \Rightarrow 128 - 40a + 4b = 0 \Rightarrow 32 + b = 10a$

Note: $128 - 48a + 4b + 8a \Rightarrow 128 + 4b = 40a \Rightarrow 32 + b = 10a$ is M1A0* (we do not see $= 0$ correctly at some point or e.g. $f(4) = 0$)

Mark scheme (b)

Scheme	Marks	AO
$f(2) = 2 \times 2^3 - 3a \times 2^2 + 2b + 8a = 0 \Rightarrow 8 + b = 2a$	M1	1.1b
Solve simultaneously $\Rightarrow a = \dots$ or $\Rightarrow b = \dots$	dM1	2.1
$a = 3$ or $b = -2$ or $k = 3$	A1	1.1b
$(f(x) =) (2x + 3)(x - 4)(x - 2)$	A1	1.1b
	(4)	

Notes

Note that there are many different equations which can be formed.

Sight of $a = 3$ or $b = -2$ or $k = 3$ scores the first 3 marks BUT answers with no working – send to review

M1: Attempts $f(2) = 0$ to form another equation in a and b . Does not need to be simplified. Condone slips substituting in 2 (but not if there is a clear intention to substitute in -2) and $= 0$ may be implied by further work. Accept attempts to divide algebraically by $x - 2$: e.g. condone slips but they must have a quadratic quotient $2x^2 \pm (\dots a \pm \dots)x$ and proceed as far as a remainder of the form $\dots a \pm \dots b \pm \dots$ which is then set equal to 0.

e.g.

$$\begin{array}{r}
 x - 2 \quad \frac{2x^2 + (-3a + 4)x + (b - 6a + 8)}{2x^3 - 3ax^2 + bx + 8a} \\
 \underline{2x^3 - 4x^2} \\
 (-3a + 4)x^2 + bx \\
 \underline{(-3a + 4)x^2 + (6a - 8)x} \\
 (b - 6a + 8)x + 8a \\
 \underline{(b - 6a + 8)x - 2b + 12a - 16} \\
 -4a + 2b + 16 = 0
 \end{array}$$

You may also see an attempt at the grid method:

	$2x^2$	$+(4 - 3a)x$	$+(8 - 6a + b)$
x	$2x^3$	$+(4 - 3a)x^2$	$+(8 - 6a + b)x$
-2	$-4x^2$	$(-8 + 6a)x$	$-16 + 12a - 2b$

$$-16 + 12a - 2b = 8a$$

In this method condone slips but they need to proceed as far as a quadratic quotient of the form $2x^2 \pm (\dots a \pm \dots)x$ and the bottom right cell of the form $\dots a \pm \dots b \pm \dots$ which is then set equal to $8a$.

Look out for other valid methods such as

- multiplying out $f(x) = (2x + k)(x - 4)(x - 2)$ to achieve a cubic where the coefficients of x^2 and x , and the constant term are all in terms of k , and equating coefficients to form two more simultaneous equations
- algebraically dividing $2x^3 - 3ax^2 + bx + 8a$ by $x^2 - 6x + 8$ and setting their remainder equal to zero. Look for a quotient of the form $2x \pm \dots a \pm \dots$
- substituting $b = 10a - 32$ into $f(x)$ so that the polynomial coefficients are in terms of a only (or equivalently all in b only) and using $f(2) = 0$ to form an equation in one variable. Send to review if you are unsure.

dM1: Attempts to solve their equations simultaneously to find a value for a or b (or they may proceed directly to finding a value for k). This may be done on a calculator. You do not need to check their method for solving. It is dependent on the previous method mark.

A1: $a = 3$ or $b = -2$ or $k = 3$

A1: $(f(x) =) (2x + 3)(x - 4)(x - 2)$ (All on one line) (Stating the values of a , b and k alone does not score this mark). Allow to be scored if seen in (c).

Alternative Further Maths method using the sum and product of the roots

M1: $\alpha + \beta + \gamma = \frac{3a}{2} \Rightarrow 4 + 2 - \frac{k}{2} = \frac{3a}{2}$ or $\alpha\beta\gamma = -\frac{8a}{2} \Rightarrow 4 \times 2 \times \left(-\frac{k}{2}\right) = -\frac{8a}{2}$

$$\mathbf{A1:} \alpha + \beta + \gamma = \frac{3a}{2} \Rightarrow 4 + 2 - \frac{k}{2} = \frac{3a}{2} \text{ and } \alpha\beta\gamma = -\frac{8a}{2} \Rightarrow 4 \times 2 \times \left(-\frac{k}{2}\right) = -\frac{8a}{2}$$

$$\text{(Less likely but possible to see } \alpha\beta + \beta\gamma + \alpha\gamma = \frac{b}{2} \Rightarrow 4 \times 2 + 2 \times \left(-\frac{k}{2}\right) + 4 \times \left(-\frac{k}{2}\right) = \frac{b}{2}\text{)}$$

dM1A1A1: See notes above

Mark scheme (c)

Scheme	Marks	AO
(i) 3	B1	1.1b
(ii) 12	B1ft	2.2a
	(2)	
	(8 marks)	

Notes

(i) **B1:** 3 (listing the actual roots only is B0)

(ii) **B1ft:** 12 only Follow through on their $2x + k \Rightarrow -\frac{3k}{2}$ if $k < -8$