

QQQ – Pure Year 1 – Chapter 7 – Polynomials (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. A student is looking for factors of the polynomial $f(x)$

They suggest that $(x - 2)$ is a factor of $f(x)$

The method they use to check this suggestion is to calculate $f(-2)$

They correctly calculate that $f(-2) = 0$

They conclude that their suggestion is correct.

(a) Make **one** comment about the student's **method**. [1 mark]

(b) Make **two** comments about the student's **conclusion**. [2 marks]

(Total 3 marks)

AQA June 2024 Paper 1 Q5

2. Simplify fully $\frac{2x^3 + x^2 - 7x - 6}{x^2 - x - 2}$. [4]

(Total 4 marks)

OCR A October 2020 Paper 2 Q2

3. Show that the expression $3x^3 + x^2 - 6x - 5$ can be written in the form $(x + 2)(ax^2 + bx + c) + d$ where a, b, c and d are constants to be determined. [5]

(Total 5 marks)

OCR MEI June 2023 Paper 1 Q6

4. It is given that $(x + 1)$ and $(x - 3)$ are two factors of $f(x)$, where

$$f(x) = px^3 - 3x^2 - 8x + q$$

(a) Find the values of p and q . [3 marks]

(b) Fully factorise $f(x)$. [2 marks]

(Total 5 marks)

AQA June 2019 Paper 1 Q3

5.

$$f(x) = 2x^3 + 5x^2 + 2x + 15$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$. **(2)**

(b) Find the constants a , b and c such that

$$f(x) = (x + 3)(ax^2 + bx + c)$$

(2)

(c) Hence show that $f(x) = 0$ has only one real root. **(2)**

(d) Write down the real root of the equation $f(x - 5) = 0$ **(1)**

(Total 7 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
States the student should have calculated $f(2)$ or States the student should not have calculated $f(-2)$	E1	2.3
	(1)	

Typical solution

The student should have calculated $f(2)$

Mark scheme (b)

Scheme	Marks	AO
Infers that $(x - 2)$ may or may not be a factor Do not accept definitive conclusion that $(x - 2)$ is not a factor	E1	2.2b
Deduces that $(x + 2)$ is a factor	E1	2.2a
	(2)	
	(3 marks)	

Typical solution

They do not know whether $(x - 2)$ is a factor or not.

They could conclude that $(x + 2)$ is a factor.

2.

Scheme	Marks	AO
Numerator $\equiv (x + 1)(x - 2)(2x + 3)$	M1	3.1a
Denominator $\equiv (x + 1)(x - 2)$	M1	1.1
	M1	1.1

Ans: $2x + 3$	A1	1.1
	[4]	

Notes

M1: Attempt factorise numerator into 3 linear factors

M1: Attempt factorise denominator into 2 linear factors

M1: “cancel” two common factors in num & denom

A1: Allow no mention of $x \neq -1$ or $x \neq 2$ conditions.

NB correct answer with no working or partial working: 4 marks

SC: Answer $x + \frac{3}{2}$ **B3**

Alternative

Scheme	Marks
$ \begin{array}{r} 2x + 3 \\ x^2 - x - 2 \overline{) 2x^3 + x^2 - 7x - 6} \\ \underline{2x^3 - 2x^2 - 4x} \\ 3x^2 - 3x - 6 \\ \underline{3x^2 - 3x - 6} \\ - \end{array} $ Attempt long division by $x^2 - x - 2$ or by $x + 1$ or by $x - 2$	M1
Obtain “ $2x$ ” in quotient	A1
Obtain “ $+ 3$ ” in quotient	A1
$2x + 3$ Answer $2x + 3$ clear (not just in the division sum)	A1

3.

Scheme	Marks	AO
$a = 3$	B1	1.1b
Attempt to divide the cubic by $(x + 2)$	M1	1.1a
$b = -5$	A1	1.1b
$c = 4$	A1	1.1b
$d = -13$	B1	1.1b
[So $f(x) = (x + 2)(3x^2 - 5x + 4) - 13$]		

Notes

M1: Allow grid method or long division as far as the linear term of the quotient

A1: (*b*) May be embedded in final expression

A1: (*c*) May be embedded in final expression FT their *b*

B1: (*d*) May also be found using the remainder theorem. May be embedded in final expression

[So $f(x) = \dots$] Need not be given explicitly if all coefficients seen

By inspection

Scheme	Marks	AO
$f(x) = (x + 2)(3x^2 - 5x + 4) - 13$	B1 M1 A1 A1 B1	

B1: B1 for *a* embedded in their expression even if incomplete

M1: Method may be implied by correct *b*, or correct FT for *c*

A1: $b = -5$

A1: $c = 4$ FT their *b* with $c = -6 - 2b$

B1: May also be found using the remainder theorem. May be embedded in their expression

Alternative method

Scheme	Marks	AO
$a = 3$	B1	
expanding and equating coefficients quadratic term $2a + b = 1$ linear term $2b + c = -6$ constant term $2c + d = -5$	M1	
$b = -5$	A1	
$c = 4$	A1	
$d = -13$	B1	
So $f(x) = (x + 2)(3x^2 - 5x + 4) - 13$		

M1: expanding and equating coefficients for at least the quadratic or linear term

A1: (b) May be embedded in final expression

A1: (c) May be embedded in final expression FT their b

B1: (d) May also be found using the remainder theorem. May be embedded in final expression

So $f(x) = \dots$: Need not be given explicitly if all coefficients seen

4.

Mark scheme (a)

Scheme	Marks	AO
Substitutes $x = -1$ or $x = 3$ into $f(x)$ to obtain one equation or uses identity eg $f(x) \equiv (x + 1)(x - 3)(ax + b)$	M1	3.1a
Obtains two correct equations by substitution method ACF or obtains $a = 2, b = 1$	A1	1.1b
Solves to find p and q CAO	A1	1.1b
	(3)	

Typical solution

$$x = -1 \quad -p - 3 + 8 + q = 0$$

$$x = 3 \quad 27p - 27 - 24 + q = 0$$

$$p = 2 \text{ and } q = -3$$

Mark scheme (b)

Scheme	Marks	AO
Uses inspection, division by quadratic factor or repeated division or finds third root $x = -\frac{1}{2}$ PI by $(x + \frac{1}{2})$	M1	1.1a
Completes factorisation	A1	1.1b
	(2)	
	(5 marks)	

Typical solution

$$(x + 1)(x - 3) = x^2 - 2x - 3$$

$$(x^2 - 2x - 3)(2x + 1)$$

$$(x + 1)(x - 3)(2x + 1)$$

5.

Mark scheme (a)

Scheme	Marks	AO
$f(-3) = 2(-3)^3 + 5(-3)^2 + 2(-3) + 15$ $= -54 + 45 - 6 + 15$	M1	1.1b
$f(-3) = 0 \Rightarrow (x + 3) \text{ is a factor}$	A1	2.4
	(2)	

Notes

M1: Attempts $f(-3)$. Attempted division by $(x + 3)$ or $f(3)$ is M0

Look for evidence of embedded values or two correct terms of $f(-3) = -54 + 45 - 6 + 15 = \dots$

A1: Achieves and states $f(-3) = 0$, and makes a suitable conclusion. Sight of $f(x) = 0$ when $x = -3$ is also acceptable.

It must follow M1. Accept, for example, $f(-3) = 0 \Rightarrow (x + 3) \text{ is a factor}$

This may be seen in a preamble before finding $f(-3) = 0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

Mark scheme (b)

Scheme	Marks	AO
At least 2 of: $a = 2, b = -1, c = 5$	M1	1.1b
All of: $a = 2, b = -1, c = 5$	A1	1.1b
	(2)	

Notes

M1: Correct method implied by values for at least 2 correct constants. Allow embedded in their $f(x)$ or within their working if they use algebraic division/other methods which may be seen in part (a) and used in part (b).

A1: All values correct. Allow embedded in their $f(x)$ or seen as the quotient from algebraic division. Isw incorrectly stated values of a b and c following a correct quadratic expression seen.

$$\begin{array}{r}
 2x^2 - x + 5 \\
 x + 3 \overline{) 2x^3 + 5x^2 + 2x + 15} \\
 \underline{2x^3 + 6x^2} \\
 -x^2 + 2x \\
 \underline{-x^2 - 3x} \quad \text{scores M1A1} \\
 5x + 15 \\
 \underline{5x + 15} \\
 0
 \end{array}$$

Mark scheme (c)

Scheme	Marks	AO
$b^2 - 4ac = (-1)^2 - 4(2)(5)$	M1	2.1
$b^2 - 4ac = -39$ which is < 0 so the quadratic has no real roots so $f(x) = 0$ has only 1 real root	A1	2.4
	(2)	

Notes

M1: Either:

- considers the discriminant using their a , b and c (does not need to be evaluated)
 $(b^2 - 4ac =) (-1)^2 - 4(2)(5)$ (the $(-1)^2$ may appear as 1^2 and condone missing brackets for this mark for -1^2). Discriminant $= -39$ is sufficient for M1
- attempts to complete the square so score for $2 \left(x \pm \frac{1}{4} \right)^2 + \dots$
- attempts to find the roots of the quadratic using the formula. The values embedded in the formula score this mark.
 $\frac{1 \pm \sqrt{(-1)^2 - 4 \times 2 \times 5}}{2 \times 2}$ (the $(-1)^2$ may appear as 1^2 and condone missing brackets for this mark for -1^2)
- Sketches a graph of the quadratic. It must be a U shaped quadratic which does not cross the x -axis.

A1: Provides a correct explanation from correct working. They must

- Have a correct calculation

- Explanation that the quadratic has no (real) roots
- Minimal conclusion stating that $f(x) = 0$ has only one root

eg $b^2 - 4ac = -39 < 0$ so only one root is M1A0 (needs to explain the quadratic has no real roots)

eg $2\left(x - \frac{1}{4}\right)^2 + \frac{39}{8} > 0$ **so no real roots** (for the quadratic) **so** ($f(x)$ has) **only one** (real) **root** is M1A1

The value of the discriminant, completed square form $2\left(x - \frac{1}{4}\right)^2 + \frac{39}{8}$ or roots of the quadratic

$\left(= \frac{1 \pm \sqrt{39}i}{4}\right)$ must be correct.

If they sketch the quadratic graph it must be a U shaped quadratic which crosses the y -axis at 5 and has a minimum in the 1st quadrant. They must explain that the graph does not cross the x -axis so no real roots for the quadratic so only one root for $f(x) = 0$.

Mark scheme (d)

Scheme	Marks	AO
$(x =) 2$	B1	2.2a
	(1)	
	(7 marks)	

Notes

B1: 2 condone (2, 0)