

QQQ – Pure Year 1 – Chapter 1 – Surds (Version 2)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. Write $\frac{8}{3 - \sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers to be found. [2]

(Total 2 marks)

OCR MEI June 2018 Paper 1 Q1

- 2.
- (a) Express $\sqrt{108}$ in the form $a\sqrt{3}$, where a is an integer. (1)
- (b) Express $(2 - \sqrt{3})^2$ in the form $b + c\sqrt{3}$, where b and c are integers to be found. (3)

(Total 4 marks)

Edexcel C1 January 2007 Q2

3. Express $\frac{\sqrt{3} + 3\sqrt{5}}{\sqrt{5} - \sqrt{3}}$ in the form $a + \sqrt{b}$, where a and b are integers.

Fully justify your answer.

[4 marks]

(Total 4 marks)

AQA June 2024 Paper 1 Q3

4. Solve the equation

$$10 + x\sqrt{8} = \frac{6x}{\sqrt{2}}$$

Give your answer in the form $a\sqrt{b}$ where a and b are integers. (4)

(Total 4 marks)

Edexcel C1 June 2014 (R) Q5

5. The expression

$$\frac{3 - \sqrt{n}}{2 + \sqrt{n}}$$

can be written in the form $a + b\sqrt{n}$, where a and b and n are rational but $\sqrt{n}$ is irrational.

Find expressions for a and b in terms of n .

[4 marks]

(Total 4 marks)

AQA June 2022 Paper 2 Q7

6.

(a) Write $\sqrt{80}$ in the form $c\sqrt{5}$, where c is a positive constant. **(1)**

A rectangle R has a length of $(1 + \sqrt{5})$ cm and an area of $\sqrt{80}$ cm².

(b) Calculate the width of R in cm. Express your answer in the form $p + q\sqrt{5}$, where p and q are integers to be found. **(4)**

(Total 5 marks)

Edexcel C1 June 2014 Q6

Solutions

1.

Scheme	Marks	AO
$\frac{8}{(3 - \sqrt{5})} \times \frac{(3 + \sqrt{5})}{(3 + \sqrt{5})} = 6 + 2\sqrt{5}$	M1 A1	1.1a 1.1b
	[2]	

Notes

M1: Attempt to rationalize the denominator

A1: Must be in correct notation

Allow full credit for correct answer

2.

Mark scheme (a)

Scheme	Marks
$6\sqrt{3}$ ($a = 6$)	B1
	(1)

Notes

$\pm 6\sqrt{3}$ also scores B1.

Mark scheme (b)

Scheme	Marks
Expanding $(2 - \sqrt{3})^2$ to get 3 or 4 separate terms	M1
$7, -4\sqrt{3}$ ($b = 7, c = -4$)	A1, A1
	(3)
	(4 marks)

Notes

M1: The 3 or 4 terms may be wrong.

1st A1 for 7, 2nd A1 for $-4\sqrt{3}$.

Correct answer $7 - 4\sqrt{3}$ with no working scores all 3 marks.

$7 + 4\sqrt{3}$ with or without working scores M1 A1 A0.

Other wrong answers with no working score no marks.

3.

Scheme	Marks	AO
Multiplies numerator and denominator by conjugate of the denominator Condone missing brackets	B1	1.1b
Obtains $18 + 4\sqrt{15}$	B1	1.1b
Obtains denominator of 2	B1	1.1b
Obtains $9 + \sqrt{60}$ Accept $a = 9, b = 60$	B1	1.1b
	(4 marks)	

Typical solution

$$\begin{aligned} & \frac{\sqrt{3} + 3\sqrt{5}}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} \\ &= \frac{3 \times 5 + 3 + \sqrt{15} + 3\sqrt{15}}{5 - 3} \\ &= \frac{18 + 4\sqrt{15}}{2} \\ &= 9 + 2\sqrt{15} \\ &= 9 + \sqrt{60} \end{aligned}$$

4.

Method 1

Scheme	Marks
$x\sqrt{8} + 10 = \frac{6x}{\sqrt{2}}$	

$\times \sqrt{2} \Rightarrow x\sqrt{16} + 10\sqrt{2} = 6x$	M1,A1
$4x + 10\sqrt{2} = 6x \Rightarrow 2x = 10\sqrt{2}$ $x = 5\sqrt{2}$ or $a = 5$ and $b = 2$	M1A1
	(4)

Notes

M1: For multiplying both sides by $\sqrt{2}$ – allow a slip e.g. $\sqrt{2}x\sqrt{8} + 10 = \frac{6x}{\sqrt{2}} \times \sqrt{2}$ or

$\sqrt{2} \times 10 + x\sqrt{8} = \frac{6x}{\sqrt{2}} \times \sqrt{2}$, where one term has an error or the correct

$\sqrt{2}(10 + x\sqrt{8}) = \frac{6x}{\sqrt{2}} \times \sqrt{2}$

NB $x\sqrt{8} + 10 = 6x\sqrt{2}$ is M0

A1: A correct equation in x with no fractional terms. Eg $x\sqrt{16} + 10\sqrt{2} = 6x$ oe.

M1: An attempt to solve their linear equation in x to produce an answer of the form $a\sqrt{2}$ or $a\sqrt{50}$

A1: $5\sqrt{2}$ oe (accept $1\sqrt{50}$)

Method 2

Scheme	Marks
$x\sqrt{8} + 10 = \frac{6x}{\sqrt{2}}$	
$2\sqrt{2}x + 10 = 3\sqrt{2}x$	M1A1
$\sqrt{2}x = 10 \Rightarrow x = \frac{10}{\sqrt{2}} = \frac{10\sqrt{2}}{2}, = 5\sqrt{2}$ oe	M1,A1
	(4)

M1: For writing $\sqrt{8}$ as $2\sqrt{2}$ or $\frac{6}{\sqrt{2}}$ as $3\sqrt{2}$

A1: A correct equation in x with no fractional terms. Eg $2\sqrt{2}x + 10 = 3\sqrt{2}x$ or $x\sqrt{8} + 10 = 3\sqrt{2}x$ oe.

M1: An attempt to solve their linear equation in x to produce an answer of the form $a\sqrt{2}$ or $a\sqrt{50}$

$\sqrt{2}x = 10 \Rightarrow x = \frac{10}{\sqrt{2}} = \frac{10\sqrt{2}}{2}, = 5\sqrt{2}$

or $\sqrt{2}x = 10 \Rightarrow 2x^2 = 100 \Rightarrow x^2 = 50 \Rightarrow x = \sqrt{50}$ or $5\sqrt{2}$

A1: $5\sqrt{2}$ oe Accept $1\sqrt{50}$

Scheme	Marks	AO
Recalls that correct step is to multiply top and bottom by $2 - \sqrt{n}$ PI by subsequent work	M1	1.2
Multiplies numerator and denominator by $(2 - \sqrt{n})$ to get correct terms (condone sign errors) Does not need to be simplified PI by correct simplification	M1	1.1a
Obtains correct simplified numerator and denominator not necessarily in a fraction	A1	1.1b
States correct expressions for a and b Or gives expression with a and b correctly identified	A1	1.1b
	(4 marks)	

Typical solution

$$\frac{3 - \sqrt{n}}{2 + \sqrt{n}} \times \frac{2 - \sqrt{n}}{2 - \sqrt{n}}$$

$$\frac{6 - 3\sqrt{n} - 2\sqrt{n} + n}{4 + 2\sqrt{n} - 2\sqrt{n} - n}$$

$$\frac{6 + n - 5\sqrt{n}}{4 - n}$$

$$a = \frac{6 + n}{4 - n}$$

$$b = \frac{-5}{4 - n}$$

6.

Mark scheme (a)

Scheme	Marks
$80 = 5 \times 16$ $\sqrt{80} = 4\sqrt{5}$	B1
	(1)

Notes

B1: Accept $4\sqrt{5}$ or $c = 4$ – no working necessary

Mark scheme (b)

Method 1

Scheme	Marks
$\frac{\sqrt{80}}{\sqrt{5}+1}$ or $\frac{c\sqrt{5}}{\sqrt{5}+1}$	B1ft
$= \frac{\sqrt{80}}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1}$ or $\frac{\sqrt{80}}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}}$	M1
$= \frac{20-4\sqrt{5}}{4}$ or $\frac{4\sqrt{5}-20}{-4}$	A1
$= 5 - \sqrt{5}$	Alcao
	(4)
	(5 marks)

Notes

B1ft: Only ft on c See $\frac{\sqrt{80}}{\sqrt{5}+1}$ or $\frac{c\sqrt{5}}{\sqrt{5}+1}$

M1: State intention to multiply by $\sqrt{5}-1$ or $1-\sqrt{5}$ in the numerator **and** the denominator

A1: Obtain denominator of 4 (for $\sqrt{5}-1$) **or** -4 (for $1-\sqrt{5}$) **or** correct simplified numerator of $20-4\sqrt{5}$ or $4(5-\sqrt{5})$ **or** $4\sqrt{5}-20$ or $4(\sqrt{5}-5)$ **So either numerator or denominator must be correct**

A1: Correct answer only. Both **numerator and denominator must have been correct and division of numerator and denominator by 4 has been performed.**

Accept $p = 5$, $q = -1$ or accept $5 - \sqrt{5}$ or $-\sqrt{5} + 5$ Also accept $5 - 1\sqrt{5}$

Method 2

Scheme	Marks
$(p+q\sqrt{5})(\sqrt{5}+1) = \sqrt{80}$	B1ft
$p\sqrt{5} + q\sqrt{5} + p + 5q = 4\sqrt{5}$	M1
$p + 5q = 0$ $p + q = 4$	A1

$$p = 5, q = -1$$

Alcao

B1ft: Only fit on $c(p + q\sqrt{5})(\sqrt{5} + 1) = \sqrt{80}$ or $c\sqrt{5}$

M1: Multiply out the lhs and replace $\sqrt{80}$ by $c\sqrt{5}$

A1: Compare rational and irrational parts to give $p + q = 4$, **and** $p + 5q = 0$

A1: Solve equations to give $p = 5, q = -1$

Common error:

$$\frac{\sqrt{80}}{1 + \sqrt{5}} \times \frac{1 - \sqrt{5}}{1 - \sqrt{5}} = \frac{4\sqrt{5} - 20}{4} = \sqrt{5} - 5 \text{ gets B1 M1 A1 (for correct numerator - denominator is}$$

wrong for their product) then A0

Correct answer with no working – send to review – have they used a calculator?

Correct answer after trial and improvement with evidence that $(5 - \sqrt{5})(\sqrt{5} + 1) = \sqrt{80}$ could earn all four marks