

QQQ – Pure Year 1 – Chapter 1 – Surds (Version 1)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. Write

$$\sqrt{(75)} - \sqrt{(27)}$$

in the form $k\sqrt{x}$, where k and x are integers.

(2)

(Total 2 marks)

Edexcel C1 June 2010 Q1

2. In this question you must show detailed reasoning.

Express $\frac{1 + 4\sqrt{3}}{2 + \sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are integers to be determined.

[3]

(Total 3 marks)

OCR MEI June 2024 Paper 2 Q4

3.

(i) Express

$$(5 - \sqrt{8})(1 + \sqrt{2})$$

in the form $a + b\sqrt{2}$, where a and b are integers.

(3)

(ii) Express

$$\sqrt{80} + \frac{30}{\sqrt{5}}$$

in the form $c\sqrt{5}$, where c is an integer.

(3)

(Total 6 marks)

Edexcel C1 January 2013 Q3

4. In this question you must show detailed reasoning.

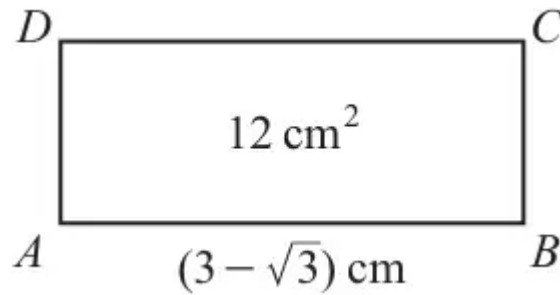
Solve the equation $x\sqrt{5} + 32 = x\sqrt{45} + 2x$. Give your answer in the form $a\sqrt{5} + b$, where a and b are integers to be determined.

[4]

(Total 4 marks)

OCR A June 2023 Paper 2 Q2

5. In this question you must show detailed reasoning.



The diagram shows the rectangle $ABCD$ where $AB = (3 - \sqrt{3}) \text{ cm}$.

The area of $ABCD$ is 12 cm^2 .

Find the perimeter of $ABCD$. Give your answer in the form $(a + b\sqrt{3}) \text{ cm}$, where a and b are integers to be determined. [4]

(Total 4 marks)

OCR A June 2025 Paper 2 Q2

6.

(a) Show that

$$\frac{1}{\sqrt{k} + \sqrt{k+1}} \equiv \sqrt{k+1} - \sqrt{k} \quad k > 0$$

[2 marks]

(b) Using the result from part (a) find the exact value of

$$\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}}$$

Fully justify your answer.

[2 marks]

(Total 4 marks)

AQA June 2025 Paper 2 Q10

Solutions

1.

Scheme	Marks
$(\sqrt{75} - \sqrt{27}) = 5\sqrt{3} - 3\sqrt{3}$	M1
$= 2\sqrt{3}$	A1
	(2 marks)

Notes

M1: for $5\sqrt{3}$ from $\sqrt{75}$ or $3\sqrt{3}$ from $\sqrt{27}$ seen anywhere

A1: for $2\sqrt{3}$ or $k = 2, x = 3$; allow $\sqrt{12}$ or allow $k = 1, x = 12$

Some Common errors

$\sqrt{75} - \sqrt{27} = \sqrt{48}$ leading to $4\sqrt{3}$ is MOAO

$25\sqrt{3} - 9\sqrt{3} = 16\sqrt{3}$ is MOAO

2.

Scheme	Marks	AO
attempt to multiply out $\frac{1 + 4\sqrt{3}}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = \dots$	M1	1.1a
$\frac{2 - \sqrt{3} + 8\sqrt{3} - 12}{2^2 - \sqrt{3}^2}$ or better with attempt at numerator	A1	1.1
$-10 + 7\sqrt{3}$	A1	1.1
	[3]	

Notes

M1: Multiplying by the correct conjugate, **and** making a valid attempt at the resulting multiplication on **either** numerator **or** denominator – condoning sign/bracket errors only

A1: A valid attempt at multiplication on the numerator must be made to score this mark (condoning one sign or coeff. error only)

If they write down $(1 + 4\sqrt{3})(2 - \sqrt{3}) = -10 + 7\sqrt{3}$ and $(2 + \sqrt{3})(2 - \sqrt{3}) = 1$ then MOAOAO as detailed reasoning required.

A1: Correct result from correct work. At least one correct intermediate line of working required before the final answer.

3.

Mark scheme (i)

Scheme	Marks
$(5 - \sqrt{8})(1 + \sqrt{2})$	
$= 5 + 5\sqrt{2} - \sqrt{8} - 4$	M1
$= 5 + 5\sqrt{2} - 2\sqrt{2} - 4$ $\sqrt{8} = 2\sqrt{2}$, seen or implied at any point.	B1
$= 1 + 3\sqrt{2}$ $1 + 3\sqrt{2}$ or $a = 1$ and $b = 3$.	A1
	(3)

Notes

M1: Multiplies out brackets correctly giving four correct terms or simplifying to correct expansion. (This may be implied by correct answer) – can appear as table

B1: $\sqrt{8} = 2\sqrt{2}$, seen or implied at any point

A1: Fully and correctly simplified to $1 + 3\sqrt{2}$ or $a = 1$ and $b = 3$.

Alternative for (i)

Scheme	Marks
$(5 - 2\sqrt{2})(1 + \sqrt{2})$ This earns the B1 mark.	
$= 5 + 5\sqrt{2} - 2\sqrt{2} - 2\sqrt{2}\sqrt{2}$ Multiplies out correctly with $2\sqrt{2}$. This may be seen or implied and may be simplified e.g. $= 5 + 3\sqrt{2} - 2\sqrt{4}$ o.e.	M1
For earlier use of $2\sqrt{2}$	B1
$= 1 + 3\sqrt{2}$ $1 + 3\sqrt{2}$ or $a = 1$ and $b = 3$.	A1 (3)

Mark scheme (ii)

Method 1

Scheme	Marks
Either $\sqrt{80} + \frac{30}{\sqrt{5}} \left(\frac{\sqrt{5}}{\sqrt{5}} \right)$	M1

$= 4\sqrt{5} + \dots$	B1
$= 4\sqrt{5} + 6\sqrt{5}$ $= 10\sqrt{5}$	A1
	(3)
	(6 marks)

Notes

M1: Rationalises denominator i.e. Multiplies $\left(\frac{k}{\sqrt{5}}\right)$ by $\left(\frac{\sqrt{5}}{\sqrt{5}}\right)$ or $\left(\frac{-\sqrt{5}}{-\sqrt{5}}\right)$, seen or implied or uses

Method 3 or similar e.g. $\left(\frac{30}{\sqrt{5}}\right) = \frac{6 \times 5}{\sqrt{5}} = 6\sqrt{5}$

B1: (Independent mark) States $\sqrt{80} = 4\sqrt{5}$ Or either $\sqrt{400} = 20$ or $\sqrt{80}\sqrt{5} = 20$ at any point if they use Method 2.

A1: $10\sqrt{5}$ or $c = 10$.

Method 2

Scheme	Marks
Or $\left(\frac{\sqrt{400} + 30}{\sqrt{5}}\right) \frac{\sqrt{5}}{\sqrt{5}}$	M1
$= \left(\frac{20 + \dots}{\dots}\right) \frac{\dots}{\dots}$	B1
$= \left(\frac{50\sqrt{5}}{5}\right)$ $= 10\sqrt{5}$	A1

Method 3

Scheme	Marks
$\sqrt{80} + \frac{\sqrt{900}}{\sqrt{5}} = \sqrt{80} + \sqrt{180}$	M1
$= 4\sqrt{5} + \dots$	B1
$= 4\sqrt{5} + 6\sqrt{5}$	A1

N.B There are other methods e.g. $\sqrt{80} = \frac{20}{\sqrt{5}}$ (B1) then add $\frac{20}{\sqrt{5}} + \frac{30}{\sqrt{5}} = \frac{50}{\sqrt{5}}$ then M1 A1 as before

Those who multiply initial expression by $\sqrt{5}$ to obtain $\sqrt{400} + 30 = 20 + 30 = 50$ earn M0 B1 A0

4.

Scheme	Marks	AO
DR $x\sqrt{5} + 32 = x\sqrt{45} + 2x$ $x(\sqrt{45} + 2 - \sqrt{5}) = 32$	M1*	1.1
$\sqrt{45} = 3\sqrt{5}$	B1	1.1
$x = \left(\frac{32}{2\sqrt{5} + 2} \right) \left(\frac{2\sqrt{5} - 2}{2\sqrt{5} - 2} \right)$	M1dep*	1.1a
$x = \frac{32(2\sqrt{5} - 2)}{20 - 4} = 4\sqrt{5} - 4$	A1	1.1
	[4]	

Notes

M1*: Re-arranging and factorising out x

B1: Replacing $\sqrt{45} = 3\sqrt{5}$

(Or $\sqrt{45} \times \sqrt{5} = 15$ if multiplying through by $\sqrt{5}$)

Could appear at any point

M1dep*: Correct method for rationalising the surd of the denominator with x taking the form

$$\frac{k_1}{k_2\sqrt{5} + k_3} \text{ o.e.}$$

A1: cao where $a = 4, b = -4$

Need to see some correct working

Alternative scheme

Scheme	Marks
$\sqrt{45} = 3\sqrt{5}$	B1
$(32 - 2x)^2 = (2\sqrt{5}x)^2 \Rightarrow 16x^2 + 128x - 1024 (= 0)$	M1*
	M1dep*
$x = 4\sqrt{5} - 4$ only	A1

B1: Replacing $\sqrt{45} = 3\sqrt{5}$

(Or $\sqrt{45} \times \sqrt{5} = 15$ if multiplying through by $\sqrt{5}$)

Could appear at any point

M1*: Rearranging and squaring leading to a 3TQ

M1dep*: Solve by completing the square or using quadratic formula

5.

Scheme	Marks	AO
DR $(BC =) \frac{12}{3 - \sqrt{3}}$	B1	3.1a
$\frac{12}{3 - \sqrt{3}} \times \frac{3 + \sqrt{3}}{3 + \sqrt{3}} = \frac{36 + 12\sqrt{3}}{9 - 3}$	M1	1.1
$2(3 - \sqrt{3}) + 2(6 + 2\sqrt{3})$	M1	1.1
$18 + 2\sqrt{3}$	A1	1.1
	[4]	

Notes

B1: Correct expression for BC

Can be implied by $6 + 2\sqrt{3}$

M1: Multiply numerator and denominator by $\pm(3 + \sqrt{3})$ to obtain a numerator with at least two terms and a denominator (not 0 or 1)

If correct, then $\frac{36 + 12\sqrt{3}}{6}$ or $6 + 2\sqrt{3}$

M1: Correct method **seen** for finding perimeter with their AD/BC

Do not award if BC is never rationalised.

A1: $a = 18$ and $b = 2$

6.

Mark scheme (a)

Scheme	Marks	AO
Uses $(\sqrt{k} + \sqrt{k+1})(\sqrt{k} - \sqrt{k+1})$ Or $(\sqrt{k} + \sqrt{k+1})(\sqrt{k+1} - \sqrt{k})$ PI by $\pm[k - (k+1)]$ on denominator	M1	1.1a

Completes a reasoned argument to show given result Must see correct simplification of the denominator AG	R1	2.1
	(2)	

Typical solution

$$\begin{aligned}
 & \frac{1}{\sqrt{k} + \sqrt{k+1}} \times \frac{\sqrt{k} - \sqrt{k+1}}{\sqrt{k} - \sqrt{k+1}} \\
 &= \frac{\sqrt{k} - \sqrt{k+1}}{k - (k+1)} \\
 &= \frac{\sqrt{k} - \sqrt{k+1}}{-1} \\
 &= \sqrt{k+1} - \sqrt{k}
 \end{aligned}$$

Mark scheme (b)

Scheme	Marks	AO
Uses part (a) to rewrite given expression with no fractions	M1	3.1a
Obtains 1 Do not accept $\sqrt{4} - 1$ CSO	A1	1.1b
	(2)	
	(4 marks)	

Typical solution

$$\begin{aligned}
 & \frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} \\
 &= (\sqrt{2} - \sqrt{1}) + (\sqrt{3} - \sqrt{2}) + (\sqrt{4} - \sqrt{3}) \\
 &= \sqrt{4} - \sqrt{1} \\
 &= 2 - 1 \\
 &= 1
 \end{aligned}$$