

QQQ – Pure Year 1 – Chapter 1 – Indices (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. Identify the expression below which is equivalent to $\left(\frac{2x}{5}\right)^{-3}$

Circle your answer.

[1 mark]

- $\frac{8x^3}{125}$
- $\frac{125x^3}{8}$
- $\frac{125}{8x^3}$
- $\frac{8}{125x^3}$

(Total 1 marks)

AQA June 2023 Paper 1 Q2

2.

(a) Evaluate $81^{\frac{3}{2}}$ **(2)**

(b) Simplify fully $x^2 \left(4x^{-\frac{1}{2}}\right)^2$ **(2)**

(Total 4 marks)

Edexcel C1 June 2014 (R) Q2

3. Given that $32\sqrt{2} = 2^a$, find the value of a . **(3)**

(Total 3 marks)

Edexcel C1 June 2009 Q2

4. Solve the equation $4x^{-\frac{1}{2}} = 7$, giving your answer as a fraction in its lowest terms. **[3]**

(Total 3 marks)

OCR MEI June 2019 Paper 2 Q1

5. In this question you should show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Given

$$\frac{9^{x-1}}{3^{y+2}} = 81$$

express y in terms of x , writing your answer in simplest form.

(3)

(Total 3 marks)

Edexcel November 2021 Paper 1 Q2

6. Solve

(a) $2^y = 8$

(1)

(b) $2^x \times 4^{x+1} = 8$

(4)

(Total 5 marks)

Edexcel C1 June 2013 (R) Q5

7. Express $\frac{a^{\frac{7}{2}} - a^{\frac{5}{2}}}{a^{\frac{3}{2}} - a}$ in the form $a^m + \sqrt{a^n}$, where m and n are integers and $a \neq 0$ or 1.

[5]

(Total 5 marks)

OCR A June 2024 Paper 1 Q9

Solutions

1.

Scheme	Marks	AO
Circles correct answer	B1	1.1b
	(1 mark)	

Typical solution

$$\frac{125}{8x^3}$$

2.

Mark scheme (a)

Scheme	Marks
$81^{\frac{3}{2}} = \left(81^{\frac{1}{2}}\right)^3 = 9^3$ or $81^{\frac{3}{2}} = \left(81^3\right)^{\frac{1}{2}} = (531441)^{\frac{1}{2}}$	M1
$= 729$	A1
	(2)

Notes

M1: Dealing with either the ‘cube’ or the ‘square root’ first. A correct answer will imply this mark.

Also accept a law of indices approach $81^{\frac{3}{2}} = 81^1 \times 81^{\frac{1}{2}} = 81 \times 9$

A1: Cao 729. Accept $(\pm)729$

Mark scheme (b)

Scheme	Marks
$\left(4x^{-\frac{1}{2}}\right)^2 = 16x^{-\frac{2}{2}}$ or $\frac{16}{x}$ or equivalent	M1
$x^2 \left(4x^{-\frac{1}{2}}\right)^2 = 16x$	A1
	(2)
	(4 marks)

Notes

M1: For correct use of power 2 on both 4 and the $x^{-\frac{1}{2}}$ term.

A1: $\text{Cao} = 16x$

3.

Scheme	Marks
$32 = 2^5$ or $2048 = 2^{11}$, $\sqrt{2} = 2^{\frac{1}{2}}$ or $\sqrt{2048} = (2048)^{\frac{1}{2}}$	B1, B1
$a = \frac{11}{2}$ (or $5\frac{1}{2}$ or 5.5)	B1
	(3 marks)

Notes

1st B1: for $32 = 2^5$ or $2048 = 2^{11}$

This should be explicitly seen: $32\sqrt{2} = 2^a$ followed by $2^5\sqrt{2} = 2^a$ is OK

Even writing $32 \times 2 = 2^5 \times 2 (= 2^6)$ is OK but simply writing $32 \times 2 = 2^6$ is NOT

2nd B1: for $2^{\frac{1}{2}}$ or $(2048)^{\frac{1}{2}}$ seen. This mark may be implied

3rd B1: for answer as written. **Need** $a = \dots$ so $2^{\frac{11}{2}}$ is B0

$a = \frac{11}{2}$ (or $5\frac{1}{2}$ or 5.5) with no working scores full marks.

If $a = 5.5$ seen then award 3/3 unless it is clear that the value follows from totally incorrect work.

Part solutions: e.g. $2^5\sqrt{2}$ scores the first B1.

Special case:

If $\sqrt{2} = 2^{\frac{1}{2}}$ is not explicitly seen, but the final answer includes $\frac{1}{2}$,

e.g. $a = 2\frac{1}{2}$, $a = 4\frac{1}{2}$, the second B1 is given by implication.

4.

Scheme	Marks	AO
$4 = 7x^{\frac{1}{2}}$ or $\frac{x^{\frac{1}{2}}}{4} = \frac{1}{7}$	M1	1.1a
Square both sides	M1	1.1a
$x = \frac{16}{49}$	A1	1.1

[3]

Notes

M1: Order of M marks may vary. For getting their x term in numerator. $\frac{4}{x^{\frac{1}{2}}} = 7$ not suff for this mark

M1: eg $\frac{x}{16} = \frac{1}{49}$

5.

Scheme	Marks	AO
$\frac{9^{x-1}}{3^{y+2}} = 81 \Rightarrow \frac{3^{2x-2}}{3^{y+2}} = 3^4$ or $\frac{9^{x-1}}{3^{y+2}} = 81 \Rightarrow \frac{9^{x-1}}{9^{\frac{1}{2}(y+2)}} = 9^2$	M1	1.1b
$\Rightarrow 2x - 2 - y - 2 = 4 \Rightarrow y =$ or $\Rightarrow x - 1 - \frac{1}{2}y - 1 = 2 \Rightarrow y =$	dM1	1.1b
$\Rightarrow y = 2x - 8$	A1	1.1b
	(3)	
	(3 marks)	

Notes

M1: Attempts to set 9^{x-1} and 81 as powers of 3. Condone $9^{x-1} = 3^{2x-1}$ and $9^{x-1} = 3^{3x-3}$.

Alternatively attempts to write each term as a logarithm of base 3 or 9. You must see the base written to award this mark.

dM1: Attempts to use the addition (or subtraction) index law, or laws or logarithms, correctly and rearranges the equation to reach y in terms of x . Condone slips in their rearrangement.

A1: $y = 2x - 8$

Alt

Scheme	Marks	AO
Eg. $\log_3 \left(\frac{9^{x-1}}{3^{y+2}} \right) = \log_3 81$	M1	1.1b
$\Rightarrow (x - 1) \log_3 9 - (y + 2) \log_3 3 = 4$ $\Rightarrow 2(x - 1) - y - 2 = 4 \Rightarrow y =$	dM1	1.1b
$\Rightarrow y = 2x - 8$	A1	1.1b

(corrected from the printed mark scheme: the Alt dM1 line is printed as

$$(x - 1) \log_3 (9^{x-1}) - (y + 2) \log_3 (3^{y+2}) = 4; \text{ it should read } (x - 1) \log_3 9 - (y + 2) \log_3 3 = 4)$$

6.

Mark scheme (a)

Scheme	Marks
$2^y = 8 \Rightarrow y = 3$	B1
(Alternative: Takes logs base 2: $\log_2 2^y = \log_2 8 \Rightarrow y \log_2 2 = 3 \log_2 2 \Rightarrow y = 3$)	
	(1)

Notes

B1: Cao (Can be implied i.e. by 2^3)

Mark scheme (b)

Way 1

Scheme	Marks
$8 = 2^3$	M1
$4^{x+1} = (2^2)^{x+1} \text{ or } (2^{x+1})^2$	M1
$2^{3x+2} = 2^3 \Rightarrow 3x + 2 = 3 \Rightarrow x = \frac{1}{3}$	M1A1
	(4)
	(5 marks)

Notes

M1: Replaces 8 by 2^3 (May be implied)

M1: Replaces 4 by 2^2 **correctly**.

M1: Adds their powers of 2 on the lhs and puts this equal to 3 leading to a solution for x .

A1: $x = \frac{1}{3}$ or $x = 0.\dot{3}$ or awrt 0.333

(b) Way 2

Scheme	Marks
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$4^{x+1} = 4 \times 4^x$	M1
$2^x \times 4^x = 8^x$	M1
$4 \times 8^x = 8 \Rightarrow 8^x = 2 \Rightarrow x = \frac{1}{3}$	M1A1

M1: Obtains 4^{x+1} in terms of 4^x **correctly**

M1: Combines their 2^x and 4^x **correctly**

M1: Solves $8^x = k$ leading to a solution for x .

A1: $x = \frac{1}{3}$ or $x = 0.\dot{3}$ or awrt 0.333

7.

Scheme	Marks	AO
DR	M1	3.1a
$\frac{a^{\frac{5}{2}}(a-1)}{a(a^{\frac{1}{2}}-1)} \times \frac{(a^{\frac{1}{2}}+1)}{(a^{\frac{1}{2}}+1)}$	M1	2.1
$= \frac{a^{\frac{3}{2}}(a-1)(a^{\frac{1}{2}}+1)}{a-1} \text{ or } \frac{a^{\frac{5}{2}}(a-1)(a^{\frac{1}{2}}+1)}{a(a-1)}$	A1	1.1
$= a^{\frac{3}{2}}(a^{\frac{1}{2}}+1) \text{ or } a^2 + a^{\frac{3}{2}}$	A1	1.1
$= a^2 + \sqrt{a^3}$	A1	2.2a
	[5]	

Notes

M1 for correct factorisation of numerator and denominator

M1 for multiplying numerator and denominator by $(a^{\frac{1}{2}}+1)$ or $a(a^{\frac{1}{2}}+1)$ or another multiple of this.

A1 for correct, simplified, denominator (dependent only on second **M1** above)

(corrected from the printed mark scheme: a stray extra bracket after $a^{\frac{5}{2}}(a-1)$ has been removed in both methods)

Alternative method

Scheme	Marks
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$\frac{a^{\frac{5}{2}}(a-1)}{a(a^{\frac{1}{2}}-1)}$	M1
$= \frac{a^{\frac{3}{2}}(a^{\frac{1}{2}}-1)(a^{\frac{1}{2}}+1)}{(a^{\frac{1}{2}}-1)}$	M1A1
$= a^{\frac{3}{2}}(a^{\frac{1}{2}}+1) \text{ or } a^2 + a^{\frac{3}{2}}$	A1
$= a^2 + \sqrt{a^3}$	A1

M1 for correct factorisation of numerator and denominator

M1 for factorising $(a-1)$; **A1** for correct factors

These two marks may be implied by the next line