

QQQ – Mechanics Year 2 – Chapter 8 – Vectors (Version 2)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. A particle P moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-2}$

At time $t = 0$, P is moving with velocity $4\mathbf{i} \text{ m s}^{-1}$

(a) Find the velocity of P at time $t = 2$ seconds. (2)

At time $t = 0$, the position vector of P relative to a fixed origin O is $(\mathbf{i} + \mathbf{j}) \text{ m}$.

(b) Find the position vector of P relative to O at time $t = 3$ seconds. (2)

(Total 4 marks)

Edexcel October 2021 Paper 3 Mechanics Q1

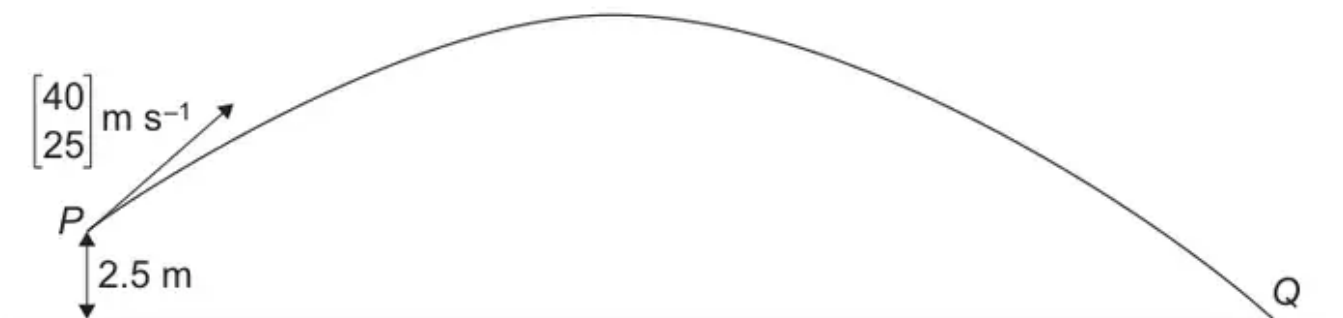
2. In this question use $g = 9.8 \text{ m s}^{-2}$

An arrow is projected from a point P which is at a height of 2.5 metres above the horizontal ground.

The arrow has an initial velocity of $\begin{bmatrix} 40 \\ 25 \end{bmatrix} \text{ m s}^{-1}$

The arrow lands on the horizontal ground at a point Q

The path of the arrow is shown in the diagram.



Find the **speed** of the arrow at point Q [4 marks]

(Total 4 marks)

AQA June 2025 Paper 2 Q14

3. A car has an initial velocity of 1 m s^{-1}

The car is moving in a straight line.

The acceleration $a \text{ m s}^{-2}$ of the car at time t seconds is given by

$$a = 3kt^2 - 2kt + 1$$

where k is a constant.

When $t = 3$ the car has a velocity of 10 m s^{-1}

Show that $k = \frac{1}{3}$

[4 marks]

(Total 4 marks)

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4. In this question the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in the x - and y -directions respectively.

The velocity $\mathbf{v} \text{ m s}^{-1}$ of a particle is given by $\mathbf{v} = 3\mathbf{i} + (6t^2 - 5)\mathbf{j}$. The initial position of the particle is $7\mathbf{j} \text{ m}$.

(a) Find an expression for the position vector of the particle at time $t \text{ s}$.

[4]

(b) Find the Cartesian equation of the path of the particle.

[2]

(Total 6 marks)

OCR MEI June 2024 Paper 1 Q12

5. At time t seconds a particle, P , has position vector $\mathbf{r}$ metres, with respect to a fixed origin, such that

$$\mathbf{r} = (t^3 - 5t^2)\mathbf{i} + (8t - t^2)\mathbf{j}$$

(a) Find the exact speed of P when $t = 2$

[4 marks]

(b) Bella claims that the magnitude of acceleration of P will never be zero.

Determine whether Bella's claim is correct.

Fully justify your answer.

[3 marks]

(Total 7 marks)

AQA June 2020 Paper 2 Q14

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with $t = 2$: $\mathbf{v} = 4\mathbf{i} + 2(2\mathbf{i} - 3\mathbf{j})$ OR integration: $\mathbf{v} = (2\mathbf{i} - 3\mathbf{j})t + 4\mathbf{i}$, with $t = 2$	M1	3.1a
$\mathbf{v} = 8\mathbf{i} - 6\mathbf{j}$	A1	1.1b
	(2)	

Notes

Accept column vectors throughout

M1: Complete method to find $\mathbf{v}$, using $\mathbf{ruvat}$ or integration
(M0 if $\mathbf{i}$ and/or $\mathbf{j}$ is missing)

A1: Apply isw if they also find the speed

Mark scheme (b)

Scheme	Marks	AO
Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ at $t = 3$: $(\mathbf{i} + \mathbf{j}) + \left[3 \times 4\mathbf{i} + \frac{1}{2} \times (2\mathbf{i} - 3\mathbf{j}) \times 3^2 \right]$ OR: find $\mathbf{v}$ at $t = 3$: $4\mathbf{i} + 3(2\mathbf{i} - 3\mathbf{j}) = (10\mathbf{i} - 9\mathbf{j})$ then use $\mathbf{r} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$ $(\mathbf{i} + \mathbf{j}) + \left[\frac{1}{2} [4\mathbf{i} + (10\mathbf{i} - 9\mathbf{j})] \times 3 \right]$ or $\mathbf{r} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ $(\mathbf{i} + \mathbf{j}) + \left[3 \times (10\mathbf{i} - 9\mathbf{j}) - \frac{1}{2} \times (2\mathbf{i} - 3\mathbf{j}) \times 3^2 \right]$ OR integration: $\mathbf{r} = (\mathbf{i} + \mathbf{j}) + \left[(2\mathbf{i} - 3\mathbf{j})\frac{1}{2}t^2 + 4t\mathbf{i} \right]$, with $t = 3$	M1	3.1a

$\mathbf{r} = 22\mathbf{i} - 12.5\mathbf{j}$	A1	2.2a
	(2)	
	(4 marks)	

Notes

Accept column vectors throughout

M1: Complete method to find the p.v. but this mark can be scored if they omit $(\mathbf{i} + \mathbf{j})$

i.e. the M1 is for the expression in the square bracket

If they integrate, the M1 is earned once the expression in the square bracket is seen with $t = 3$

(M0 if $\mathbf{i}$ and/or $\mathbf{j}$ is missing)

A1: cao

2.

Scheme	Marks	AO
Substitutes $u = 25$, $a = \pm 9.8$ and $s = \pm 2.5$ into $v^2 = u^2 + 2as$ PI $\left[\frac{k}{674} \right]$ or $\left[\frac{\sqrt{k}}{\sqrt{674}} \right]$ Or Substitutes $u = 25$, $a = -9.8$ and $s = -2.5$ into $s = ut + \frac{1}{2}at^2$ to find t and substitutes their t into $v = u + at$	M1	3.3
Obtains correct $v^2 = 674$ or $v = \sqrt{674}$ If inconsistent signs are used for u , a and s score A0 PI AWRT ± 25.96 If vector form is used, both components must be correct	A1	1.1b
Finds the magnitude of the resultant vector of their vertical component of velocity and 40 Do not allow 25 for their vertical component of velocity	M1	1.1a

Obtains the correct speed of the arrow. Accept AWR 48 If inconsistent signs are used for u , a and s score A0	A1	1.1b
	(4 marks)	

Typical solution

$$v^2 = 25^2 + 2 - 9.8 \times -2.5$$

$$v^2 = 674$$

$$\text{Speed} = \sqrt{40^2 + 674}$$

$$\text{Speed} = 47.7 \text{ m s}^{-1}$$

$$\text{Speed} = 48 \text{ m s}^{-1}$$

3.

Scheme	Marks	AO
Integrates a with at least one term correct.	M1	3.4
Obtains a fully correct expression for v ACF Coefficients can be unsimplified. Condone omission of constant	A1	1.1b
Uses given initial conditions to find their constant of integration. This must be done before $v = 10$ and $t = 3$ are substituted.	M1	3.4
Completes reasoned argument by substituting $v = 10$ and $t = 3$ into $v = kt^3 - kt^2 + t + 1$ to show $k = \frac{1}{3}$ Must include at least more one intermediate step after substituting. AG	A1	1.1b
	(4 marks)	

Typical solution

$$v = \int a \, dt$$

$$v = kt^3 - kt^2 + t + c$$

$$v = 1 \text{ when } t = 0 \text{ then } c = 1$$

$$v = 10 \text{ and } t = 3$$

$$10 = 27k - 9k + 3 + 1$$

$$18k = 6$$

$$k = \frac{1}{3}$$

4.

Mark scheme (a)

Scheme	Marks	AO
$\mathbf{r} = \int (3\mathbf{i} + (6t^2 - 5)\mathbf{j}) \, dt$	M1	1.1a
$= 3t\mathbf{i} + (2t^3 - 5t)\mathbf{j} + \mathbf{c}$	A1	1.1b
When $t = 0$, $\mathbf{r}_0 = 0\mathbf{i} + 7\mathbf{j}$	M1	1.1a
So position is $= 3t\mathbf{i} + (2t^3 - 5t + 7)\mathbf{j}$	A1	2.5
	[4]	

Notes

M1: Attempt to integrate velocity either as a vector or 2 separate components

A1: Condone missing constant

M1: Either as a vector constant or 2 separate components evaluated.

May be implied by correct vector answer

A1: Must be in vector form (could be column vector but must be exact vector notation eg brackets and not $\mathbf{i}$ and $\mathbf{j}$ as well)

Allow $3t\mathbf{i} + (2t^3 - 5t)\mathbf{j} + 7\mathbf{j}$

Mark scheme (b)

Scheme	Marks	AO
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Using $x = 3t$ and $y = 2t^3 - 5t + 7$	M1	3.1a
We get $y = 2\left(\frac{x}{3}\right)^3 - 5\left(\frac{x}{3}\right) + 7$	A1	1.1
	[2]	

Notes

M1: Attempt to eliminate t from the parametric equations

A1: FT their (a)

IsW

5.

Mark scheme (a)

Scheme	Marks	AO
Differentiates to find expression for $\mathbf{v}$ with at least one component correct	M1	3.4
Substitutes $t = 2$ into their $\mathbf{v}$ which has at least one component correct.	M1	1.1a
Uses correct process to find the magnitude of their expression for $\mathbf{v}$ provided both of their components are non-zero	M1	1.1a
Obtains correct answer $4\sqrt{5}$ ISW ACF CSO	A1	1.1b
	(4)	

Typical solution

$$\mathbf{v} = (3t^2 - 10t)\mathbf{i} + (8 - 2t)\mathbf{j}$$

So when $t = 2$

$$\mathbf{v} = -8\mathbf{i} + 4\mathbf{j}$$

$$Speed = |\mathbf{v}| = \sqrt{8^2 + 4^2}$$

$$= 4\sqrt{5} \text{ m s}^{-1}$$

Mark scheme (b)

Scheme	Marks	AO
Differentiates <i>their</i> vector $\mathbf{v}$ to find expression for $\mathbf{a}$ with at least one component correct. Do not award for $\mathbf{a} = \mathbf{r}$	M1	3.4
Uses a valid explanation as to why the magnitude must be positive for all values of t This could be earned by: forming a quadratic from acceleration components, allow sign error or Stating that the $\mathbf{j}$ component is always -2	E1	2.4
Deduces that $ \mathbf{a} \geq 2$ OE or that Bella is correct or $ \mathbf{a} > 0$. Deduction must be from completely correct reasoning.	R1	2.2a
	(3)	
	(7 marks)	

Typical solution

$$\mathbf{a} = (6t - 10)\mathbf{i} - 2\mathbf{j}$$

$$|\mathbf{a}| = \sqrt{(6t - 10)^2 + 2^2}$$

Since $|\mathbf{a}| \geq 2$ for any value of t the magnitude of the acceleration is never zero

Therefore Bella's claim is correct.