

QQQ – Mechanics Year 2 – Chapter 8 – Vectors (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. A particle P moves with constant acceleration $(3\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-2}$. At time $t = 4$ seconds, P has velocity $6\mathbf{i} \text{ m s}^{-1}$.

Determine the speed of P at time $t = 0$ seconds.

[4]

(Total 4 marks)

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2. The velocity $v \text{ m s}^{-1}$ of a particle at time $t \text{ s}$ is given by

$$v = 0.5t(7 - t).$$

Determine whether the **speed** of the particle is increasing or decreasing when $t = 8$.

[4]

(Total 4 marks)

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3. The position vector $\mathbf{r}$ metres of a particle at time t seconds is given by

$$\mathbf{r} = (1 + 12t - 2t^2) \mathbf{i} + (t^2 - 6t) \mathbf{j}.$$

- (i) Find an expression for the velocity of the particle at time t .

[2]

- (ii) Determine whether the particle is ever stationary.

[2]

(Total 4 marks)

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4. In this question the x - and y -directions are horizontal and vertically upwards respectively and the origin is on horizontal ground.

A ball is thrown from a point 5 m above the origin with an initial velocity $\begin{pmatrix} 14 \\ 7 \end{pmatrix} \text{ m s}^{-1}$.

- (a) Find the position vector of the ball at time $t \text{ s}$ after it is thrown.

[3]

- (b) Find the distance between the origin and the point at which the ball lands on the ground.

[3]

(Total 6 marks)

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5. [In this question position vectors are given relative to a fixed origin O]

At time t seconds, where $t \geq 0$, a particle, P , moves so that its velocity $\mathbf{v}$ m s⁻¹ is given by

$$\mathbf{v} = 6t\mathbf{i} - 5t^{\frac{3}{2}}\mathbf{j}$$

When $t = 0$, the position vector of P is $(-20\mathbf{i} + 20\mathbf{j})$ m.

(a) Find the acceleration of P when $t = 4$ **(3)**

(b) Find the position vector of P when $t = 4$ **(3)**

(Total 6 marks)

Solutions

1.

Scheme	Marks	AO
$6\mathbf{i} = \mathbf{u} + 4(3\mathbf{i} - 2\mathbf{j})$	M1*	3.3
$\mathbf{u} = -6\mathbf{i} + 8\mathbf{j}$	A1	2.5
$u = \sqrt{(-6)^2 + 8^2}$	M1dep*	1.1
$u = 10 \text{ (ms}^{-1}\text{)}$	A1	1.1
	[4]	

Notes

M1*: Applying $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ correctly - working must imply that $\mathbf{v}$ and $\mathbf{a}$ are vectors

Or for $\mathbf{v} = 3t\mathbf{i} - 2t\mathbf{j} + \mathbf{c}$ and using $t = 4, \mathbf{v} = 6\mathbf{i}$ to find $\mathbf{c}$

M0 if $\mathbf{u} = -6\mathbf{i} \pm 14\mathbf{j}$

A1: or for $\mathbf{v} = (3t - 6)\mathbf{i} + (-2t + 8)\mathbf{j}$ and setting $t = 0$ to obtain correct $\mathbf{u}$

M1dep*: Correctly taking the magnitude of their $\mathbf{u}$ but condone $\sqrt{-6^2 + 8^2} = \sqrt{\pm 36 + 64}$

Correct answer following $-6\mathbf{i} + 8\mathbf{j}$ (with no wrong working) scores full marks

A1: www

2.

Scheme	Marks	AO
$v = 3.5t - 0.5t^2 \Rightarrow \frac{dv}{dt} = 3.5 - t$	M1	3.1b
When $t = 8, a = 3.5 - 8 = -4.5$	A1	2.1
And $v = 3.5 \times 8 - 0.5 \times 8^2 = -4$	B1	3.1b
Either "Velocity is negative and decreasing, so the speed is increasing". Or "As both the velocity and acceleration are negative, the speed is increasing" or similar	A1	3.2a
	[4]	

Notes

M1: Attempt to differentiate to find a

A1: Evaluating a when $t = 8$

Allow A1 for establishing that $a < 0$ from correct expression without evaluating.

B1: Evaluating v when $t = 8$

Allow B1 for establishing $v < 0$ from correct expression without evaluating.

A1: Argued from negative a and v

Special case

Scheme	Marks
When $t = 8, v = 3.5 \times 8 - 0.5 \times 8^2 = -4$	B1
Evaluating v either side of $t = 8$	B1
Statement referring to correct values	B1

B1: Evaluating v when $t = 8$

B1: Two correct values

B1: Argued from correct working

Method is not a full argument so max 3/4 marks.

t	7	7.5	7.9	8	8.1	8.5	9
v	0	-1.875	-3.555	-4	-4.455	-5.88	-9

3.

Mark scheme (i)

Scheme	Marks	AO
$\mathbf{v} = \frac{d\mathbf{r}}{dt} = (12 - 2 \times 2t)\mathbf{i} + (2t - 6)\mathbf{j}$	M1 A1	1.1a 2.5
	[2]	

Notes

M1: Attempt to differentiate at least one coefficient

A1: Must use vector notation

Mark scheme (ii)

Scheme	Marks	AO
When $t = 3$ both components of velocity are zero,	M1	3.1a
so the particle is stationary at $t = 3$.	E1	2.2a
	[2]	

Notes

M1: Equating at least one component of their vector velocity to zero

Do not allow M1 for solving $12 - 4t = 2t - 6$ unless at least one zero subsequently established

E1: Must be argued from two zero components

4.

Mark scheme (a)

Scheme	Marks	AO
$x = 14t$	B1	1.1b
$y = 7t - \frac{1}{2}gt^2 + 5$	M1	1.1b
So the position vector is $\begin{pmatrix} 14t \\ 7t - 4.9t^2 + 5 \end{pmatrix}$	A1	2.5
	[3]	

Notes

B1: must be $x = \dots$ or seen as the first component of the vector.

Do not award for an expression that adds a vector to a scalar

M1: allow without $+5$, or if -5 seen

Do not award for an expression that adds a vector to a scalar

A1: Must be a single vector.

Accept $\frac{1}{2}g$ in final answer

Accept $14t \mathbf{i} + (7t - 4.9t^2 + 5) \mathbf{j}$

SC1: for $\begin{pmatrix} 7t - 4.9t^2 + 5 \\ 14t \end{pmatrix}$ or $\begin{pmatrix} 14t - 4.9t^2 + 5 \\ 7t \end{pmatrix}$

Mark scheme (b)

Scheme	Marks	AO
Lands when $y = 0$ $7t - 4.9t^2 + 5 = 0$	M1	3.1b
$t = 1.95$	A1	1.1b
gives $x = 14t = 27.3$ m	B1	1.1b
	[3]	

Notes

M1: Award for correct quadratic or an attempt to find value of t when their quadratic $y = 0$

A1: cao

B1: FT their t and their linear expression for x

ISW where candidates find the distance from the point of projection

5.

Mark scheme (a)

Scheme	Marks	AO
Differentiate $\mathbf{v}$	M1	1.1a
$(\mathbf{a} =) 6\mathbf{i} - \frac{15}{2}t^{\frac{1}{2}}\mathbf{j}$	A1	1.1b
$= 6\mathbf{i} - 15\mathbf{j} \text{ (m s}^{-2}\text{)}$	A1	1.1b
	(3)	

Notes

N.B. Accept column vectors throughout and condone missing brackets in working but they must be there in final answers

M1: Use of $\mathbf{a} = \frac{d\mathbf{v}}{dt}$ with attempt to differentiate (both powers decreasing by 1)

M0 if $\mathbf{i}$'s and $\mathbf{j}$'s omitted and they don't recover

A1: Correct differentiation in any form

A1: Correct and simplified.

Ignore subsequent working (ISW) if they go on and find the magnitude.

Mark scheme (b)

Scheme	Marks	AO
Integrate $\mathbf{v}$	M1	1.1a
$(\mathbf{r} =) (\mathbf{r}_0) + 3t^2\mathbf{i} - 2t^{\frac{5}{2}}\mathbf{j}$	A1	1.1b
$= (-20\mathbf{i} + 20\mathbf{j}) + (48\mathbf{i} - 64\mathbf{j}) = 28\mathbf{i} - 44\mathbf{j} \text{ (m)}$	A1	2.2a
	(3)	
	(6 marks)	

Notes

N.B. Accept column vectors throughout and condone missing brackets in working but they must be there in final answers

M1: Use of $\mathbf{r} = \int \mathbf{v} \, dt$ with attempt to integrate (both powers increasing by 1)

M0 if $\mathbf{i}$'s and $\mathbf{j}$'s omitted and they don't recover

A1: Correct integration in any form. Condone $\mathbf{r}_0$ not present

A1: Correct and simplified.