

QQQ – Mechanics Year 2 – Chapter 6 – Projectiles (Version 2)

Total Marks: 22

(22 = Platinum, 20 = Gold, 18 = Silver, 15 = Bronze)

1. In this question use $g = 9.8 \text{ m s}^{-2}$

A ball is projected from a point on horizontal ground with an initial velocity of 7 m s^{-1} at an angle θ above the horizontal.

The ball reaches a maximum vertical height of h metres above the ground.

(a) Show that

$$h = 2.5 \sin^2 \theta$$

[3 marks]

(b) Hence, given that $0^\circ \leq \theta \leq 60^\circ$, find the maximum value of h .

[2 marks]

(c) Nisha claims that the larger the size of the ball, the greater the maximum vertical height will be.

State whether Nisha is correct, giving a reason for your answer.

[1 mark]

(Total 6 marks)

2.

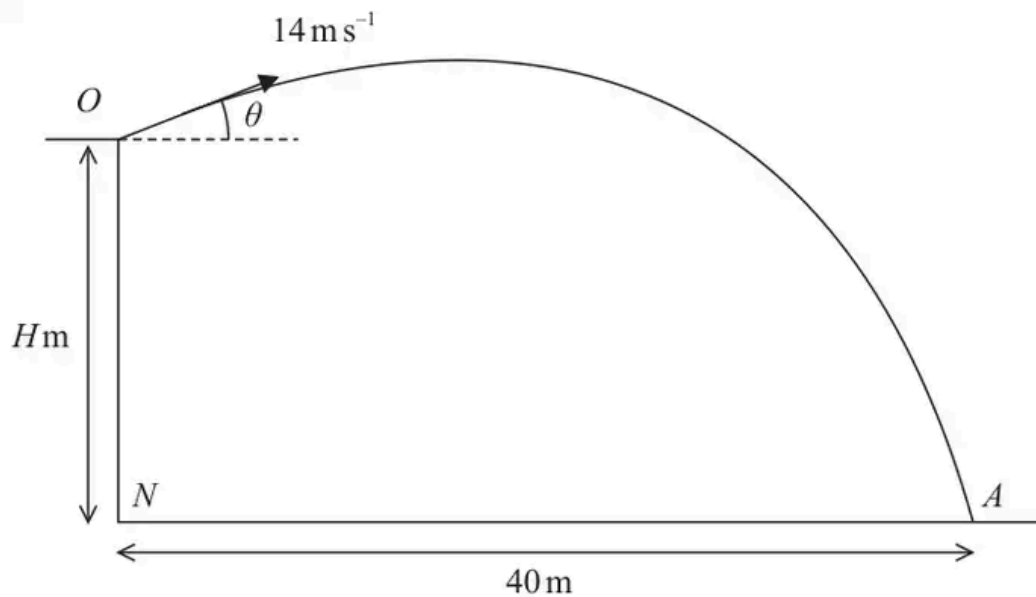


Figure 3

A small stone is projected with speed 14 m s^{-1} from a point O on the top of a cliff.

The point O is H metres vertically above the point N .

Point N is on horizontal ground.

The stone is projected at an angle θ above the horizontal, where $\tan \theta = \frac{1}{2}$

The stone strikes the horizontal ground at the point A , where $NA = 40\text{ m}$, as shown in Figure 3.

The stone is modelled as a particle moving freely under gravity.

Using this model, find

(a) the value of H (5)

(b) the maximum height of the stone above the horizontal ground. (3)

(Total 8 marks)

3. A vertical cliff is 73.5 m high. Two stones A and B are projected simultaneously. Stone A is projected horizontally from the top of the cliff with speed 28 m s^{-1} . Stone B is projected from the bottom of the cliff with speed 35 m s^{-1} at an angle α above the horizontal. The stones move freely under gravity in the same vertical plane and collide in mid-air. By considering the horizontal motion of each stone,

(a) prove that $\cos \alpha = \frac{4}{5}$. **(4)**

(b) Find the time which elapses between the instant when the stones are projected and the instant when they collide. **(4)**

(Total 8 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
Obtains $7 \sin \theta$ for vertical component of initial velocity	B1	1.1b
Uses $v^2 = u^2 + 2as$ with $v = 0$ Or uses appropriate constant acceleration equations that form a complete method to obtain h eg finds t then subs into $s = ut + \frac{1}{2}at^2$	M1	3.3
Completes argument substituting $s = h, v = 0, u = 7 \sin \theta$ and $a = -9.8$ to show the given result. Accept negative values used consistently AG	R1	2.1
	(3)	

Typical solution

$$u = 7 \sin \theta$$

$$v^2 = u^2 + 2as$$

$$0 = 49 \sin^2 \theta - 19.6h$$

So

$$h = 2.5 \sin^2 \theta$$

Mark scheme (b)

Scheme	Marks	AO
Substitutes any value of θ in the range $0 < \theta \leq 60$ to obtain a height greater than zero	M1	1.1a

Deduces $h = 1.9$ AWRT 1.9 or $\frac{15}{8}$	A1	2.2a
	(2)	

Typical solution

$$\theta = 60^\circ$$

$$h = 2.5 \sin^2 60$$

$$h = 1.9$$

Mark scheme (c)

Scheme	Marks	AO
States that Nisha is incorrect and refers to air resistance or ball modelled as a particle	E1	3.5a
	(1)	
	(6 marks)	

Typical solution

Nisha is incorrect

The model ignores air resistance which gets greater as ball gets larger

2.

Mark scheme (a)

Scheme	Marks	AO
Using horizontal motion	M1	3.3
$14 \cos \theta \times t = 40$	A1	1.1b
Using vertical motion	M1	3.4
$(s =) 14 \sin \theta \times t - \frac{1}{2}gt^2$ oe with down as positive.	A1	1.1b
OR		
Using the equation of the path		

$(s =) 40 \tan \theta - \frac{g \times 40^2}{2 \times 14^2 \cos^2 \theta}$ oe	M2 A2	
$(H =) 30$ or 30.0	A1	1.1b
	(5)	

Notes

N.B. Penalise the use of $g = 9.81$ **once** (the first time it is used) for the whole question.

All marks available if they use a different letter for θ

M1: Complete method, using the horizontal motion, to give an equation in t and θ only, condone sin/cos confusion and sign errors

(θ may or may not be substituted for)

A1: Correct equation (t and/or θ may or may not be substituted for)

M1: Complete method, using the vertical motion, to give an equation in t and θ only, condone sin/cos confusion and sign errors

(t and/or θ may or may not be substituted for)

A1: Correct equation (t and/or θ may or may not be substituted for)

N.B. Just mark the RHS of the equation for this A mark.

OR

M2 A2: Quoting and using correctly the equation of the path.

N.B. No marks if they misquote the equation and no marks if they put in 14 and/or 40 incorrectly.

These 4 marks are indivisible i.e. all 4 or 0.

A1: $H = 30$ or 30.0 to 2 or 3 sf, after use of $g = 9.8$, **obtained correctly**.

Mark scheme (b)

Scheme	Marks	AO
Using vertical motion OR Using cons. of energy	M1	2.1
$0^2 = (14 \sin \theta)^2 - 2gh$ OR $\frac{1}{2}m(14^2 - (14 \cos \theta)^2) = mgh$	A1	1.1b
32 (m)	A1ft	1.1b
	(3)	
	(8 marks)	

Notes

N.B. Penalise the use of $g = 9.81$ **once** (the first time it is used) for the whole question.

All marks available if they use a different letter for θ

M1: Complete method to give equation in h and θ only, condone sin/cos confusion and sign errors

They may find t first: $0 = 14 \sin \theta - gt \Rightarrow t = \frac{14 \sin \theta}{g}$

$$\text{then } h = (14 \sin \theta) \frac{14 \sin \theta}{g} - \frac{1}{2}g \left(\frac{14 \sin \theta}{g} \right)^2$$

$$\text{or } h = -\frac{1}{2}(-g) \left(\frac{14 \sin \theta}{g} \right)^2$$

$$\text{or } h = \frac{(14 \sin \theta + 0)}{2} \left(\frac{14 \sin \theta}{g} \right)$$

A1: Correct equation

A1ft: |their H | + 2, to 2sf or 3sf

3.

Mark scheme (a)

Scheme	Marks
$x_A = 28t$ $x_B = 35 \cos \alpha t$	B1 B1
Meet $\Rightarrow 28t = 35 \cos \alpha t \Rightarrow \cos \alpha = 28/35 = 4/5$ *	M1 A1
	(4)

Mark scheme (b)

Scheme	Marks
$y_A = 73.5 - \frac{1}{2}gt^2$ $y_B = 21t - \frac{1}{2}gt^2$	B1 B1
Meet $\Rightarrow 73.5 = 21t \Rightarrow t = \underline{3.5 \text{ s}}$	M1 A1
	(4)
	(8 marks)