

QQQ – Mechanics Year 2 – Chapter 6 – Projectiles (Version 1)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. A cannonball is fired from a point on horizontal ground at 100 m s^{-1} at an angle of 25° above the horizontal. Ignoring air resistance, calculate

(a) the greatest height the cannonball reaches, [3]

(b) the range of the cannonball. [4]

(Total 7 marks)

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2. A pebble is thrown horizontally at 14 m s^{-1} from a window which is 5 m above horizontal ground. The pebble goes over a fence 2 m high d m away from the window as shown in Fig. 9. The origin is on the ground directly below the window with the x -axis horizontal in the direction in which the pebble is thrown and the y -axis vertically upwards.

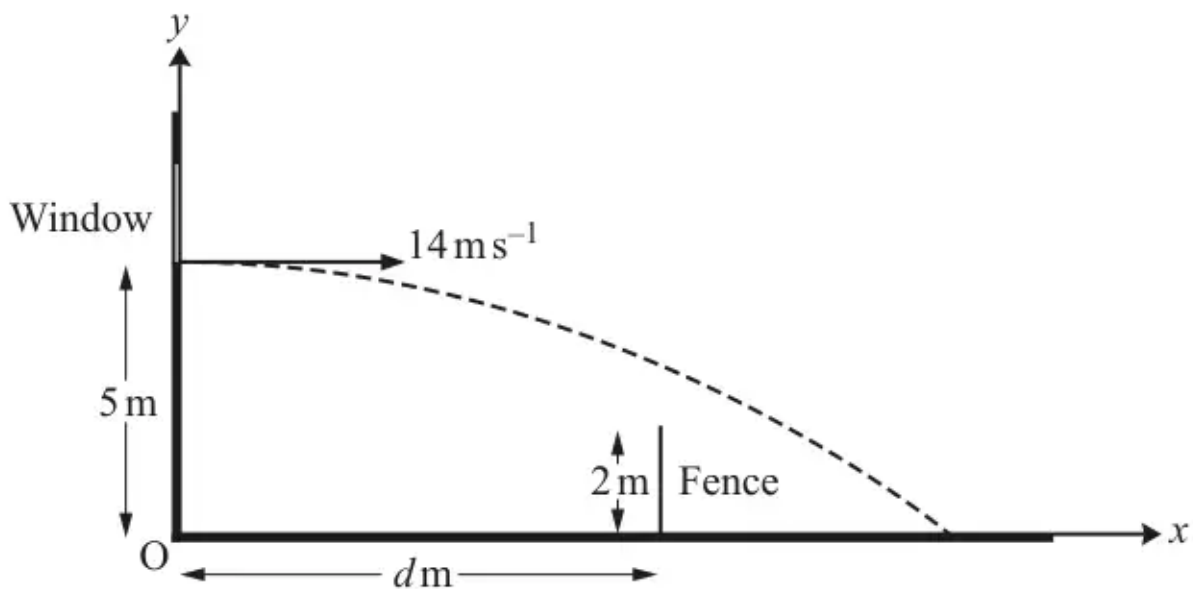


Fig. 9

(i) Find the time the pebble takes to reach the ground. [3]

(ii) Find the cartesian equation of the trajectory of the pebble. [4]

(iii) Find the range of possible values for d . [3]

(Total 10 marks)

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3. A ball is projected forward from a fixed point, P , on a horizontal surface with an initial speed $u \text{ m s}^{-1}$, at an acute angle θ above the horizontal.

The ball needs to first land at a point at least d metres away from P .

You may assume the ball may be modelled as a particle and that air resistance may be ignored.

Show that

$$\sin 2\theta \geq \frac{dg}{u^2}$$

[6 marks]

(Total 6 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
Vertical motion $u_y = 100 \sin 25^\circ$, $v_y = 0$	M1	3.3
$v^2 = u^2 + 2as$	B1	1.1a
$0 = (100 \sin 25^\circ)^2 - 2 \times 9.8 \times s$		
$s = \frac{42.26^2}{19.6} = 91.1 \text{ m (3sf)}$	A1	3.2a
	[3]	

Notes

M1: Use of *suvat* equation(s) with $v = 0$ leading to a value for s . Allow sign errors
eg from $t = 4.31 \text{ s}$

B1: Correct component of velocity so

A1: cao

Mark scheme (b)

Scheme	Marks	AO
Vertical motion $u_y = 100 \sin 25^\circ$, $y = 0$	M1	3.3
$s = ut + \frac{1}{2}at^2 \Rightarrow 0 = (100 \sin 25^\circ)t - 4.9t^2$	A1	1.1a
$t(100 \sin 25^\circ - 4.9t) = 0 \Rightarrow t = 0 \text{ or } 8.62 \dots$		
$x = (100 \cos 25^\circ) \times 8.62 \dots = 781.678 \dots$	M1	1.1a
Range is 782 m (3sf)	A1	1.1b
	[4]	

Notes

M1: Use of *suvat* equation(s) with $y = 0$ and their $u_y \neq 100$ leading to a value for t . Allow sign errors

A1: Correct value or expression for t from correct working

Can be BC

M1: Use of horizontal motion equation with $u_x = 100 \cos 25^\circ$

Allow for sin/cos interchange used throughout part (b).

A1: FT their value for t

2.

Mark scheme (i)

Scheme	Marks	AO
Vertical motion $u = 0$	B1	3.3
$s = ut + \frac{1}{2}at^2$ $-5 = 0 - \frac{9.8}{2}t^2$	M1	3.4
$t = \sqrt{\frac{10}{9.8}} = 1.01 \text{ s}$	A1	1.1b
	[3]	

Notes

B1: Using $u = 0$ in the vertical direction to model horizontal motion so i

M1: Using suvat equation(s) to find t . Allow sign errors and incorrect value for u .

A1: Must follow from working where the signs are consistent.

Mark scheme (ii)

Scheme	Marks	AO
$x = 14t$	B1	3.3
$y = 5 - 4.9t^2$	B1	3.3
So cartesian equation is $y = 5 - 4.9 \left(\frac{x}{14} \right)^2 \left[= 5 - \frac{x^2}{40} \right]$	M1 A1	1.1a 1.1b
	[4]	

Notes

B1: May be implied

B1: May be implied

M1: Attempt to eliminate t

A1: Any form

Mark scheme (iii)

Scheme	Marks	AO
EITHER When $y = 2$ $y = 5 - \frac{x^2}{40} = 2$ m $\frac{x^2}{40} = 3$	M1	3.4
$x = \sqrt{120} = 10.9544 \dots$	A1	1.1b
$[0 <] d < 11.0$ m	E1	3.2a
	[3]	

Notes

M1: Using their equation of trajectory and $y = 2$

A1: Must be 11.0 or better

E1: Allow “Fence must be less than 10.95 m from the origin.” FT their value

SC2 for $d < \sqrt{80}$ [= 8.94]

SC1 for $d = \sqrt{80}$ [= 8.94]

OR

Scheme	Marks
When $y = 2$ $2 = 5 - 4.9t^2$ $t = 0.782$ When $t = 0.782$ $x = 14 \times 0.782 = 10.95$	M1
$[0 <] d < 11.0$ m	A1 A1

M1: Both steps required for M1

A1: Must be 11.0 or better

A1: Allow “Fence must be less than 10.95 m from the origin.”

Allow if the origin is taken to be at window height and the top of the wall is 3m below the window.

Signs must be consistent for A1

3.

Scheme	Marks	AO
Models vertical motion using a suitable constant acceleration equation, to find the time of flight. Condone consistent sine/ cosine error	M1	3.3
Obtains correct equation or inequality for t Must use g not numerical value	A1	1.1b
Models horizontal displacement Condone consistent sine/ cosine error	M1	3.3
Obtains correct equation or inequality to model horizontal displacement	A1	1.1b
Eliminates t , where t is time of flight, from their horizontal and vertical models to obtain an expression in terms of u , g and θ only.	M1	3.4
Completes reasoned argument to obtain stated result. Must justify stated inequality must have used correct resolution of u throughout	R1	2.1
	(6 marks)	

Typical solution

Vertically:

$$0 = t \times u \sin \theta - 0.5 \times g \times t^2$$

$$t = \frac{2u \sin \theta}{g}$$

Horizontally:

$$x = u t \cos \theta$$

$$x = u \frac{2u \sin \theta}{g} \cos \theta,$$

using $\sin 2\theta = 2 \sin \theta \cos \theta$

$$x = \frac{u^2 \sin 2\theta}{g}$$

Since $x \geq d$

$$\frac{u^2 \sin 2\theta}{g} \geq d$$

$$\sin 2\theta \geq \frac{dg}{u^2}$$