

QQQ – Mechanics Year 2 – Chapter 4 – Moments (Version 2)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

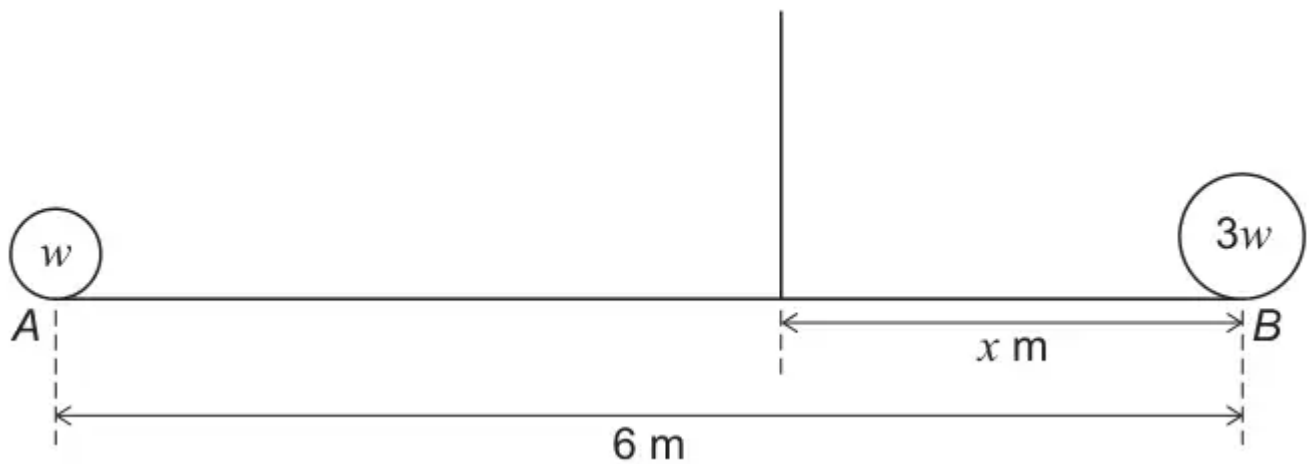
1. A straight uniform rod, AB , has length 6 m and mass 0.2 kg

A particle of weight w newtons is fixed at A .

A second particle of weight $3w$ newtons is fixed at B .

The rod is suspended by a string from a point x metres from B .

The rod rests in equilibrium with AB horizontal and the string hanging vertically as shown in the diagram below.



Show that

$$x = \frac{3w + 0.3g}{2w + 0.1g}$$

[4 marks]

(Total 4 marks)

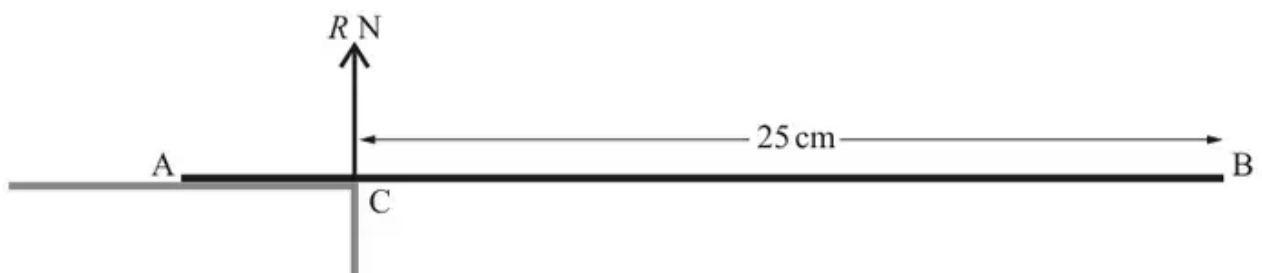
2. A uniform ruler AB has mass 28 g and length 30 cm. As shown in Fig. 6, the ruler is placed on a horizontal table so that it overhangs a point C at the edge of the table by 25 cm.

A downward force of F N is applied at A. This force just holds the ruler in equilibrium so that the contact force between the table and the ruler acts through C.



Fig. 6

- (a) Complete the force diagram in the Printed Answer Booklet, labelling the forces and all relevant distances.



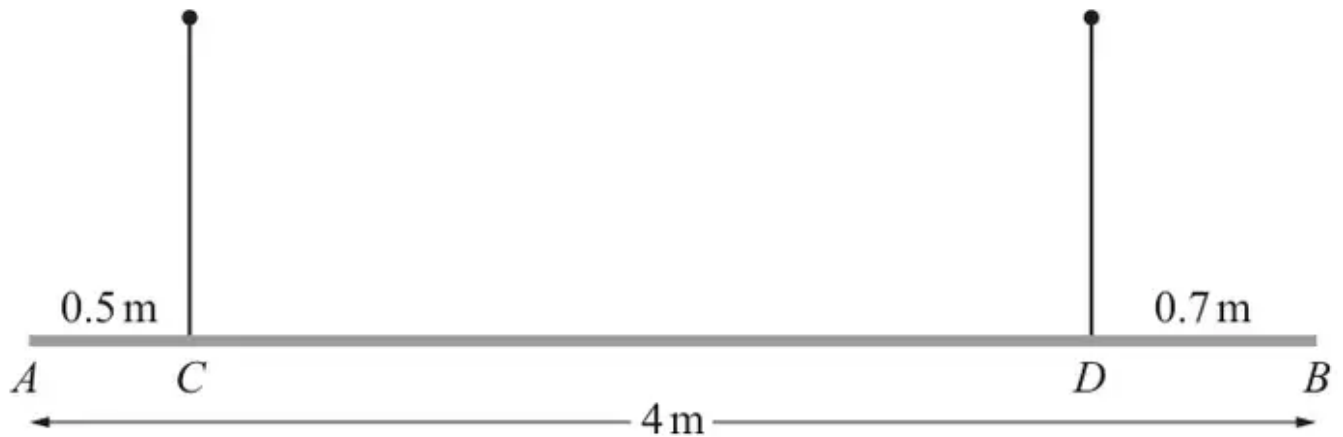
[2]

- (b) Calculate the value of F .

[2]

(Total 4 marks)

3.



A beam, AB , has length 4 m and mass 20 kg. The beam is suspended horizontally by two vertical ropes. One rope is attached to the beam at C , where $AC = 0.5$ m. The other rope is attached to the beam at D , where $DB = 0.7$ m (see diagram).

The beam is modelled as a non-uniform rod and the ropes as light inextensible strings.

It is given that the tension in the rope at C is three times the tension in the rope at D .

(a) Determine the distance of the centre of mass of the beam from A . [5]

A particle of mass m kg is now placed on the beam at a point where the magnitude of the moment of the particle's weight about C is $3.5mg$ N m. The beam remains horizontal and in equilibrium.

(b) Determine the largest possible value of m . [2]

(Total 7 marks)

4.

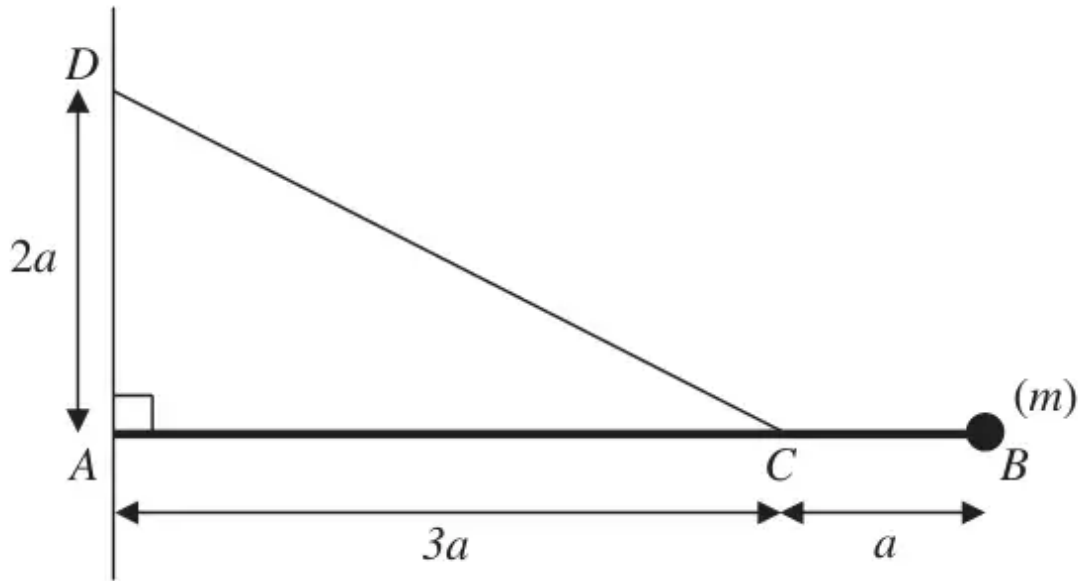


Figure 2

Figure 2 shows a uniform rod AB of mass m and length $4a$. The end A of the rod is freely hinged to a point on a vertical wall. A particle of mass m is attached to the rod at B . One end of a light inextensible string is attached to the rod at C , where $AC = 3a$. The other end of the string is attached to the wall at D , where $AD = 2a$ and D is vertically above A . The rod rests horizontally in equilibrium in a vertical plane perpendicular to the wall and the tension in the string is T .

(a) Show that $T = mg\sqrt{13}$. (5)

The particle of mass m at B is removed from the rod and replaced by a particle of mass M which is attached to the rod at B . The string breaks if the tension exceeds $2mg\sqrt{13}$. Given that the string does not break,

(b) show that $M \leq \frac{5}{2}m$. (3)

(Total 8 marks)

Solutions

1.

Scheme	Marks	AO
Takes moments about the point of suspension to form a moments equation involving w and x with one term correctly expressed. or Takes moments about any other point to form a moments equation involving w, x and T with one term correctly expressed. If moments taken about A then $0.2g(3) + 3w(6) = T(6 - x)$ If moments taken about B then $w(6) + 0.2g(3) = Tx$	M1	3.3
Forms a dimensionally correct moments equation with two terms correctly expressed	M1	1.1a
Forms a fully correct equation involving w, x and g only If moments taken about A or B then $T = 4w + 0.2g$ must have been substituted at this point	A1	1.1b
Obtains given answer showing at least the step $x(4w + 0.2g) = 6w + 0.6g$ OE or $x = \frac{6w + 0.6g}{4w + 0.2g}$ OE Must have fully corrected any error involving weight and mass throughout AG	R1	2.1
	(4 marks)	

Typical solution

Take moments about the point of suspension

$$3wx = (3 - x)0.2g + (6 - x)w$$

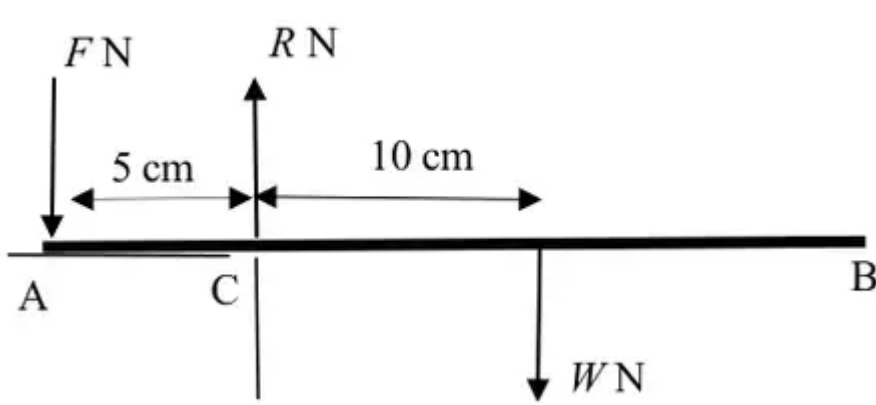
$$4wx + 0.2gx = 6w + 0.6g$$

$$2(2w + 0.1g)x = 6w + 0.6g$$

$$\therefore x = \frac{3w + 0.3g}{2w + 0.1g}$$

2.

Mark scheme (a)

Scheme	Marks	AO
	B1	1.1a
	B1	1.1
	[2]	

Notes

B1: F and their weight even if not labelled in roughly correct position

R is already drawn in Printed Answer Booklet

B1: Distances labelled. Allow for 5cm **and** any measurements that show that the weight is acting in the centre of the ruler.

Mark scheme (b)

Scheme	Marks	AO
Take moments about C $10 \times 0.028g = 5F$	M1	1.1a
$F = 0.549 \text{ N}$	A1	1.1
	[2]	

Notes

M1: Moments about any point with all relevant forces seen. Allow their weight and one incorrect distance
[Value of R is 0.8232 N]

A1: allow $\frac{7}{125}g$ oe

3.

Mark scheme (a)

Scheme	Marks	AO
$0.5T_C + (4 - 0.7)T_D = 20g\bar{x}$ (moments about A) $(\bar{x} - 0.5)(20g) = (4 - 1.2)T_D$ (moments about C) $(4 - 1.2)T_C = 20g(4 - 0.7 - \bar{x})$ (moments about D) $0.7T_D + (4 - 0.5)T_C = 20g(4 - \bar{x})$ (moments about B) $(\bar{x} - 0.5)T_C = (4 - 0.7 - \bar{x})T_D$ (moments about com)	M1* A1FT	3.3 1.1
$T_C = 3T_D$ or $T_C = 3T$ and $T_D = T$	B1	3.3
$T_C = 147 (= 15g)$ and/or $T_D = 49 (= 5g)$ (from $T_C + T_D = 20g$) used in relevant moment equation(s)	M1dep*	1.1
$\bar{x} = 1.2$ (m)	A1	2.2a
	[5]	

Notes

M1*: Moments (with correct number of terms) about A, C, D, B or com

$\bar{x}$ is the centre of mass of the rod from A

Allow g to be absent

A1FT: Follow though their T_C and T_D (allow just T) and allow W for $20g$

B1: Correct relationship(s) for the tensions at C and D (soi)

M1dep*: Equation in $\bar{x}$ only, e.g.,

- $W = 4T$ and any one mom. eq.
- $4T = 20g$ and any one mom. eq.
- $W = 20g$ and any two mom. eq.

M0 if tension used are the same in both ropes

A1: Must be 1.2 as question asks for com from A

Alternative: **B1** as main scheme (soi), then **M1** for splitting 2.8 in the ratio 1: 3 or 3:1 then **A1** for correct 1:3 then **B2** for $0.5 + 0.25(2.8) = 1.2$ (m)

Mark scheme (b)

Scheme	Marks	AO
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$0.7mg = 20g(4 - \bar{x} - 0.7)$ (moments about D) $20g(\bar{x} - 0.5) + 3.5mg = (4 - 0.5 - 0.7)(20g + mg)$ (moments about C)	M1	3.1b
$m = 60$	A1	2.2a
	[2]	

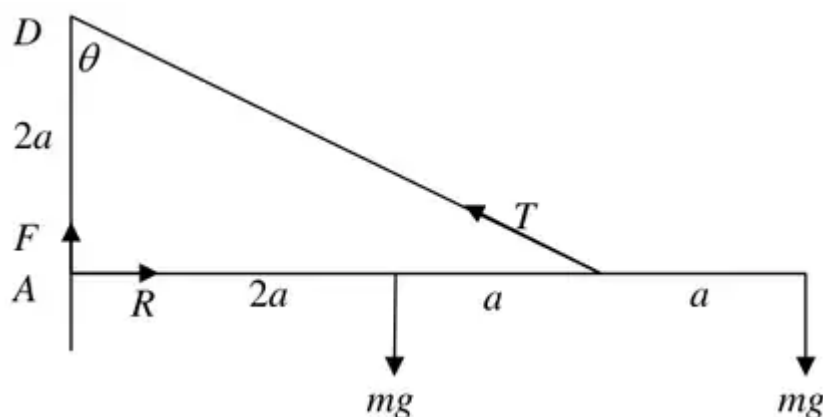
Notes

M1: Moments about D (oe) – correct number of terms – if taking moments about another point then must set $T_C = 0$ and $T_D = 20g + mg$

Allow g to be absent

4.

Mark scheme (a)



Scheme	Marks
M(A) $3a \times T \cos \theta = 2amg + 4amg$	M1 A1 A1
$\cos \theta = \left(\frac{2}{\sqrt{9+4}} \right) \frac{2}{\sqrt{13}}$	B1
$\frac{6}{\sqrt{13}}T = 6mg$	
$T = mg\sqrt{13} *$	A1
	(5)

Mark scheme (b)

Scheme	Marks
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$3a \times T \times \cos \theta = 2amg + 4aMg$	M1
$T = \frac{(2mg + 4Mg)}{6} \sqrt{13} \leq 2mg\sqrt{13}$	A1
$mg + 2Mg < 6mg$	
$M \leq \frac{5}{2}m * \cos \theta$	A1
	(3)
	(8 marks)