

QQQ – Mechanics Year 2 – Chapter 4 – Moments (Version 1)

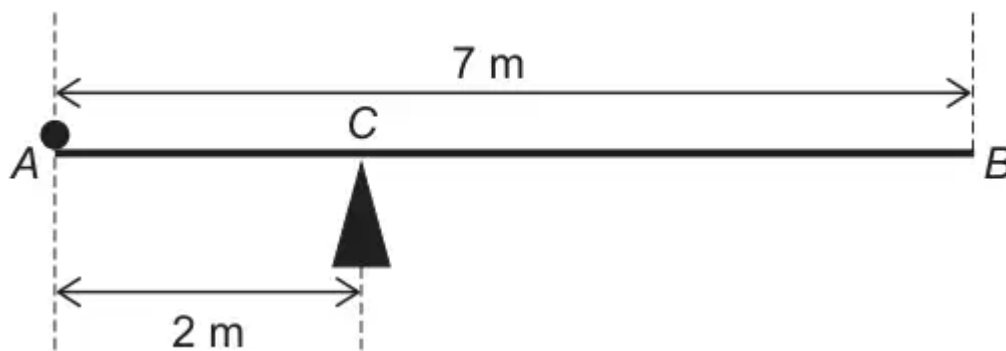
Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. A uniform rod, AB , has length 7 metres and mass 4 kilograms.

The rod rests on a single fixed pivot point, C , where $AC = 2$ metres.

A particle of weight W newtons is fixed at A , as shown in the diagram.



The system is in equilibrium with the rod resting horizontally.

(a) Find W , giving your answer in terms of g . [2 marks]

(b) Explain how you have used the fact that the rod is uniform in part (a). [1 mark]

(Total 3 marks)

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2. Fig. 4 shows a uniform beam of mass 4 kg and length 2.4 m resting on two supports P and Q. P is at one end of the beam and Q is 0.3 m from the other end.

Determine whether a person of mass 50 kg can tip the beam by standing on it. [3]

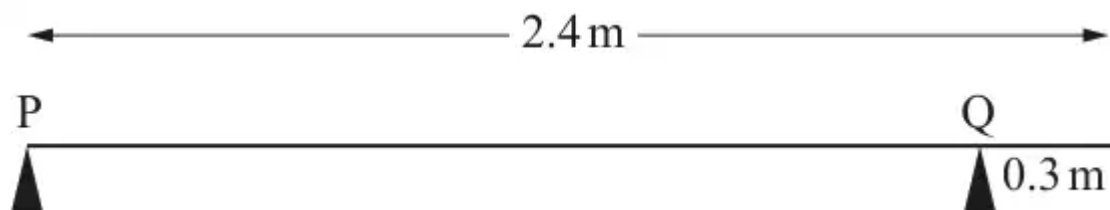


Fig. 4

(Total 3 marks)

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3.

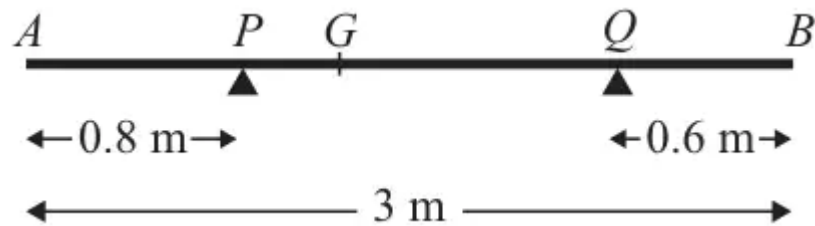


Figure 1

A non-uniform rod AB has length 3 m and mass 4.5 kg . The rod rests in equilibrium, in a horizontal position, on two smooth supports at P and at Q , where $AP = 0.8\text{ m}$ and $QB = 0.6\text{ m}$, as shown in Figure 1. The centre of mass of the rod is at G . Given that the magnitude of the reaction of the support at P on the rod is twice the magnitude of the reaction of the support at Q on the rod, find

(a) the magnitude of the reaction of the support at Q on the rod, (3)

(b) the distance AG . (4)

(Total 7 marks)

4.

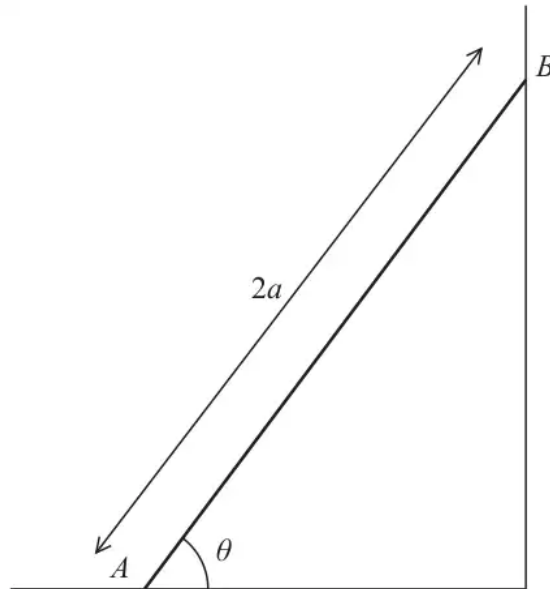


Figure 2

A beam AB has mass m and length $2a$.

The beam rests in equilibrium with A on rough horizontal ground and with B against a smooth vertical wall.

The beam is inclined to the horizontal at an angle θ , as shown in Figure 2.

The coefficient of friction between the beam and the ground is μ

The beam is modelled as a uniform rod resting in a vertical plane that is perpendicular to the wall.

Using the model,

(a) show that $\mu \geq \frac{1}{2} \cot \theta$ (5)

A horizontal force of magnitude kmg , where k is a constant, is now applied to the beam at A .

This force acts in a direction that is perpendicular to the wall and towards the wall.

Given that $\tan \theta = \frac{5}{4}$, $\mu = \frac{1}{2}$ and the beam is now in limiting equilibrium,

(b) use the model to find the value of k . (5)

(Total 10 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
Finds a moment of a force about any point. Must have the form force x distance. Can be awarded for example for 2w, 7w, 5R, 2R Condone use of 9.8, 9.81, 10 instead of g	M1	3.4
Obtains $w = 3g$ CAO	A1	1.1b
	(2)	

Typical solution

$$2W = 1.5 \times 4g$$

$$W = 3g$$

Mark scheme (b)

Scheme	Marks	AO
Explains that the weight of the rod acts through the midpoint or the centre of mass is at the midpoint (of the rod) OE	E1	3.5a
	(1)	
	(3 marks)	

Typical solution

The rod is uniform so its weight acts at the centre

2.

Scheme	Marks	AO
Let R_P and R_Q be the normal reactions upwards Assume the person stands on the end beyond Q and that the beam	M1	3.1b

is in equilibrium Taking moments about Q:		
$50g \times 0.3 + R_P \times 2.1 = 4g \times 0.9$	A1	1.1b
$R_P = \frac{3.6g - 15g}{2.1} [= -53.2] < 0$ So the beam will tip.	A1	3.2a
	[3]	

Notes

M1: Finding moment of a force about any point

If the weight of the person shown between P and Q allow M1 maximum.

A1: Correct equation from moments about Q

Could also be obtained from moments about P and resolving to evaluate R_P

A1: Conclusion must be clear from correct working (R_P need not be evaluated but must be clear that it is negative)

Alternative solution

Scheme	Marks
If the person stands x m beyond Q so that the beam is on the point of tipping $R_P = 0$ Taking moments about Q	M1
$4g \times 0.9 = 50gx$ giving $x = 0.072$	A1
$0.072 < 0.3$ so this is possible while standing on the beam	A1

M1: Finding moment of a force about any point

A1: Correct equation from moments about Q

A1: Conclusion must be clear from correct working and reference to 0.3 m

Second alternative solution

Scheme	Marks
Largest clockwise moment of the weight of the person about Q is $50g \times 0.3 = 15g [= 147]$	M1
Anticlockwise moment of the weight of the beam is $4g \times 0.9 = 3.6g [= 35.28]$	A1
The moment of the person's weight is larger so the beam will tip	A1

M1: Finding moment of a force about any point

A1: Both correct moments about Q required

A1: Conclusion must be clear from a comparison of moments of two forces

SC 2 (omitting g)

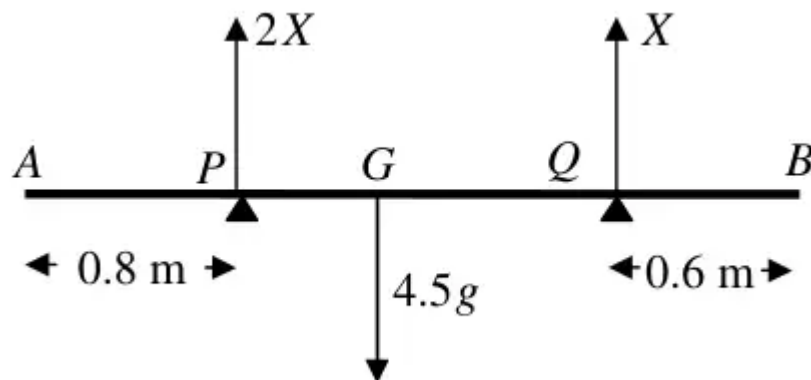
$$0.9 \times 4 < 0.3 \times 50$$

$$3.6 < 15$$

So the beam will tip.

3.

Mark scheme (a)



Scheme	Marks
$\uparrow 2X + X = 4.5g$	M1 A1
Leading to $X = \frac{3g}{2}$ or 14.7 or 15 (N)	A1
	(3)

Notes

First M1 for a complete method for finding R_Q , either by resolving vertically, or taking moments twice, with usual criteria (allow M1 even if $R_P = 2R_Q$ not substituted)

First A1 for a correct equation in either R_Q or R_P ONLY.

Second A1 for 1.5g or 14.7 or 15 (A0 for a negative answer)

Mark scheme (b)

Scheme	Marks
$M(A) \quad 4.5g \times AG = (2X) \times 0.8 + X \times 2.4$	M1 A2 ft (1,0)
$AG = \frac{4}{3}$ (m), 1.3, 1.33,...	A1

	(4)
	(7 marks)

Notes

First M1 for taking moments about any point, with usual criteria.

A2 ft for a correct equation (A1A0 one error, A0A0 for two or more errors, ignoring consistent omission of g's) in terms of X and their x (which may not be AG at this stage)

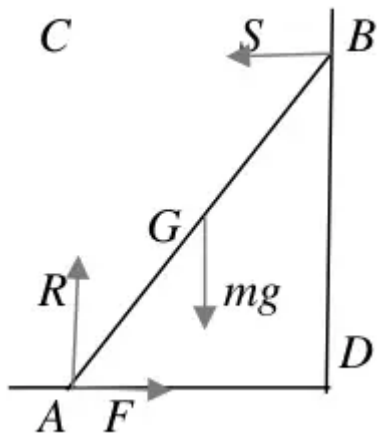
Third A1 for $AG = 4/3, 1.3, 1.33, \dots$ (any number of decimal places, since g cancels) need ' $AG =$ ' or x marked on diagram

N.B. if $R_Q = 2R_P$ throughout, mark as a misread as follows:

(a) M1A1A0 (resolution method) (b) M1A0A1A1, assuming all work follows through correctly..

4.

Mark scheme (a)

Scheme	Marks	AO
Part (a) is a 'Show that..' so equations need to be given in full to earn A marks		
		
Moments equation: (M1A0 for a moments inequality)	M1	3.3
$M(A), mga \cos \theta = 2Sa \sin \theta$ $M(B), mga \cos \theta + 2Fa \sin \theta = 2Ra \cos \theta$ $M(C), F \times 2a \sin \theta = mga \cos \theta$ $M(D), 2Ra \cos \theta = mga \cos \theta + 2Sa \sin \theta$ $M(G), Ra \cos \theta = Fa \sin \theta + Sa \sin \theta.$	A1	1.1b

$(\updownarrow) R = mg$ OR $(\leftrightarrow) F = S$	B1	3.4
Use their equations (<u>they must have enough</u>) and $F \leq \mu R$ to give an inequality in μ and θ only (allow DM1 for use of $F = \mu R$ to give an <i>equation</i> in μ and θ only)	DM1	2.1
$\mu \geq \frac{1}{2} \cot \theta$ *	A1*	2.2a
	(5)	

Notes

M1: Any moments equation with correct terms, condone sign errors and sin/cos confusion

A1: Correct equation

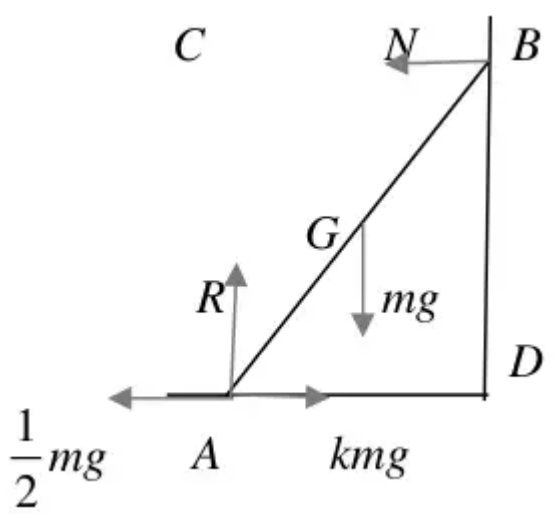
B1: Correct equation

DM1: Dependent on M1, for using their equations (they must have enough), and $F \leq \mu R$ to give an inequality in μ and θ only

(allow M1 for use of $F = \mu R$ to give an equation in μ and θ only)

A1*: Given answer correctly obtained with no wrong working seen (e.g. if they use $F = \mu R$ anywhere, A0)

Mark scheme (b)

Scheme	Marks	A0
		
Moments equation:	M1	3.4

$M(A), mga \cos \theta = 2Na \sin \theta$ $M(B),$ $mga \cos \theta + 2kmga \sin \theta = 2Ra \cos \theta + \frac{1}{2}mg2a \sin \theta$ $M(D), 2Ra \cos \theta = mga \cos \theta + N2a \sin \theta$ $M(G), kmga \sin \theta + Na \sin \theta = \frac{1}{2}mga \sin \theta + Ra \cos \theta$	A1	1.1b
S.C. $M(C), mga \cos \theta + \frac{1}{2}mg2a \sin \theta = kmg2a \sin \theta$ M1A1B1 $1 + \frac{5}{4} = \frac{5k}{2}$ M1 $k = 0.9$ A1		
$N = kmg - F$ OR $R = mg$	B1	3.3
Use their equations (<u>they must have enough</u>) to solve for k (numerical)	DM1	3.1b
$k = 0.9$ oe	A1	1.1b
	(5)	
	(10 marks)	

Notes

M1: Any moments equation with correct terms, condone sign errors

A1: Correct equation

B1: Correct equation

DM1: Dependent on M1, for using their equations (they must have enough) with trig substituted, to solve for k , which must be numerical.

A1: cao