QQQ – Mechanics Year 1 – Chapter 11 – Variable Acceleration (Calculus) (Version 2)

Total Marks: 26

(26 = Platinum, 23 = Gold, 21 = Silver, 18 = Bronze)

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1. In this question you must show detailed reasoning.

A particle moves in a straight line. Its velocity v m s^{-1} after t s is given by $v = t^3 - 5t^2$.

(a) Find the times at which the particle is stationary. **[2]**

(b) Find the total distance travelled by the particle in the first 6 seconds. **[3]**

(Total 5 marks)

2. A particle, P, is moving along a straight line such that its acceleration a m s^{-2}, at any time, t seconds, may be modelled by

$$a = 3 + 0.2t$$

When $t = 2$, the velocity of P is k m s^{-1}

(a) Show that the initial velocity of P is given by the expression $(k - 6.4)$ m s^{-1} **[4 marks]**

(b) The initial velocity of P is one fifth of the velocity when $t = 2$

Find the value of k. **[2 marks]**

(Total 6 marks)

3. A particle P is moving in a straight line. At time t seconds P has velocity v m s^{-1} where $v = (2t + 1)(3 - t)$.

(a) Find the deceleration of P when $t = 4$. **[2]**

(b) State the positive value of t for which P is instantaneously at rest. **[1]**

(c) Find the total distance that P travels between times $t = 0$ and $t = 4$. **[3]**

(Total 6 marks)

4. A car, starting from rest, is driven along a horizontal track.

The velocity of the car, v m s^{-1}, at time t seconds, is modelled by the equation

$$v = 0.48t^2 - 0.024t^3 \quad \text{for } 0 \leqslant t \leqslant 15$$

(a) Find the distance the car travels during the first 10 seconds of its journey. **[3 marks]**

(b) Find the maximum speed of the car.

Give your answer to three significant figures. **[4 marks]**

(c) Deduce the range of values of t for which the car is modelled as decelerating. **[2 marks]**

(Total 9 marks)

Solutions

1. OCR MEI October 2020 Paper 1 Q6

Mark scheme (a)

Scheme	Marks	AO
DR		
$v = t^3 - 5t^2 = 0$ $t^2(t-5) = 0$	M1	1.1a
So at rest when $t = 0$ or 5 s	A1	1.1
	[2]	

Notes

M1: Attempt to solve $v = 0$

Allow M1 for at least one root found by substitution

A1: Both roots and no others

Mark scheme (b)

Scheme	Marks	AO
DR		
There is a change of direction when $t = 5$	M1	3.4
$\int_0^5 (t^3 - 5t^2)\,\mathrm{d}t = \left[\frac{t^4}{4} - \frac{5t^3}{3}\right]_0^5 = -\frac{625}{12}$	M1	3.4
$\int_5^6 (t^3 - 5t^2)\,\mathrm{d}t = \frac{193}{12}$		
Total distance $\frac{625 + 193}{12} = \frac{409}{6} = 68.2$ m	A1	1.1
	[3]	

Notes

M1: Considering signed areas either side of $t = 5$ s.

M1: Algebraic integration seen attempted

A1: Correct to at least 2 sf

SPECIAL CASE

$$\int_0^6 \left(t^3 - 5t^2\right) \mathrm{d}t = \left[\frac{t^4}{4} - \frac{5t^3}{3}\right]_0^6 = -36$$

SCM1 for algebraic integration seen attempted

SCA1 for –36 m seen or distance 36 given

(corrected from the printed mark scheme: the integral was printed as $\left[\frac{t^4}{4} - \frac{5t^2}{3}\right]$, here and in the special case)

2. AQA June 2021 Paper 1 Q14

Mark scheme (a)

Scheme	Marks	AO
Integrates given expression to find v with at least one term correct	M1	3.4
Obtains an expression for v with both terms correct condone omission of $+c$	A1	1.1b
Substitutes $t = 2$ and $v = k$ into their integrated expression (must include constant of integration)	M1	1.1a
Completes rigorous argument with no slips to obtain $v = k - 6.4$ when $t = 0$	R1	2.1
	(4)	

Typical solution

$$v = \int a\, dt$$

$$= 3t + 0.1t^2 + c$$

When $t = 2, v = k$

$$k = 6 + 0.4 + c$$

$$c = k - 6.4$$

$$v = 3t + 0.1t^2 + k - 6.4$$

Since $v = c$ when $t = 0$ the initial velocity is $k - 6.4$

Mark scheme (b)

Scheme	Marks	AO
Forms equation using k, $k - 6.4$, and 0.2 or 5	M1	3.1b
Obtains $k = 8$	A1	1.1b
	(2)	
	(6 marks)	

Typical solution

$$0.2k = k - 6.4$$

$$k = 8$$

3. OCR A October 2020 Paper 2 Q10

Mark scheme (a)

Scheme	Marks	AO
Value of $\frac{\mathrm{d}v}{\mathrm{d}t}$ at $t = 4$ is -11	B1	3.4
Deceleration of P is $11\ \mathrm{m\ s^{-2}}$	B1	2.5
	[2]	

Notes

B1: BC

B1: Correct positive value, with units

Alternative solution

Scheme	Marks
$\frac{\mathrm{d}v}{\mathrm{d}t} = 5 - 4t$ evaluated at $t = 4$	M1
Deceleration of P is $11\ \mathrm{m\ s^{-2}}$	A1

A1: Correct positive value, with units

Mark scheme (b)

Scheme	Marks	AO
$t = 3$ s	B1	3.4
	[1]	

Notes

$-\frac{1}{2}$, if present, must be rejected

Mark scheme (c)

Scheme	Marks	AO
$\int_0^3 (2t+1)(3-t)\,\mathrm{d}t = \frac{27}{2}$	B1	3.1b
$\int_3^4 (2t+1)(3-t)\,\mathrm{d}t = -\frac{25}{6}$	B1	1.1
Total distance is $\frac{27}{2} + \frac{25}{6} = \frac{53}{3}$ m or 17.7 m	B1	1.1
	[3]	

Notes

B1: BC

B1: BC

B1: Correct answer (exact, or at least 3sf)

If no marks, then award **SC B1** for either correctly integrating v or for an answer of 28/3 or 9.33 (at least 3 sf)

4. AQA June 2019 Paper 1 Q13

Mark scheme (a)

Scheme	Marks	AO
Integrates given velocity equation, with at least one term correct PI	M1	3.4
Obtains fully correct integral PI	A1	1.1b
Substitutes correct limits to obtain correct answer. CAO. Condone omission of units.	A1	1.1b

	(3)	

Typical solution

$$s = \int_0^{10} 0.48t^2 - 0.024t^3 \, \mathrm{d}t$$

$$s = \left[0.16t^3 - 0.006t^4\right]_0^{10}$$

$$s = 100 \text{ m}$$

Mark scheme (b)

Scheme	Marks	AO
Differentiates v (at least one term correct) PI	M1	3.4
Equates their $\frac{\mathrm{d}v}{\mathrm{d}t}$ equation to zero PI	M1	1.1a
Finds correct non-zero value for t	A1	1.1b
Finds correct maximum speed 28.4 Condone exact answer $28\frac{4}{9}$. Condone omission of units.	A1	1.1b
	(4)	

Typical solution

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 0.96t - 0.072t^2$$

$$0.96t - 0.072t^2 = 0$$

$$t = 0 \text{ or } t = \frac{40}{3}$$

When $t = \frac{40}{3}$ then $v = 28.4 \text{ m s}^{-1}$

Mark scheme (c)

Scheme	Marks	AO
Deduces lower critical value of t with correct associated inequality	R1F	2.2a

Follow through their value of t from part (b) provided $0 < t < 15$		
States $t \leqslant 15$	R1	2.5
	(2)	
	(9 marks)	

Typical solution

$$\frac{40}{3} < t \leqslant 15$$