

QQQ – Further Statistics 2 – Chapter 7 – Confidence Intervals and Tests Using the t-Distribution (Version 2)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. A production line is designed to fill bottles with oil. The amount of oil placed in a bottle is normally distributed and the mean is set to 100 ml.

The amount of oil, x ml, in each of 8 randomly selected bottles is recorded, and the following statistics are obtained.

$$\bar{x} = 92.875 \quad s = 8.3055$$

Malcolm believes that the mean amount of oil placed in a bottle is less than 100 ml.

Stating your hypotheses clearly, test, at the 5% significance level, whether or not Malcolm's belief is supported. **(5)**

(Total 5 marks)

Edexcel S4 June 2014 Q1

2. Students studying for their Mathematics GCSE are assessed by two examination papers. A teacher believes that on average the score on paper I is more than 1 mark higher than the score on paper II. To test this belief the scores of 8 randomly selected students are recorded. The results are given in the table below.

Student	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>
Score on paper I	57	63	68	81	43	65	52	31
Score on paper II	53	62	61	78	44	64	43	29

Assuming that the scores are normally distributed and stating your hypotheses clearly, test at the 5% level of significance whether or not there is evidence to support the teacher's belief. **(8)**

(Total 8 marks)

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3. Brickland and Goodbrick are two manufacturers of bricks. The lengths of the bricks produced by each manufacturer can be assumed to be normally distributed. A random sample of 20 bricks is taken from Brickland and the length, x mm, of each brick is recorded. The mean of this sample is 207.1 mm and the variance is 3.2 mm^2 .

(a) Calculate the 98% confidence interval for the mean length of brick from Brickland. **(4)**

A random sample of 10 bricks is selected from those manufactured by Goodbrick. The length of each brick, y mm, is recorded. The results are summarised as follows.

$$\sum y = 2046.2 \quad \sum y^2 = 418\,785.4$$

The variances of the length of brick for each manufacturer are assumed to be the same.

(b) Find a 90% confidence interval for the value by which the mean length of brick made by Brickland exceeds the mean length of brick made by Goodbrick. **(8)**

(Total 12 marks)

Solutions

1.

Scheme	Marks
$H_0 : \mu = 100 \quad H_1 : \mu < 100$	B1
$t = \frac{ \bar{x} - \mu }{s/\sqrt{n}} = \frac{ 92.875 - 100 }{8.3055/\sqrt{8}} = 2.4264 \dots \text{ or }$ $\frac{c - 100}{8.3055/\sqrt{8}} = -1.895 \therefore \text{CR } c < 94.435$	M1A1
$t_7(5\%) = \pm 1.895$	B1
There is evidence to reject H_0 . <u>Malcolm's belief is supported</u> or there is evidence that the amount of <u>oil</u> placed in bottles is <u>less</u> than <u>100ml</u>	A1ft
	(5)

Notes

B1 both hypotheses

M1 either $\frac{|92.875 - 100|}{8.3055/\sqrt{8}}$ or $p = 0.0228$ or $\frac{c - 100}{8.3055/\sqrt{8}} = -(\text{a } t \text{ value})$

A1 awrt 2.43 or awrt 94.4 or awrt 0.0228

B1 ± 1.895 or $0.0228 < 0.05$ (must have correct comparison for hypotheses)

A1ft Do Not allow contradictions

(Corrected from the printed mark scheme: the conclusion printed "100mm" for 100 ml.)

2.

Scheme	Marks
$D = \text{Paper I score} - \text{paper II score}$	
$H_0 : \mu_D = 1 \quad H_1 : \mu_D > 1$	B1
$d: 4, 1, 7, 3, -1, 1, 9, 2$	M1
$\bar{d} = 3.25; \quad s^2 = \frac{162 - 8 \times 3.25^2}{7} = 11.07 \dots \quad (s = 3.32)$	M1;M1

$t_7 = \frac{3.25 - 1}{3.32/\sqrt{8}} = 1.9126 \dots$ awrt 1.91	M1A1
$t_7(5\%) = 1.895$	B1
There is evidence to support the teacher's belief or the score on paper I is more than one mark higher than on paper II	A1 ft
	(8)
	(8 marks)

Notes

1st M1 for attempting differences

2nd M1 for attempting $\bar{d}$

3rd M1 for attempting s_d or s_d^2 , correct expression with their $\sum d^2$ and $\bar{d}$ or correct calculation (to 2 sf or better)

4th M1 for use of $\frac{\bar{d} - 1}{s/\sqrt{8}}$, ft their values.

1st A1 awrt 1.91

2nd B1 for 1.895

2nd A1 contextual conclusion ft their values.

SC if they use a 2 sample test they may get the first B1 for $H_0 : \mu_I - \mu_{II} = 1$ and $H_1 : \mu_I - \mu_{II} > 1$

3.

Mark scheme (a)

Scheme	Marks
Confidence interval is given by $\bar{x} \pm t_{19} \times \frac{s}{\sqrt{n}}$ 2.539	B1
i.e. $207.1 \pm 2.539 \times \sqrt{\frac{3.2}{20}}$ Using $\bar{x} \pm t \times \frac{s}{\sqrt{n}}$	M1
All correct	A1
i.e. 207.1 ± 1.0156	A1
i.e. (206.08 ..., 208.1156) awrt (206, 208)	
	(4)

Mark scheme (b)

Scheme	Marks
$s_p^2 = \frac{19 \times 3.2 + 9 \times 10.2173}{28}$ 10.2173 awrt 10.2	B1 M1
= 5.45557... awrt 5.46	A1
Confidence interval is given by $\bar{x}_B - \bar{x}_G \pm t_{28} \times \sqrt{5.45557 \left(\frac{1}{20} + \frac{1}{10} \right)}$ 1.701	B1
i.e. $(207.1 - 204.62) \pm 1.701 \sqrt{5.45557 \left(\frac{1}{20} + \frac{1}{10} \right)}$ i.e. 2.48 ± 1.53875 Using $\bar{x} - \bar{y} \pm t \sqrt{s^2 \left(\frac{1}{n_x} + \frac{1}{n_y} \right)}$	M1
All correct	A1
i.e. (0.94125, 4.0187) awrt 0.941; 4.02	A1; A1
	(8)