

QQQ – Further Statistics 2 – Chapter 7 – Confidence Intervals and Tests Using the t-Distribution (Version 1)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. Jemima makes jam to sell in a local shop. The jam is sold in jars and the weight of jam in a jar is normally distributed.

Jemima takes a random sample of 8 of her jars of jam and weighs the contents of each jar, x grams. Her results are summarised as follows

$$\sum x = 3552 \quad \sum x^2 = 1\,577\,314$$

(a) Calculate a 95% confidence interval for the mean weight of jam in a jar. **(5)**

The labels on the jars state that the average contents weigh 440 grams.

(b) State, giving a reason, whether or not Jemima should be concerned about the labels on her jars of jam. **(1)**

(Total 6 marks)

Edexcel A2 October 2020 Q2

2. Alexa believes that students are equally likely to achieve the same percentage score on each of two tests, paper I and paper II. She randomly selects 8 students and gives them each paper I and paper II. The percentage scores for each paper are recorded.

The following paired data are collected.

Student	A	B	C	D	E	F	G	H
Paper I (%)	70	70	84	80	64	65	65	90
Paper II (%)	64	76	72	74	68	64	58	76

Test, at the 1% significance level, whether or not there is evidence to support Alexa's belief.

State your hypotheses clearly and show your working. **(7)**

(Total 7 marks)

Edexcel A2 June 2019 Q5

3. A farmer is investigating the milk yields of two breeds of cow. He takes a random sample of 9 cows of breed A and an independent random sample of 12 cows of breed B . For a 5 day period he measures the amount of milk, x gallons, produced by each cow. The results are summarised in the table below.

Breed	Sample size	Mean ($\bar{x}$)	Standard deviation (s_x)
A	9	6.23	2.98
B	12	7.13	2.33

The amount of milk produced by each cow can be assumed to follow a normal distribution.

- (a) Use a two-tail test to show, at the 10% level of significance, that the variances of the yields of the two breeds can be assumed to be equal. State your hypotheses clearly. **(4)**
- (b) Stating your hypotheses clearly, test, at the 5% level of significance, whether or not there is a difference in the mean yields of the two breeds of cow. **(7)**
- (c) Explain briefly the importance of the test in part (a) for the test in part (b). **(1)**

(Total 12 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
$\bar{x} = 444$	M1	2.1
$s_x^2 = \frac{1\,577\,314 - 8 \times 444^2}{7} = \frac{226}{7} = 32.2857\dots$	A1	1.1b
$t_7(5\%)$ 2-tail cv = 2.365	B1	1.1b
95% CI for μ is: $444 \pm 2.365 \times \sqrt{\frac{32.2857\dots}{8}}$	M1	2.1
$= (439.248\dots, 448.75\dots) = \text{awrt } (439, 449)$	A1	1.1b
	(5)	

Notes

1st M1 for finding mean and attempting s^2

1st A1 for correct mean and s^2 (accept awrt 3sf)

B1 for a correct cv of 2.365 or better

2nd M1 for use of correct formula, ft their mean, s_x and cv for t (use of 1.96 is M0)

2nd A1 for awrt (439, 449)

Mark scheme (b)

Scheme	Marks	AO
440 is in CI so the average contents statement is OK	B1	2.2b
	(1)	
	(6 marks)	

Notes

1st B1 for correct statement about 440 and interval and conclusion

2.

Scheme	Marks	AO
$d: 6 \quad -6 \quad 12 \quad 6 \quad -4 \quad 1 \quad 7 \quad 14$	M1	3.1b
$\bar{d} = \pm 4.5 \quad s_d = \sqrt{50.285 \dots} = 7.09 \dots$	M1	1.1b
$H_0 : \mu_d = 0 \quad H_1 : \mu_d \neq 0$	B1	3.3
$t = \pm \frac{\text{"4.5"} \sqrt{8}}{\text{"7.09 \dots"}} \text{ oe}$	M1	1.1b
$= \pm 1.7948 \dots \text{ awrt } \pm 1.79/1.8$	A1	1.1b
Critical value $t_7 = \pm 3.499$	B1	1.1b
There is insufficient evidence that the papers are of a different level of difficulty or Alexa's belief is correct	A1ft	2.2b
	(7)	
	(7 marks)	

Notes

M1: for realising that the model to use is the paired t -test and finding the differences ($\pm$) At least 3 correct

M1: correct method for finding $\bar{d}$ and s_d .

B1: Using a correct model for difference and both hypotheses correct using the notation μ_d or μ
Condone $\mu_I = \mu_{II}$ and $\mu_I \neq \mu_{II}$

M1: Using the correct method to find test statistics ie $t = \pm \frac{\text{"their 4.5"} \sqrt{8}}{\text{"their 7.09 \dots"}}$

A1: awrt 1.79 or 1.8

B1: for correct critical value $t = \pm 3.499$ with compatible sign

A1ft: Drawing a correct inference in context using their CV and their value of t

NB difference of means test gets MOMOB1M0A0BOAO

3.

Mark scheme (a)

Scheme	Marks
$H_0 : \sigma_A^2 = \sigma_B^2 \quad H_1 : \sigma_A^2 \neq \sigma_B^2$	B1

$(F_{8,11} =) \frac{2.98^2}{2.33^2} = (1.6357 \dots)$	M1
$F_{8,11}$ 10% (two-tail) cv = 2.95 (or prob. = awrt 0.22)	B1
Not significant so can accept the assumption that variances are equal.	A1
	(4)

Notes

1st B1 allow σ or σ^2

M1 for use of the correct test statistic

Mark scheme (b)

Scheme	Marks
$H_0 : \mu_A = \mu_B \quad H_1 : \mu_A \neq \mu_B$	B1
$s_p^2 = \frac{8 \times 2.98^2 + 11 \times 2.33^2}{19}, = 6.88216 \dots$ or $s_p = 2.62338 \dots$	M1, A1
$(t_{19} =) (\pm) \frac{7.13 - 6.23}{s_p \sqrt{\frac{1}{9} + \frac{1}{12}}} = (\pm) 0.7780047 \dots = \text{awrt } \underline{\underline{0.778}}$	M1 A1
$t_{19}(0.05)$ two-tail cv = 2.093	B1
[Not significant] Insufficient evidence of a <u>difference in mean</u> milk <u>yields</u> between the two <u>breeds</u>	A1
	(7)

Notes

1st M1 for attempting s_p or s_p^2

1st A1 for awrt 6.90 or 2.63

2nd M1 for use of a correct test statistic

2nd A1 for awrt 0.77 (accept $\pm$)

2nd B1 for 2.093 (allow ± 1.729 for one-tailed H_1)

Mark scheme (c)

Scheme	Marks
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Test in part(b) requires the variances to be equal. The test in part (a) showed that the variances could be assumed to be equal.	B1
	(1)
	(12 marks)