

QQQ – Further Statistics 2 – Chapter 6 – Further Hypothesis Tests (Version 2)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. Find the value of the constant a such that

$$P(a < F_{8,10} < 3.07) = 0.94$$

(2)

(Total 2 marks)

Edexcel S4 June 2011 Q1

2. The sample variance of the lengths of a random sample of 9 paving slabs sold by a builders' merchant is 36 mm^2 . The sample variance of the lengths of a random sample of 11 paving slabs sold by a second builders' merchant is 225 mm^2 . Test at the 10% significance level whether or not there is evidence that the lengths of paving slabs sold by these builders' merchants differ in variability. State your hypotheses clearly.

(You may assume the lengths of paving slabs are normally distributed.)

(5)

(Total 5 marks)

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3. The concentration of an air pollutant is measured in micrograms/m³

Samples of air were taken at two different sites and the concentration of this particular air pollutant was recorded.

For Site *A* the summary statistics are shown below.

	number of samples	s_A^2
Site A	13	6.39

For Site *B* there were 9 samples of air taken.

A test of the hypothesis $H_0 : \sigma_A^2 = \sigma_B^2$ against the hypothesis $H_1 : \sigma_A^2 \neq \sigma_B^2$ is carried out using a 2% level of significance.

(a) State a necessary assumption required to carry out the test. **(1)**

Given that the assumption in part (a) holds,

(b) find the set of values of s_B^2 that would lead to the null hypothesis being rejected, **(4)**

(c) find a 99% confidence interval for the variance of the concentration of the air pollutant at **Site A**. **(3)**

(Total 8 marks)

4. A researcher is investigating the accuracy of IQ tests. One company offers IQ tests that it claims will give any individual's IQ with a standard deviation of 5

The researcher takes these tests 9 times with the following results

123, 118, 127, 120, 134, 120, 118, 135, 121

(a) Find the sample mean, $\bar{x}$, and the sample variance, s^2 , of these scores. (2)

Given that any individual's IQ scores on these tests are independent and have a normal distribution,

(b) use the hypotheses

$$H_0 : \sigma^2 = 25 \quad \text{against} \quad H_1 : \sigma^2 > 25$$

to test the company's claim at the 5% significance level. (4)

Gurdip works for the company and has taken these IQ tests 12 times. Gurdip claims that the sample variance of these 12 scores is $s^2 = 8.17$

(c) Use this value of s^2 to calculate a 95% confidence interval for the variance of Gurdip's IQ test scores.

[You may use $P(\chi_{11}^2 > 3.816) = 0.975$ and $P(\chi_{11}^2 > 21.920) = 0.025$] (2)

(d) Assuming that $\sigma^2 = 25$, comment on Gurdip's claim. (1)

(Total 9 marks)

Solutions

1.

Scheme	Marks
$P(F_{8,10} > 3.07) = 0.05$	
So need $P(F_{10,8} > x) = 0.01$ so $x = 5.81$	B1
So $a = \frac{1}{5.81} = \underline{\underline{0.172}}$ awrt 0.172	B1
	(2 marks)

2.

Scheme	Marks
$H_0 : \sigma_A^2 = \sigma_B^2; H_1 : \sigma_A^2 \neq \sigma_B^2$	B1
$S_A^2/S_B^2 = \frac{225}{36} = 6.25 \quad \left(\frac{36}{225} = 0.16 \right)$	M1A1
CR: $F_{10,8} > 3.35 \quad \left(\frac{1}{F_{10,8}} = 0.299 \right)$	B1
Since 6.25 is in the critical region we can assume that the lengths of paving slabs sold by the builders merchant differ in variability.	A1ft
	(5)
	(5 marks)

Notes

B1 both correct. Must use σ . May use different notation to A and B

M1 $\frac{225}{36}$ or $\frac{36}{225}$ allow $\frac{15}{6}$ or $\frac{6}{15}$

A1 either 6.25 or 0.16

B1 CR must match their method

A1 context must include “lengths of slabs”

3.

Mark scheme (a)

Scheme	Marks	A0
The concentration of <u>air pollutant</u> (for each site) follows a normal distribution	B1	2.4
	(1)	

Notes

B1: Correct modelling assumption. Allow air pollutant samples are independent.
Samples of air pollutant are normally distributed is B0

Mark scheme (b)

Scheme	Marks	A0
$F_{12,8(0.01)} = 5.67$ or $F_{8,12(0.01)} = 4.5(0)$	B1	3.3
$\frac{6.39}{s_B^2} > '5.67'$ $\frac{s_B^2}{6.39} > '4.50'$	M1M1	1.1b 2.1
$(0 <) s_B^2 < 1.12698 \dots, s_B^2 > 28.755$	A1	1.1b
	(4)	

Notes

B1: Setting up either F statistic

M1: Either correct inequality (condone equation)

M1: Understanding both correct inequalities are needed (condone equation)

A1: Correct range of values with awrt 1.13 and awrt 28.8

$s_B^2 < 1.12698 \dots$ and $s_B^2 > 28.755$ is A0

Mark scheme (c)

Scheme	Marks	A0
$\chi_{12,0.005}^2 = 28.3$ or $\chi_{12,0.995}^2 = 3.074$	M1	3.3
$\frac{12 \times 6.39}{\text{"28.3"}} < \sigma_A^2 < \frac{12 \times 6.39}{\text{"3.074"}}$	M1	1.1b
$2.7095 \dots < \sigma_A^2 < 24.94469 \dots$	A1	1.1b
	(3)	
	(8 marks)	

Notes

M1: For realising a χ^2 distribution must be used as a model with at least one value correct. Allow $\chi^2_{12,0.01} = 26.217$ or $\chi^2_{12,0.99} = 3.571$ for this mark.

M1: Setting up 99% CI

A1: Correct CI with awrt 2.71 and awrt 24.9

4.

Mark scheme (a)

Scheme	Marks
$\bar{x} = \frac{\sum x}{n} = \frac{1116}{9} = 124$	B1
$s^2 = \frac{9}{8} \left(\frac{138728}{9} - 124^2 \right) = 43$ Or $s^2 = \frac{1}{8} \left(138728 - \frac{1116^2}{9} \right) = 43$	B1
	(2)

Notes

B1 124

B1 43

Mark scheme (b)

Scheme	Marks
Test stat $\chi^2 = \frac{8 \times 43}{25} = 13.76$	M1A1
Critical value $\chi^2 = 15.507$	B1
Therefore not in critical region, insufficient evidence to reject H_0 There is evidence at the 5% level that the company's claim is supported	B1d
	(4)

Notes

M1 $\frac{8 \times \text{their } 43}{25}$

A1 awrt 13.8

B1 15.507

B1 dep on previous M1 being awarded. Allow the standard deviation of the **IQ** scores is 5 oe. Must have IQ

Mark scheme (c)

Scheme	Marks
CI given by $\frac{11 \times 8.17}{21.920} < \sigma^2 < \frac{11 \times 8.17}{3.816}$	M1
Therefore $4.0999 \dots < \sigma^2 < 23.55 \dots$ awrt 4.10 and 23.6	A1
	(2)

Notes

M1 $\frac{11 \times 8.17}{3.816 \text{ or } 21.92}$

A1 both correct

Mark scheme (d)

Scheme	Marks
$\sigma^2 = 25$ is not in CI which suggests Gurdip's(his) claim may not be true.	B1ft
	(1)
	(9 marks)

Notes

B1ft their interval from part(c). Gurdip's claim may not be true

NB, no interval in (c) then B0