

QQQ – Further Statistics 2 – Chapter 5 – Estimation, Confidence Intervals and Tests Using a Normal Distribution (Version 2)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. A biased coin has a probability p of landing on heads, where $0 < p < 1$

Simon spins the coin n times and the random variable X represents the number of heads.

Taruni spins the coin m times, $m \neq n$, and the random variable Y represents the number of heads.

Simon and Taruni want to combine their results to find unbiased estimators of p .

Simon proposes the estimator $S = \frac{X + Y}{m + n}$ and Taruni proposes $T = \frac{1}{2} \left[\frac{X}{n} + \frac{Y}{m} \right]$

- (a) Show that both S and T are unbiased estimators of p . (3)

- (b) Prove that, for all values of m and n , S is the better estimator. (4)

(Total 7 marks)

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2. The waiting times, in minutes, of patients at a doctor's surgery follows a normal distribution with unknown mean μ and known standard deviation σ

A random sample of 120 patients was taken.

- (a) Find, in the form $k\sigma$, the width of a 99% confidence interval for μ based on this sample.
Give the value of k to 2 decimal places. (3)

A further random sample of 100 patients from the surgery gave a 90% confidence interval for μ of (5.14, 6.25)

- (b) Use this confidence interval to determine whether or not it provides evidence that $\mu = 6$
State the hypotheses being tested here and write down the significance level being used.
You do not need to carry out any further calculations. (3)
- (c) Find the value of σ (3)

(Total 9 marks)

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3. An engineer has developed a new battery. She claims that the new battery will last more than 8 hours longer, on average, than the old battery. To test the claim, the engineer randomly selects a sample of 50 new batteries and 40 old batteries. She records how long each battery lasts, x hours for the new batteries and y hours for the old batteries. The results are summarised in the table below.

	n	Sample mean	s^2
New battery	50	$\bar{x} = 83$	7
Old battery	40	$\bar{y} = 74$	6

- (a) Test, at the 5% level of significance, whether or not there is evidence to support the engineer's claim. State your hypotheses and show your working clearly. (7)
- (b) Explain the relevance of the Central Limit Theorem to the test in part (a). (2)

(Total 9 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
$X \sim B(n, p)$ so $E(X) = np$ and $Y \sim B(m, p)$ so $E(Y) = mp$	M1	3.3
$E(S) = \frac{E(X + Y)}{n + m} = \frac{np + mp}{n + m} = p$ so S is unbiased	M1	3.4
$E(T) = \frac{1}{2} \left[\frac{E(X)}{n} + \frac{E(Y)}{m} \right] = \frac{1}{2} \left[\frac{np}{n} + \frac{mp}{m} \right] = \frac{1}{2} \times 2p = p$ so T is unbiased	Alcso	1.1b
	(3)	

Notes

1st M1 for selecting correct models for X and Y

2nd M1 for using these models to show that either S or T is unbiased

Alcso for correctly showing that both are unbiased.

Mark scheme (b)

Scheme	Marks	AO
$\text{Var}(S) = \frac{np(1-p) + mp(1-p)}{(n+m)^2} = \frac{p(1-p)}{n+m}$	M1	2.1
$\text{Var}(T) = \frac{1}{4} \left[\frac{np(1-p)}{n^2} + \frac{mp(1-p)}{m^2} \right] = \frac{p(1-p)(m+n)}{4nm}$	A1	1.1b
$\text{Var}(S) < \text{Var}(T) \Rightarrow \frac{p(1-p)}{n+m} < \frac{p(1-p)(m+n)}{4mn} \Leftrightarrow 4mn < (n+m)^2$	M1	1.1b
$\Leftrightarrow 0 < m^2 + 2mn + n^2 - 4mn \Leftrightarrow 0 < (m-n)^2$ So S always has the smaller variance and is the better estimator	Alcso	2.2a

	(4)	
	(7 marks)	

Notes

1st M1 for a correct attempt at $\text{Var}(S)$ or $\text{Var}(T)$ (need not be simplified)

1st A1 for both correct variances

2nd M1 for a correct inequality in m and n and a first step to clear denominators

2nd A1cso for a correct proof and conclusion

2.

Mark scheme (a)

Scheme	Marks
$2 \times 2.5758 \times \frac{\sigma}{\sqrt{120}} = 0.47027 \dots \sigma$	M1B1A1
	(3)

Notes

1st M1 Use of $2z \frac{\sigma}{\sqrt{n}}$ with $z > 2$

1st B1 2.58 or better

1st A1 awrt 0.47σ

Mark scheme (b)

Scheme	Marks
$H_0 : \mu = 6 \quad H_1 : \mu \neq 6$	B1
(Significance level =)10%	B1
(6 is in the interval so not significant, do not reject H_0) $\mu = 6$	B1
	(3)

Notes

1st B1 Both hypotheses in terms of μ .

2nd B1 10%

3rd B1 Correct comment leading to accepting H_0 **Mark scheme (c)**

Scheme	Marks
$1.6449 \times \frac{\sigma}{\sqrt{100}} = (6.25 - 5.14)/2 (= 0.555)$	M1B1
$\sigma = 3.374 \dots$	A1
	(3)
	(9 marks)

Notes

1st M1 for $z \frac{\sigma}{\sqrt{100}} = 0.555$ oe, using $n = 100$ and where $|z| > 1.5$

1st B1 for 1.6449 or better in an attempt (could be $1.6449\sigma = c$ or even $1.6449 \sigma^2 = c$)

1st A1 awrt 3.37. Allow awrt 3.38 from use of $z = 1.64$

3.**Mark scheme (a)**

Scheme	Marks
$H_0 : \mu_{\text{new}} - \mu_{\text{old}} = 8$	B1
$H_1 : \mu_{\text{new}} - \mu_{\text{old}} > 8$	B1
$z = \frac{\pm(83 - 74 - 8)}{\sqrt{\frac{7}{50} + \frac{6}{40}}}$	M1
$z = \frac{\pm 1}{0.5385 \dots} = 1.86$	dM1A1
cv $z = 1.6449$	B1
$(1.86 > 1.6449)$ so reject H_0	
Evidence to support engineer's claim (that the new battery will last more than 8 hours longer than the old battery).	A1cso
	(7)

Notes

1st B1 Accept equivalent rearranged equation. Definitions of parameters must be clear e.g. use of 1 and 2 without definitions scores B0. Accept ' x ' for 'new' and ' y ' for 'old' as defined in the question.

2nd B1 Accept equivalent rearranged strict inequality. Definitions of parameters must be clear e.g. use of 1 and 2 without definitions scores B0. Accept ' x ' for 'new' and ' y ' for 'old' as defined in the question.

M1 for attempting standard error. Condone swapping 7 and 6. Accept 6.86 and 5.85 for 7 and 6.

dM1 for 1/ "their standard error"

A1 for awrt 1.86

NB -1.86 is A0.

If 8 missing from H_1 then accept $z = \frac{9}{0.5385 \dots} = \text{awrt } 16.7$ and must be consistent with their H_1 .

B1 Accept $\pm$ or probability of 0.9686

(Or $0.95 < 0.9686$)

Alcso Correct comment in context. Must mention "engineers claim" or "battery", "old", "new" and "8".

Mark scheme (b)

Scheme	Marks
Sample sizes are large	B1
CLT guarantees sample means ($\bar{X}$ and $\bar{Y}$) are approximately normally distributed.	B1
	(2)
	(9 marks)

Notes

2nd B1 Must mention **means** and **normal**. No assumptions are being made so B0 if key to answer.