

QQQ – Further Statistics 2 – Chapter 5 – Estimation, Confidence Intervals and Tests Using a Normal Distribution (Version 1)

Total Marks: 22

(22 = Platinum, 20 = Gold, 18 = Silver, 15 = Bronze)

1. A random sample of the daily sales (in £s) of a small company is taken and, using tables of the normal distribution, a 99% confidence interval for the mean daily sales is found to be

$$(123.5, 154.7)$$

Find a 95% confidence interval for the mean daily sales of the company.

(6)

(Total 6 marks)

Edexcel S3 June 2007 Q6

2. A population has mean μ and variance σ^2 .

A random sample of size 3 is to be taken from this population and $\bar{X}$ denotes its sample mean.

A second random sample of size 4 is to be taken from this population and $\bar{Y}$ denotes its sample mean.

- (a) Show that unbiased estimators for μ are given by

(i) $\hat{\mu}_1 = \frac{1}{3}\bar{X} + \frac{2}{3}\bar{Y},$ **(2)**

(ii) $\hat{\mu}_2 = \frac{5\bar{X} + 4\bar{Y}}{9}.$ **(2)**

- (b) Calculate $\text{Var}(\hat{\mu}_1)$ **(2)**

- (c) Given that $\text{Var}(\hat{\mu}_2) = \frac{37}{243}\sigma^2$, state, giving a reason, which of these two estimators should be used. **(1)**

(Total 7 marks)

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3. A company produces two colours of candles, blue and white. The standard deviation of the burning times of the blue candles is 2.6 minutes and the standard deviation of the burning times of the white candles is 2.4 minutes.

Nissim claims that the mean burning time of blue candles is more than 5 minutes greater than the mean burning time of white candles.

A random sample of 90 blue candles is found to have a mean burning time of 39.5 minutes. A random sample of 80 white candles is found to have a mean burning time of 33.7 minutes.

- (a) Stating your hypotheses clearly, use a suitable test to assess Nissim's belief. Use a 1% level of significance. **(6)**
- (b) Explain how the hypothesis test in part (a) would be carried out differently if the variances of the burning times of candles were unknown. **(1)**

The burning times for the candles may not follow a normal distribution.

- (c) Describe the effect this would have on the calculations in the hypothesis test in part (a). Give a reason for your answer. **(2)**

(Total 9 marks)

Solutions

1.

Scheme	Marks
$\bar{x} = \frac{1}{2}(123.5 + 154.7) = \underline{139.1}$	B1
2.5758	B1
“their 2.5758” $\frac{\sigma}{\sqrt{n}} = 154.7 - 139.1 = 15.6$	M1
AWRT 1.96	B1
“their 1.96” $\frac{\sigma}{\sqrt{n}} = \frac{15.6 \times 1.96}{2.5758} = (11.87 \dots)$	M1
So 95% C.I. = $139.1 \pm 11.87 \dots = (127.22 \dots, 150.97 \dots)$ AWRT (<u>127, 151</u>)	A1
	(6)
	(6 marks)

Notes

1st B1 for mean = 139.1 only

1st M1 for UL – mean or mean – LL set equal to z value times standard error or some equivalent expression for standard error. Follow through their 2.5758 provided a z value.

May be implied by $\frac{\sigma}{\sqrt{n}} = 6.056 \dots$ [N.B. $\frac{15.6}{2.3263} = 6.705 \dots$]

Condone poor notation for standard error if it is being used correctly to find CI.

2nd M1 for full method for semi-width (or width) of 95% interval

Follow through their z values for both M marks

N.B. Use of 2.60 instead of 2.5758 should just lose 2nd B1 since it leads to AWRT (127, 151)

2.

Mark scheme (a)

Scheme	Marks
(i) $E\left(\frac{1}{3}\bar{X} + \frac{2}{3}\bar{Y}\right) = \frac{1}{3}E\left(\frac{X_1 + X_2 + X_3}{3}\right) + \frac{2}{3}E\left(\frac{Y_1 + Y_2 + Y_3 + Y_4}{4}\right)$	M1

$= \frac{1}{3}\mu + \frac{2}{3}\mu$ $= \mu \text{ therefore unbiased estimator}$	A1
	(2)
$(ii) E\left(\frac{5\bar{X} + 4\bar{Y}}{9}\right) = \frac{1}{9} (5E(\bar{X}) + 4E(\bar{Y}))$	M1
$= \frac{1}{9}(5\mu + 4\mu)$ $= \mu \text{ therefore unbiased estimator}$	A1
	(2)

Mark scheme (b)

Scheme	Marks
$\text{Var}(\bar{X}) = \frac{\sigma^2}{3}, \quad \text{Var}(\bar{Y}) = \frac{\sigma^2}{4}$ $\text{Var}(\hat{\mu}_1) = \frac{1}{9} \cdot \frac{\sigma^2}{3} + \frac{4}{9} \cdot \frac{\sigma^2}{4} = \frac{4\sigma^2}{27}$	M1 A1
	(2)

Mark scheme (c)

Scheme	Marks
$\frac{4}{27}\sigma^2 < \frac{37}{243}\sigma^2 \text{ so use } \hat{\mu}_1.$	B1
	(1)
	(7 marks)

3.

Mark scheme (a)

Scheme	Marks	A0
$H_0 : \mu_{b(lue)} = \mu_{w(hite)} + 5 \quad H_1 : \mu_{b(lue)} > \mu_{w(hite)} + 5$	B1	2.1
$s.e. = \sqrt{\frac{2.6^2}{90} + \frac{2.4^2}{80}}$	M1	1.1b

$z = \frac{39.5 - 33.7 - 5}{\sqrt{\frac{2.6^2}{90} + \frac{2.4^2}{80}}} = 2.085773 \dots$ awrt 2.09	M1 A1	3.1b 1.1b
CV = 2.3263 [or p -value = 0.01849...]	B1	1.1b
Not significant, insufficient evidence to support Nissim's claim.	A1	2.2b
	(6)	

Notes

B1: Both hypotheses (oe) correct with correct notation (if using μ_x and μ_y these must be defined).

M1: Calculation of standard error

M1: Standardising using normal distribution test statistic for difference of two means with known variance

A1: awrt 2.09

B1: Correct critical value 2.3263 or better

A1: Drawing a correct inference in context

Mark scheme (b)

Scheme	Marks	AO
Use the t -test or (Since sample sizes are large,) use s^2 as an approximation to σ^2	B1	2.4
	(1)	

Notes

B1: Correct explanation

Mark scheme (c)

Scheme	Marks	AO
Since sample size is large, by the Central Limit Theorem, sample means will be (approximately) normally distributed	B1	2.4
...so no effect as the calculations in part (a) can still be carried out.	dB1	3.2b
	(2)	

	(9 marks)	
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Notes

B1: Understanding that the assumptions required for the hypothesis test, by CLT sample means follow a normal distribution

dB1: (dep on previous B1) Correct evaluation