

QQQ – Further Statistics 2 – Chapter 4 – Combinations of Random Variables (Version 2)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. The random variable $X \sim N(5, 0.4^2)$ and the random variable $Y \sim N(8, 0.1^2)$

X and Y are independent random variables.

A random sample of a independent observations is taken from the distribution of X and one observation is taken from the distribution of Y

The random variable $W = X_1 + X_2 + X_3 + \dots + X_a + bY$ and has the distribution $N(169, 2^2)$

Find the value of a and the value of b **(6)**

(Total 6 marks)

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2. A company packs chickpeas into small bags and large bags.

The weight of a small bag of chickpeas is normally distributed with mean 500 g and standard deviation 5 g

A random sample of 3 small bags of chickpeas is taken.

- (a) Find the probability that the total weight of these 3 bags of chickpeas is between 1490 g and 1530 g **(3)**

The weight of a large bag of chickpeas is normally distributed with mean 1020 g and standard deviation 20 g

One large bag and one small bag of chickpeas are chosen at random.

- (b) Calculate the probability that the weight of the large bag of chickpeas is at least 30 g more than twice the weight of the small bag of chickpeas.
Show your working clearly. **(6)**

(Total 9 marks)

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3. The weights of eggs are normally distributed with mean 60 g and standard deviation 5 g

Sairah chooses 2 eggs at random.

(a) Find the probability that the difference in weight of these 2 eggs is more than 2 g **(5)**

Sairah is packing eggs into cartons. The weight of an empty egg carton is normally distributed with mean 40 g and standard deviation 1.5 g

(b) Find the distribution of the total weight of a carton filled with 12 randomly chosen eggs. **(3)**

(c) Find the probability that a randomly chosen carton, filled with 12 randomly chosen eggs, weighs more than 800 g **(2)**

(Total 10 marks)

Solutions

1.

Scheme	Marks	AO
$5a + 8b = 169$	B1	1.1b
$0.16a + 0.01b^2 = 4$	B1	1.1b
$a = 33.8 - 1.6b \rightarrow 0.16(33.8 - 1.6b) + 0.01b^2 = 4$	M1	2.1
$0.01b^2 - 0.256b + 1.408 = 0 \rightarrow b = \frac{0.256 \pm \sqrt{0.256^2 - 4(0.01)(1.408)}}{2(0.01)}$	M1	1.1b
$b = 8, a = 21$ (reject $b = 17.6, a = 5.64$ since $a \in \mathbb{Z}^+$)	A1A1	1.1b 2.2a
	(6 marks)	

Notes

B1: Correct equation for the means

B1: Correct equation for the variances (allow $0.4^2a + 0.1^2b^2 = 4$)

M1: Attempt to solve two simultaneous equations in a and b by eliminating one variable

M1: Attempt to solve their quadratic (must be seen if answers are incorrect)

A1: $b = 8$ or $a = 21$ or both sets of values of a and b without rejecting

A1: Choosing correct pair of solutions $b = 8, a = 21$ only

2.

Mark scheme (a)

Scheme	Marks	AO
Let $T = S_1 + S_2 + S_3$ then $E(T) = 1500$	M1	3.3
$\text{Var}(T) = 75$	M1	2.1
$P(1490 < T < 1530) = 0.8756 \dots$	A1	1.1b
	(3)	

Notes

M1: Selecting and using the appropriate model and attempting 3×500

M1: For realising the need to use $\text{Var}(S) + \text{Var}(S) + \text{Var}(S) = 3 \times 5^2$

A1: awrt 0.876

Mark scheme (b)

Scheme	Marks	AO
Let $W = \pm(L - 2S - 30)$ then $E(W) = \pm(1020 - 2 \times 500 - 30)$ or Let $X = \pm(L - 2S)$ then $E(X) = \pm(1020 - 2 \times 500)$	M1	3.3
$E(W) = -10$ (or 10) or $E(X) = 20$ (or -20)	A1	1.1b
$\text{Var}(\dots) = 20^2 + 4 \times 5^2$	M1	2.1
$\text{Var}(\dots) = 500$	A1	1.1b
$P(W > 0)$ or $P(X > 30)$ (or $P(W < 0)$ or $P(X < 30)$)	M1	2.1
$= 0.3273 \dots$	A1	1.1b
	(6)	
	(9 marks)	

Notes

M1: Selecting and using the appropriate model $\pm(L - 2S - 30)$ or $\pm(L - 2S)$ in an attempt to find the expected value

A1: -10 or 20 (or 10 or -20)

M1: For realising they need to use $\text{Var}(L) + 4\text{Var}(S) = 20^2 + 4 \times 5^2$

A1: 500 only

M1: dependent on using an appropriate model and realising that $P(W > 0)$ (or $P(W < 0)$) or $P(X > 30)$ (or $P(X < 30)$) is required. May be implied by awrt 0.327

Using standardisation look for e.g.

$$P\left(Z > \frac{0 - \text{“} - 10\text{”}}{\sqrt{500}}\right) \text{ or } P\left(Z > \frac{30 - \text{“} 20\text{”}}{\sqrt{500}}\right) (= P(Z > 0.4472 \dots))$$

A1: awrt 0.327

3.

Mark scheme (a)

Scheme	Marks
X_i be rv 'weight of i th randomly chosen egg'	
$E(X_1 - X_2) = 0$	B1
$\text{Var}(X_1 - X_2) = 2 \times 5^2 = 50$	B1
$P(X_1 - X_2 > 2) = 2P(X_1 - X_2 > 2)$	M1
$= 2P\left(Z > \frac{2}{\sqrt{50}}\right)$ $= 2P(Z > 0.2828\dots)$	dM1
$= 2(1 - 0.6103) = 0.7794$ awrt 0.777-0.779	A1
	(5)

Notes

B1 for 0

B1 for 50

M1 for $|X_1 - X_2| > 2$ seen. Accept $X_1 - X_2 > 2$ provided a subsequent doubling of the probability is seen. i.e. 0.3897×2 .

dM1 standardise with their 0 and their $\sqrt{50}$ dependent on previous M.

A1 awrt 0.777-0.779

Mark scheme (b)

Scheme	Marks
$W = C + X_1 + X_2 + \dots + X_{12}$	
$E(W) = 40 + 12 \times 60 = 760$	B1
$\text{Var}(W) = 1.5^2 + 12 \times 5^2$	M1
$= 302.25$	A1
Distribution is $N(760, 302.25)$	
	(3)

Notes

B1 for 760

M1 requires squares

A1 cao

Mark scheme (c)

Scheme	Marks
$P(W > 800) = P\left(Z > \frac{800 - 760}{\sqrt{302.25}}\right)$ $= 1 - P(Z < 2.3007 \dots)$	M1
$= 0.0107 \quad \text{awrt } 0.0107$	A1
	(2)
	(10 marks)

Notes

Must be finding correct probability (ie $P(W > 800)$ or $P(Z > 2.3007 \dots)$ etc) and standardise with 800 and their 760 and their $\sqrt{302.25}$

A1 awrt 0.0107 from correct working.