

QQQ – Further Statistics 2 – Chapter 4 – Combinations of Random Variables (Version 1)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. The random variable A is defined as

$$A = B + 4C - 3D$$

where B , C and D are independent random variables with

$$B \sim N(6, 2^2) \quad C \sim N(7, 3^2) \quad D \sim N(4, 1.5^2)$$

Find $P(A < 45)$

(6)

(Total 6 marks)

Edexcel S3 June 2014 Q4

2. The weights of eggs, E grams, follow a normal distribution, $N(60, 3^2)$

The weights of empty small boxes, S grams, follow a normal distribution, $N(24, 1.8^2)$

The weights of empty large boxes, L grams, follow a normal distribution, $N(40, 2.1^2)$

Small boxes of eggs contain 6 randomly selected eggs.

Large boxes of eggs contain 12 randomly selected eggs.

(a) Find the probability that the total weight of a randomly selected small box of 6 eggs weighs less than 387 grams. **(3)**

(b) Find the probability that a randomly selected large box of 12 eggs weighs more than twice a randomly selected small box of 6 eggs. **(5)**

(Total 8 marks)

Edexcel A2 June 2023 Q4

3. Philip and James are racing car drivers. Philip's lap times, in seconds, are normally distributed with mean 90 and variance 9. James' lap times, in seconds, are normally distributed with mean 91 and variance 12. The lap times of Philip and James are independent. Before a race, they each take a qualifying lap.

(a) Find the probability that James' time for the qualifying lap is less than Philip's. **(4)**

The race is made up of 60 laps. Assuming that they both start from the same starting line and lap times are independent,

(b) find the probability that Philip beats James in the race by more than 2 minutes. **(5)**

(Total 9 marks)

Solutions

1.

Scheme	Marks
$E(A) = E(B) + 4E(C) - 3E(D)$	M1
$= 22$	A1
$\text{Var}(A) = \text{Var}(B) + 16\text{Var}(C) + 9\text{Var}(D)$	M1
$= 168.25$	A1
$P(A < 45) = P\left(Z < \frac{45 - 22}{\sqrt{168.25}}\right)$	M1
$= P(Z < 1.773)$ $= 0.9616$ awrt 0.962	A1
	(6 marks)

Notes

1st M1 for $E(4C) = 4E(C)$ and $-E(3D) = -3E(D)$

1st A1 for 22 cao

2nd M1 for use of $\text{Var}(aX) = a^2\text{Var } X$ and + their ' $9\text{Var}(D)$ '

2nd A1 for 168.25 cao

3rd M1 for standardising using their mean and their sd

3rd A1 for awrt 0.962. NB Calculator gives 0.961899....

2.

Mark scheme (a)

Scheme	Marks	A0
$W = E_1 + E_2 + \dots + E_6 + S \sim N(6 \times 60 + 24, 6 \times 3^2 + 1.8^2)$ $[W \sim N(384, 57.24)]$	M1M1	3.3 1.1b
$P(W < 387) = 0.6541 \dots$ awrt 0.654	A1	3.4
	(3)	

Notes

M1: setting up normal model with correct mean

M1: use of correct variance

A1: awrt **0.654**

Mark scheme (b)

Scheme	Marks	A0
$X = E_1 + E_2 + \dots + E_{12} + L \sim N(12 \times 60 + 40, 12 \times 3^2 + 2.1^2)$ [$X \sim N(760, 112.41)$]	M1	1.1b
$2W \sim N("384" \times 2, 2^2 \times "57.24") [= N(768, 228.96)]$	M1	1.1b
$X - 2W \sim N(760 - 768, 112.41 + 228.96) \rightarrow N(-8, 341.37)$	M1A1	3.3 1.1b
$P(X - 2W > 0) = 0.3325 \dots$ awrt 0.333	A1	3.4
	(5)	
	(8 marks)	

Notes

M1: setting up normal model for large box of eggs, sight of $12 \times 60 + 40$

M1: sight or use of 2 times their mean from (a) and 4 times their variance from (a)

M1: setting up distribution for the difference in weights, with mean of their "760" – "768"

A1: correct mean and variance

A1: awrt **0.333**

3.

Mark scheme (a)

Scheme	Marks
$[P \sim N(90, 9) \text{ and } J \sim N(91, 12)]$ $(J - P) \sim N(1, 21)$	M1, A1

$P(J < P) = P(J - P < 0)$ $= P\left(Z < \frac{0 - 1}{\sqrt{21}}\right)$	dM1
$= P(Z < -0.2182 \dots)$ $= 1 - 0.5871 = 0.4129 \quad \text{awrt } (0.413 \sim 0.414)$ calculator (0.4136....)	A1
	(4)

Notes

1st M1 for attempting $J - P$ and $E(J - P)$ or $P - J$ and $E(P - J)$

1st A1 for variance of 21 (Accept $9 + 12$). Ignore any slip in μ here.

2nd dM1 for attempting the correct probability and standardising with their mean and sd.

This mark is dependent on previous M so if $J - P$ (or $P - J$) is not being used score M0

If their method is not crystal clear then they must be attempting $P(Z < \text{-ve value})$ or $P(Z > \text{+ve value})$ i.e. their probability after standardisation should lead to a prob. < 0.5 so e.g. $P(J - P < 0)$ leading to 0.5871 is MOA0 unless the M1 is clearly earned.

2nd A1 for awrt 0.413 or 0.414

The first 3 marks may be implied by a correct answer

Mark scheme (b)

Scheme	Marks
$X = (J_1 + J_2 + \dots + J_{60}) - (P_1 + P_2 + \dots + P_{60})$	M1
$E(X) = 60 \times 91 - 60 \times 90 = 60$ [stated as $E(X) = 60$ or $X \sim N(60, \dots)$]	B1
$\text{Var}(X) = 60 \times 9 + 60 \times 12 = 1260$	A1
$P(X > 120) = P\left(Z > \frac{120 - 60}{\sqrt{1260}}\right)$	M1
$= P(Z > 1.69030 \dots)$ $= 1 - 0.9545 = 0.0455 \quad \text{awrt } (0.0455)$	A1
	(5)
	(9 marks)

Notes

1st M1 for a clear attempt to identify a correct form for X . This may be implied by correct variance of 1260

B1 for $E(X) = 60$. Can be awarded even if they are using $X = 60J - 60P$. Allow $P - J$ and -60

1st A1 for a correct variance. If 1260 is given the M1 is scored by implication.

2nd M1 for attempting a correct probability and standardising with 120 and their 60 and 1260

If the answer is incorrect a full expression must be seen following through their values for M1 e.g.

$$P\left(Z > \frac{120 - \text{their } 60}{\sqrt{\text{their variance}}}\right). \text{ If using } -60, \text{ should get } P\left(Z < \frac{-120 - -60}{\sqrt{\text{their variance}}}\right)$$

Use of means Attempt to use $\bar{J} - \bar{P}$ for 1st M1, $E(\bar{J} - \bar{P}) = 1$ for B1 and $\text{Var}(\bar{J} - \bar{P}) = 0.35$ for A1

Then 2nd M1 for standardisation with 2, and their 1 and 0.35