

# QQQ – Further Statistics 2 – Chapter 3 – Continuous Distributions (Version 2)

**Total Marks: 24**

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

---

1. A continuous random variable  $X$  has cumulative distribution function  $F(x)$  given by

$$F(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{5}(x+1)^2 & -1 \leq x \leq 0 \\ 1 - \frac{1}{20}(4-x)^2 & 0 < x \leq 4 \\ 1 & x > 4 \end{cases}$$

- (a) Find the probability density function,  $f(x)$  (2)
- (b) (i) Sketch  $f(x)$  (2)
- (ii) Hence describe the skewness of the distribution. (1)
- (c) Find, to 3 significant figures, the value of  $c$  such that

$$P(1 < X < c) = P(c < X < 2)$$

(3)

**(Total 8 marks)**

**2.** A continuous random variable  $X$  is uniformly distributed over the interval  $[b, 4b]$  where  $b$  is a constant.

(a) Write down  $E(X)$ . (1)

(b) Use integration to show that  $\text{Var}(X) = \frac{3b^2}{4}$ . (3)

(c) Find  $\text{Var}(3 - 2X)$ . (2)

Given that  $b = 1$  find

(d) the cumulative distribution function of  $X$ ,  $F(x)$ , for all values of  $x$ , (2)

(e) the median of  $X$ . (1)

**(Total 9 marks)**

Edexcel S2 June 2013 Q4

**3.** A rectangle is to have an area of  $40 \text{ cm}^2$

The length of the rectangle,  $L$  cm, follows a continuous uniform distribution over the interval  $[4, 10]$

Find the expected value of the perimeter of the rectangle.

Use algebraic integration, rather than your calculator, to evaluate any definite integrals. (7)

**(Total 7 marks)**

Edexcel A2 June 2022 Q7

# Solutions

1.

## Mark scheme (a)

| Scheme                                                                                                                                                           | Marks | AO   |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| $\frac{d}{dx} \left[ \frac{1}{5}(x+1)^2 \right] = \frac{2}{5}(x+1) \quad \text{or}$<br>$\frac{d}{dx} \left[ 1 - \frac{1}{20}(4-x)^2 \right] = \frac{1}{10}(4-x)$ | M1    | 2.1  |
| $f(x) = \begin{cases} \frac{2}{5}(x+1) & -1 \leq x \leq 0 \\ \frac{1}{10}(4-x) & 0 < x \leq 4 \\ 0 & \text{otherwise} \end{cases}$                               | A1    | 1.1b |
|                                                                                                                                                                  | (2)   |      |

### Notes

**M1:** Attempt to differentiate 2<sup>nd</sup> line of cdf in the form  $k(x+1)$  (oe)  
or 3<sup>rd</sup> line of cdf in the form  $m(4-x)$  (oe)

**A1:** Correct probability density function.  
(Condone missing “0 otherwise” and mis-use of  $<$  or  $\leq$  etc )

## Mark scheme (b)

| Scheme                                                                                             | Marks | AO   |
|----------------------------------------------------------------------------------------------------|-------|------|
| (i) Shape: Triangle with longer slant in 1 <sup>st</sup> quadrant                                  | B1    | 1.1b |
| Labels: -1, 4 on horizontal axis and $\frac{2}{5}$ on vertical axis                                | B1    | 1.1b |
|                                                                                                    | (2)   |      |
| (ii) (Probability density function has a longer tail to right,) so there is <b>positive skew</b> . | dB1   | 2.4  |
|                                                                                                    | (1)   |      |

### Notes

(i) **1<sup>st</sup> B1:** Correct shape. Triangle must have both ends on axis

**2<sup>nd</sup> B1:** Correct labels. All three needed.

(ii) **dB1:** Correct description of positive skew.

**Dep** on sketch that clearly shows positive skew.

### Mark scheme (c)

| Scheme                                                                                                                                                                                                       | Marks            | AO   |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------|------|
| $P(1 < X < 2) = F(2) - F(1)$ <u>or</u> $2F(c) = F(2) + F(1)$<br>$= \left(1 - \frac{4}{20}\right) - \left(1 - \frac{9}{20}\right) = \frac{5}{20}$ <u>or</u> $\frac{1}{4}$ <u>or</u> stating<br>$F(c) = 0.675$ | M1               | 3.1a |
| So e.g. $P(c < X < 2) = \frac{1}{8} \Rightarrow \frac{1}{8} = \frac{(4-c)^2}{20} - \frac{4}{20}$                                                                                                             | M1               | 1.1b |
| $c = \frac{8 - \sqrt{26}}{2}$ <u>or</u> 1.45049... awrt <b>1.45</b>                                                                                                                                          | A1               | 1.1b |
|                                                                                                                                                                                                              | <b>(3)</b>       |      |
|                                                                                                                                                                                                              | <b>(8 marks)</b> |      |

#### Notes

**1<sup>st</sup> M1:** Suitable start to problem e.g. showing  $P(1 < X < 2) = 0.25$  o.e.

e.g.  $F(c) = 0.675$  or  $2F(c) = 0.8 + 0.55$

**2<sup>nd</sup> M1:** Rearranging to form quadratic equation in  $c$  can fit their "0.25"

e.g.  $2c^2 - 16c + 19 = 0$

Allow even if wrong part of  $F(x)$  is used so MOM1A0 is possible.

**A1:** for awrt 1.45 only

NB Other root is 6.549... and if this is not rejected score A0

## 2.

### Mark scheme (a)

| Scheme                | Marks      |
|-----------------------|------------|
| $E(X) = \frac{5b}{2}$ | B1         |
|                       | <b>(1)</b> |

### Mark scheme (b)

| Scheme | Marks |
|--------|-------|
|--------|-------|

|                                                                 |            |
|-----------------------------------------------------------------|------------|
| $\text{Var}(X) = E(X^2) - (E(X))^2$                             |            |
| $= \int_b^{4b} \frac{x^2}{3b} dx - \left(\frac{5b}{2}\right)^2$ | M1         |
| $= \left[\frac{x^3}{9b}\right]_b^{4b} - \frac{25b^2}{4}$        | M1d        |
| $= \frac{63b^3}{9b} - \frac{25b^2}{4}$                          |            |
| $= \frac{3b^2}{4}$                                              | Alcso      |
|                                                                 | <b>(3)</b> |

## Notes

### Alt 4(b)

| Scheme                                                                 | Marks      |
|------------------------------------------------------------------------|------------|
| $\text{Var}(X) = \int_a^b \frac{(x - \bar{x})^2}{b - a} dx$            |            |
| $= \int_b^{4b} \frac{4x^2 - 20bx + 25b^2}{12b} dx$                     | M1         |
| $= \left[ \frac{\frac{4x^3}{3} - 10bx^2 + 25b^2x}{12b} \right]_b^{4b}$ | M1         |
| $= \frac{9b^3}{12b}$                                                   |            |
| $= \frac{3b^2}{4}$                                                     | Alcso      |
|                                                                        | <b>(3)</b> |

**NB remember the answer is given (AG) so they must show their working**

**1st M1** for using  $\int \frac{x^2}{3b} dx - (\text{their } (a))^2$  limits not needed and condone missing  $dx$ . NB need not use the letter  $x$  but if they use  $b$  instead do not award if they cancel down to  $\frac{b}{3}$

NB Check they have subtracted  $(\text{their}(a))^2$

**2nd M1** dependent on previous M being awarded. For some correct integration  $x^n \rightarrow x^{n+1}$  and correct limits substituted at some point. condone  $4b^3$  instead of  $(4b)^3$

**A1** for correct solution with no incorrect working seen.

**Mark scheme (c)**

| Scheme                                | Marks      |
|---------------------------------------|------------|
| $\text{Var}(3 - 2X) = 4\text{Var}(X)$ | M1         |
| $= 3b^2$                              | A1         |
|                                       | <b>(2)</b> |

**Notes**

**M1** for writing or using  $4\text{Var}(X)$

**Mark scheme (d)**

| Scheme                                                                                       | Marks      |
|----------------------------------------------------------------------------------------------|------------|
| $F(x) = \begin{cases} 0 & x < 1 \\ \frac{x-1}{3} & 1 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$ | B1B1       |
|                                                                                              | <b>(2)</b> |

**Notes**

**1st B1** top **and** bottom line. Allow use of  $\leq$  instead of  $<$  and  $\geq$  instead of  $>$

**2nd B1** middle row. Allow use of  $<$  instead of  $\leq$

**Mark scheme (e)**

| Scheme                             | Marks            |
|------------------------------------|------------------|
| $\frac{x-1}{3} = 0.5$ so $x = 2.5$ | B1               |
|                                    | <b>(1)</b>       |
|                                    | <b>(9 marks)</b> |

**3.**

| Scheme                                  | Marks | AO   |
|-----------------------------------------|-------|------|
| $P = 2 \left( L + \frac{40}{L} \right)$ | M1    | 3.1a |
| $f(l) = \frac{1}{6}$                    | B1    | 1.1b |

|                                                                                                                   |           |              |
|-------------------------------------------------------------------------------------------------------------------|-----------|--------------|
| $E(P) = E\left(2\left(L + \frac{40}{L}\right)\right) = \int_4^{10} \frac{1}{6} \left(2l + \frac{80}{l}\right) dl$ | M1        | 2.1          |
| $= \left[\frac{1}{6}l^2 + \frac{40}{3} \ln  l \right]_4^{10}$                                                     | M1A1      | 1.1b<br>1.1b |
| $= \left(\frac{1}{6}(100) + \frac{40}{3} \ln 10\right) - \left(\frac{1}{6}(16) + \frac{40}{3} \ln 4\right)$       | M1        | 1.1b         |
| $= 26.217 \dots \quad \text{awrt } 26.2$                                                                          | A1        | 1.1b         |
|                                                                                                                   | (7 marks) |              |

### Notes

**M1:** Finding an expression for the perimeter in terms of  $L$

**B1:** Correct distribution for  $L$  (may be implied by  $E(L) = 7$ )

**M1:** Setting up integral for expectation of perimeter

(allow 2 separate integrals e.g.  $2(7) + \int_4^{10} \frac{1}{6} \left(\frac{80}{l}\right) dl$ )

**M1:** Attempt to integrate an expression for expectation of perimeter (allow two separate expressions)

**A1:** Correct integration

**depM1:** (dep on previous M1) Use of correct limits 10 and 4

**A1:** awrt 26.2 Note: exact value is  $14 + \frac{40 \ln(2.5)}{3}$