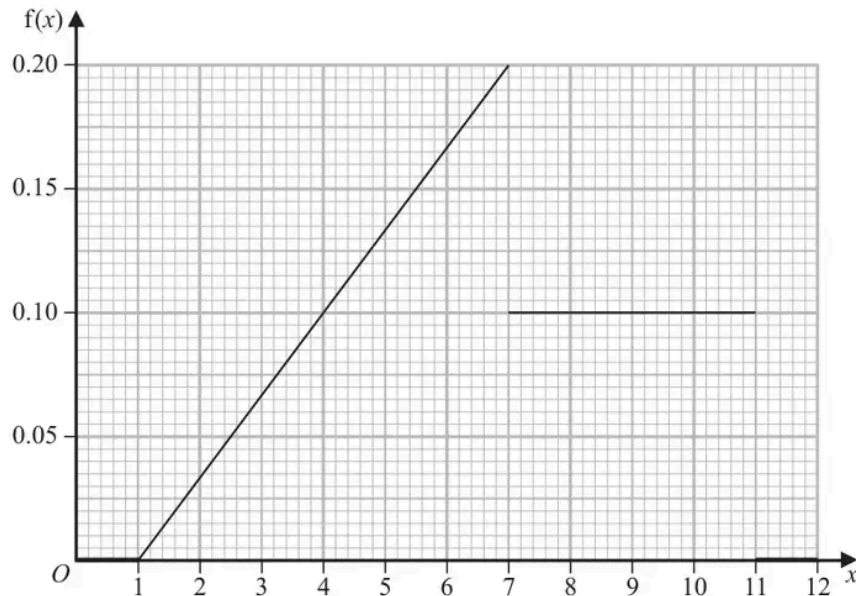


QQQ – Further Statistics 2 – Chapter 3 – Continuous Distributions (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. The graph shows the probability density function $f(x)$ of the continuous random variable X



- (a) Find $P(X < 4)$ (2)
- (b) Specify the cumulative distribution function of X for $7 \leq x \leq 11$ (3)

(Total 5 marks)

Edexcel AS June 2022 Q2

2. The random variable R has a continuous uniform distribution over the interval $[5, 9]$

- (a) Specify fully the probability density function of R . (1)
- (b) Find $P(7 < R < 10)$ (1)

The random variable A is the area of a circle radius R cm.

- (c) Find $E(A)$ (4)

(Total 6 marks)

Edexcel S2 June 2016 Q3

3. A rectangle has a perimeter of 20 cm. The length, X cm, of one side of this rectangle is uniformly distributed between 1 cm and 7 cm.

Find the probability that the length of the longer side of the rectangle is more than 6 cm long.

(5)

(Total 5 marks)

Edexcel S2 June 2010 Q3

4. The continuous random variable X has cumulative distribution function given by

$$F(x) = \begin{cases} 0 & x \leq 0 \\ k \left(x^3 - \frac{3}{8}x^4 \right) & 0 < x \leq 2 \\ 1 & x > 2 \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{2}$ **(1)**

(b) Showing your working clearly, use calculus to find

(i) $E(X)$

(ii) the mode of X

(6)

(c) Describe, giving a reason, the skewness of the distribution of X **(1)**

(Total 8 marks)

Edexcel A2 June 2019 Q4

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
$P(X < 4) = \frac{(4 - 1) \times 0.1}{2} \quad \text{or} \quad \int_1^4 \frac{1}{30}(x - 1) \, dx$	M1	2.1
$= 0.15$	A1	1.1b
	(2)	

Notes

M1: Use of area of triangle or integration with limits to find required area

Condone $\frac{4 \times 0.1}{2}$ or $\int_1^4 \frac{1}{30}(x) \, dx$ for M1

A1: 0.15 oe

Mark scheme (b)

Scheme	Marks	AO
$\frac{(7 - 1) \times 0.2}{2} +$ $\dots$ $\dots 0.1(x - 7)$	M1	3.1a
$\int_1^7 \frac{1}{30}(x - 1) \, dx + \dots$ $\dots \int_7^x 0.1 \, dt$	M1	1.1b
$\int 0.1 \, dx$ $F(7) = 0.1x + c$ or $F(11) = 0.1x + c$		
$F(x) = 0.1x - 0.1 \quad [\text{for } 7 \leq x \leq 11]$	A1	1.1b
	(3)	
	(5 marks)	

Notes

M1: Complete method for finding the cdf including area from $1 \leq x < 7$

Condone one slip for the area from $1 \leq x < 7$

M1: Attempt at area from $7 \leq x \leq 11$

A1: $0.1x - 0.1$

2.

Mark scheme (a)

Scheme	Marks
$f(r) = \begin{cases} \frac{1}{4} & 5 \leq r \leq 9 \\ 0 & \text{otherwise} \end{cases}$	B1
	(1)

Notes

B1: Allow $r < 5$ and $r > 9$ instead of 0 otherwise

Allow $<$ instead of $\leq$ signs.

Any letter may be used - condone mixed letters

Must have $f(r)$ – condone $F(r)$

Mark scheme (b)

Scheme	Marks
$P(7 < R < 10) = 2 \times \frac{1}{4}$ $= \frac{1}{2}$	B1
	(1)

Notes

B1: oe

Mark scheme (c)

Scheme	Marks
$[E(A) = E(\pi R^2)]$ $E(R^2) = \text{Var}(R) + [E(R)]^2 \quad \text{or} \quad \int_5^9 \frac{r^2}{4} dr$	M1

$E(R) = 7, \quad \text{Var}(R) = \frac{4}{3} \text{ or } \left[\frac{r^3}{12} \right]_5^9$	B1
$= 50\frac{1}{3}$	A1
$E(A) = 50\frac{1}{3}\pi$ oe	A1
	(4)
	(6 marks)

Notes

M1: Using correct formula for $E(R^2)$. This may be in any order or written in words

B1: $\text{Var}(R) = \frac{4}{3}$ or awrt 1.33 **and** $E(R) = 7$ **or** $\left[\frac{r^3}{12} \right]_5^9$. These may be implied by a correct answer

A1: Allow awrt 50.3

A1: Allow **exact** multiple of π eg $50.\dot{3}\pi$ or awrt 158

Do Not allow 50.3π

NB If both $E(R)^2$ and $[E(R)]^2$ are both worked out and neither is selected they lose the final A marks.

The best they can get is M1 B1 A1A0

3.

Scheme	Marks
Method 1	B1
$P(X > 6) = \frac{1}{6}$	M1
$P(X < 4) = \frac{1}{2}$	A1
$\text{total} = \frac{1}{6} + \frac{1}{2} = \frac{2}{3}$	M1dep B A1
	(5)
	(5 marks)

Notes

Method 2

Scheme	Marks
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$P(4 < X < 6) = \frac{1}{3}$	B1 M1 A1
$1 - \frac{1}{3} = \frac{2}{3}$	M1dep B A1

Method 3

Scheme	Marks
$P(X > 6) = \frac{1}{6}$	B1 M1
$Y \sim U[3, 9] \quad P(Y > 6) = \frac{1}{2}$	A1
total = $\frac{1}{6} + \frac{1}{2} = \frac{2}{3}$	M1dep B A1

Methods 1 and 2

B1 for 6 and 4 (allow if seen on a diagram on x -axis)

M1 for $P(X > 6)$ or $P(6 < X < 7)$; or $P(X < 4)$ or $P(1 < X < 4)$; or $P(4 < X < 6)$ Allow $\leq$ and $\geq$ signs

A1 $\frac{1}{6}$; or $\frac{1}{2}$; $\frac{1}{3}$ must match the probability statement

M1 for adding their “ $P(X > 6)$ ” and their “ $P(X < 4)$ ” or 1 - their “ $P(4 < X < 6)$ ” dep on getting first B mark

A1 cao $\frac{2}{3}$

Method 3 $Y \sim U[3, 9]$

B1 for 6 with $U[1,7]$ and 6 with $U[3,9]$

M1 for $P(X > 6)$ or $P(6 < X < 7)$ or $P(6 < Y < 9)$

A1 $\frac{1}{6}$; or $\frac{1}{2}$; must match the probability statement

M1 for adding their “ $P(X > 6)$ ” and their “ $P(Y > 6)$ ” dep on getting first B mark

A1 cao $\frac{2}{3}$

4.

Mark scheme (a)

Scheme	Marks	AO
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$k \left(2^3 - \frac{3}{8} 2^4 \right) = 1$ $2k = 1$ $k = \frac{1}{2}^*$	B1*	1.1b
Or $\frac{1}{2} \left(2^3 - \frac{3}{8} 2^4 \right) = 1 \quad \therefore k = \frac{1}{2}^*$	(B1*)	
	(1)	

Notes

B1*: substituting $x = 2$ into $F(x)$ and equating to 1 leading to $k = \frac{1}{2}$ with no errors.

Minimum subst seen is $k(8 - 6) = 1$ or $0.5(8 - 6) = 1$

Mark scheme (b)

Scheme	Marks	AO
$f(x) = k \left[3x^2 - \frac{3}{2}x^3 \right]$	M1	2.1
(i) $\int_0^2 xf(x) dx = k \int_0^2 \left(3x^3 - \frac{3}{2}x^4 \right) dx$	M1d	1.1b
$= \left[\frac{3x^4}{8} - \frac{3x^5}{20} \right]_0^2$		
$= \frac{6}{5}$ or 1.2	A1	1.1b
(ii) $3x - \frac{9x^2}{4} = 0$	M1d	3.1a
$x \left(3 - \frac{9x}{4} \right) = 0$	M1d	1.1b
$x = 0$ or $\frac{4}{3} \quad \therefore \text{mode} = \frac{4}{3}$	A1	1.1b
	(6)	

Notes

M1: Realising they need to find the pdf and attempting to differentiate $k \left[x^3 - \frac{3}{8}x^4 \right]$ at least 1 correct term

(i) **M1d:** dep on 1st M1 Attempting to find $\int_0^2 x(\text{their } f(x)) \, dx$ At least one correct term ft their pdf

A1: $\frac{6}{5}$ or 1.2 oe NB 1.2 with no working gains MOMOAO

(ii) **M1d:** dep on 1st M1 for realising they need to differentiate their pdf. At least one correct term but ft their pdf

M1d: Dep on 3rd M1. correct method for solving their differential of their pdf = 0 pdf must be of the form $ax^2 + bx$

A1: $\therefore$ mode = $\frac{4}{3}$ only. They must eliminate 0

Mark scheme (c)

Scheme	Marks	AO
Mode > mean implies it is negative skew	B1ft	2.4
	(1)	
	(8 marks)	

Notes

B1ft: ft their mode and mean or a correct sketch.