

QQQ – Further Statistics 1 – Chapter 8 – Quality of Tests (Version 2)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. The number of errors made by a secretary is modelled by a Poisson distribution with a mean of 2.4 per 100 words.

A 100-word piece of work completed by the secretary is selected at random.

- (a)** Find the probability that
- (i)** there are exactly 3 errors,
 - (ii)** there are fewer than 2 errors. **(2)**

After a long holiday, a randomly selected piece of work containing 250 words completed by the secretary is examined to see if the rate of errors has changed.

- (b)** Stating your hypotheses clearly, and using a 5% level of significance, find the critical region for a suitable test. **(4)**
- (c)** Find $P(\text{Type I error})$ for the test in part (b) **(1)**

(Total 7 marks)

2. A machine fills cartons with juice.

The amount of juice in a carton is normally distributed with mean μ ml and standard deviation 8 ml.

A manager wants to test whether or not the amount of juice in the cartons, X ml, is less than 330 ml. The manager takes a random sample of 25 cartons of juice and calculates the mean amount of juice $\bar{x}$ ml.

(a) Using a 5% level of significance, find the critical region of $\bar{X}$ for this test.

State your hypotheses clearly.

(4)

The Director is concerned about the machine filling the cartons with more than 330 ml of juice as well as less than 330 ml of juice. The Director takes a sample of 55 cartons, records the mean amount of juice $\bar{y}$ ml and uses a test with a critical region of

$$\{\bar{Y} < 328\} \cup \{\bar{Y} > 332\}$$

(b) Find $P(\text{Type I error})$ for the Director's test.

(2)

When $\mu = 325$ ml

(c) find $P(\text{Type II error})$ for the test in part (a)

(2)

(Total 8 marks)

3. The number of houses sold per week by a firm of estate agents follows a Poisson distribution with mean 2. The firm believes that the appointment of a new salesman will increase the number of houses sold. The firm tests its belief by recording the number of houses sold, x , in the week following the appointment. The firm sets up the hypotheses $H_0 : \lambda = 2$ and $H_1 : \lambda > 2$, where λ is the mean number of houses sold per week, and rejects the null hypothesis if $x \geq 3$

(a) Find the size of the test. **(2)**

(b) Show that the power function for this test is

$$1 - \frac{1}{2}e^{-\lambda}(2 + 2\lambda + \lambda^2)$$

(3)

The table below gives the values of the power function to 2 decimal places.

λ	2.5	3.0	3.5	4.0	5.0	7.0
Power	0.46	r	0.68	s	0.88	0.97

Table 1

(c) Calculate the values of r and s . **(2)**

(d) Draw a graph of the power function. **(2)**

(e) Find the range of values of λ for which the power of this test is greater than 0.6 **(1)**

(Total 10 marks)