

QQQ – Further Statistics 1 – Chapter 8 – Quality of Tests (Version 2)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. The number of errors made by a secretary is modelled by a Poisson distribution with a mean of 2.4 per 100 words.

A 100-word piece of work completed by the secretary is selected at random.

- (a)** Find the probability that
- (i)** there are exactly 3 errors,
 - (ii)** there are fewer than 2 errors. **(2)**

After a long holiday, a randomly selected piece of work containing 250 words completed by the secretary is examined to see if the rate of errors has changed.

- (b)** Stating your hypotheses clearly, and using a 5% level of significance, find the critical region for a suitable test. **(4)**
- (c)** Find $P(\text{Type I error})$ for the test in part (b) **(1)**

(Total 7 marks)

2. A machine fills cartons with juice.

The amount of juice in a carton is normally distributed with mean μ ml and standard deviation 8 ml.

A manager wants to test whether or not the amount of juice in the cartons, X ml, is less than 330 ml. The manager takes a random sample of 25 cartons of juice and calculates the mean amount of juice $\bar{x}$ ml.

(a) Using a 5% level of significance, find the critical region of $\bar{X}$ for this test.

State your hypotheses clearly.

(4)

The Director is concerned about the machine filling the cartons with more than 330 ml of juice as well as less than 330 ml of juice. The Director takes a sample of 55 cartons, records the mean amount of juice $\bar{y}$ ml and uses a test with a critical region of

$$\{\bar{Y} < 328\} \cup \{\bar{Y} > 332\}$$

(b) Find $P(\text{Type I error})$ for the Director's test.

(2)

When $\mu = 325$ ml

(c) find $P(\text{Type II error})$ for the test in part (a)

(2)

(Total 8 marks)

3. The number of houses sold per week by a firm of estate agents follows a Poisson distribution with mean 2. The firm believes that the appointment of a new salesman will increase the number of houses sold. The firm tests its belief by recording the number of houses sold, x , in the week following the appointment. The firm sets up the hypotheses $H_0 : \lambda = 2$ and $H_1 : \lambda > 2$, where λ is the mean number of houses sold per week, and rejects the null hypothesis if $x \geq 3$

(a) Find the size of the test. **(2)**

(b) Show that the power function for this test is

$$1 - \frac{1}{2}e^{-\lambda}(2 + 2\lambda + \lambda^2)$$

(3)

The table below gives the values of the power function to 2 decimal places.

λ	2.5	3.0	3.5	4.0	5.0	7.0
Power	0.46	r	0.68	s	0.88	0.97

Table 1

(c) Calculate the values of r and s . **(2)**

(d) Draw a graph of the power function. **(2)**

(e) Find the range of values of λ for which the power of this test is greater than 0.6 **(1)**

(Total 10 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
(i) 0.20901... awrt <u>0.209</u>	B1	3.4
(ii) 0.30844.. awrt <u>0.308</u>	B1	1.1b
	(2)	

Notes

1st B1 for awrt 0.209

2nd B1 for awrt 0.308

Mark scheme (b)

Scheme	Marks	AO
$H_0 : \lambda = 2.4$ (or $\mu = 6$) $H_1 : \lambda \neq 2.4$ (or $\mu \neq 6$)	B1	2.5
[E = no. of errors] $E \sim \text{Po}(6)$	M1	3.3
$P(E \leq 1) = 0.0174$ <u>or</u> $P(E \leq 2) = 0.0620$ and $P(E \leq 11) = 0.980$ <u>or</u> $P(E \geq 12) = 0.0201$	M1	3.4
Critical region: $E \leq 1$ or $E \geq 12$	A1	1.1b
	(4)	

Notes

B1 for both hypotheses correct in terms of λ or μ (allow $\lambda = 6$ etc)

1st M1 for selecting the correct model. Sight or use of $\text{Po}(6)$

2nd M1 for use of the correct model with two probs correct to 2.s.f. (accept $P(E \geq 12) = 0.02$)

Must see attempt at lower **and** upper limit. Probabilities may be seen in (c).

A1 for correct critical region (both parts). Allow $E \leq 1$ and $E \geq 12$ or $E \leq 1, E \geq 12$ etc

Writing CR as probability statements is A0

NB: Completely correct CR implies M1M1A1

SC: 1-tailed test

B0 as hypotheses are incorrect

M1 for sight or use of $Po(6)$

M1 (dep on H_1) for sight of $P(E \leq 1) = 0.0174$ or $P(E \geq 11) = 0.0426$, in line with their H_1

A1 for CR: $E \leq 1$ or CR: $E \geq 11$, in line with their hypotheses

Mark scheme (c)

Scheme	Marks	AO
$[P(\text{Type I error}) = 0.0174 + 0.0201 =] \quad \underline{\mathbf{0.0375}}$ (Calc gives: $0.017351\dots + 0.0200919\dots = 0.037443\dots$)	B1ft	1.2
	(1)	
	(7 marks)	

Notes

B1ft for 0.0375 or 0.0374 or summing their two appropriate probs (ft their CR)

NB: If candidate uses a 1-tailed test, this mark cannot be gained

2.

Mark scheme (a)

Scheme	Marks	AO
$H_0 : \mu = 330 \quad H_1 : \mu < 330$	B1	2.5
$[\bar{X} \sim] N \left(330, \left(\frac{8}{\sqrt{25}} \right)^2 \right)$	M1	3.3
$P(\bar{X} < C) = 0.05 \Rightarrow \frac{C - 330}{8/\sqrt{25}} = -1.6449$	M1	3.4
So $C = 327.368\dots$ and critical region is: $\bar{X} < \text{awrt } \underline{\mathbf{327}}$	A1	1.1b
	(4)	

Notes

B1 for both hypotheses in terms of μ

1st M1 for stating or using the correct model – may be implied by use in later line.

Condone X or any letter for $\bar{X}$

2nd M1 for a correct equation for C Allow any z value that satisfies $1.6 < |z| < 1.7$

If standardisation equation not seen, this mark may be implied by $CV = \text{awrt } 327$ or $CR: < \text{awrt } 327$

A1 for a correct CR allow just “ $< \text{awrt } 327$ ” Condone e.g. $X < 327$ rather than $\bar{X} < 327$

Condone $\leq$

Mark scheme (b)

Scheme	Marks	AO
$\bar{Y} \sim N\left(330, \left(\frac{8}{\sqrt{55}}\right)^2\right)$ and require $2 \times P(\bar{Y} < 328)$ (o.e.)	M1	3.3
$= 0.063732 \dots$ awrt 0.0637	A1	1.1b
	(2)	

Notes

M1 for sight of correct model and attempt at $P(\bar{Y} < 328)$ (o.e.) Condone missing $2 \times$

A1 for awrt 0.0637 (correct answer scores 2 out of 2)

Mark scheme (c)

Scheme	Marks	AO
$P(\bar{X} > \text{“}327.368\dots\text{”} \mid \mu = 325)$ or $1 - P(\bar{X} < \text{“}327.368\dots\text{”} \mid \mu = 325)$	M1	3.4
$= 0.0694233 \dots$ awrt 0.0694	A1	1.1b
	(2)	
	(8 marks)	

Notes

M1 for a correct (ft) statement may be implied by sight of e.g. $Z > \frac{\text{“}327.36\dots\text{”} - 325}{8/5} = 1.48\dots$

For $\mu = 325$ allow $\bar{X} \sim N(325, \dots)$

Allow ft from a 2-tailed test in part (a)

A1 for awrt 0.0694 (correct answer scores 2 out of 2)

SC Sight of $P(328 < \bar{X} < 332 \mid \mu = 325)$ or $1 - P(\bar{X} < 328 \cup \bar{X} > 332 \mid \mu = 325)$ scores M1A0

Mark scheme (a)

Scheme	Marks
$X \sim \text{Po}(2)$	
Size = $P(X \geq 3/\lambda = 2)$	
$= 1 - 0.6767$	M1
$= 0.3233$ awrt 0.323	A1
	(2)

Notes

M1 for correct expression for size using Po(2)

Mark scheme (b)

Scheme	Marks
Power = $1 - P(0) - P(1) - P(2)$	M1
$= 1 - e^{-\lambda} - \lambda e^{-\lambda} - \frac{\lambda^2 e^{-\lambda}}{2!}$	A1
$= 1 - \frac{1}{2} e^{-\lambda} (2 + 2\lambda + \lambda^2)$	A1 cso
	(3)

Notes

1st M1 for a correct expression in terms of probabilities. Allow $1 - P(X \leq 2)$ or $1 - P(X < 3)$

1st A1 for correct equation in λ

2nd A1 cso

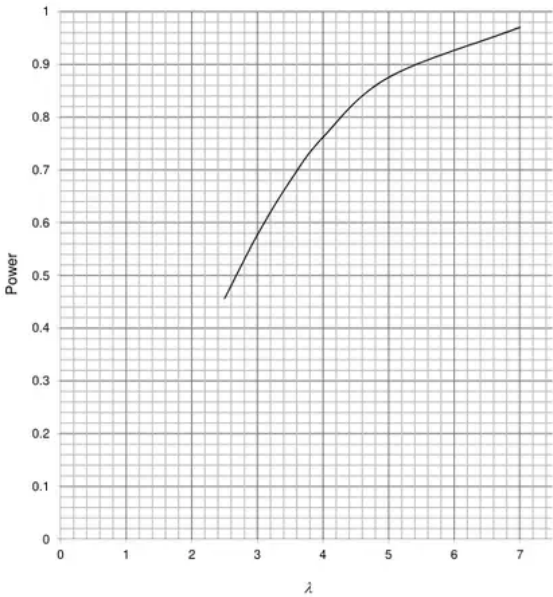
Mark scheme (c)

Scheme	Marks
$r = 0.58$ $s = 0.76$	B1, B1
	(2)

Notes

SC if both correct but not to 2dp award B1B0

Mark scheme (d)

Scheme	Marks
	B1ft points B1ft curve
	(2)

Notes
1st B1ft points
2nd B1ft curve (or straight lines) through points

Mark scheme (e)

Scheme	Marks
$\lambda > 3.1$ allow numbers in range 3.1–3.2	B1
	(1)
	(10 marks)