

QQQ – Further Statistics 1 – Chapter 8 – Quality of Tests (Version 1)

Total Marks: 22

(22 = Platinum, 20 = Gold, 18 = Silver, 15 = Bronze)

1.

(a) Define

(i) a Type I error,

(ii) a Type II error. **(2)**

Rolls of material, manufactured by a machine, contain defects at a mean rate of 6 per roll.

The machine is modified. A single roll is selected at random and a test is carried out to see whether or not the mean number of defects per roll has decreased. The significance level is chosen to be as close as possible to 5%.

(b) Calculate the probability of a Type I error for this test. **(3)**

(c) Given that the true mean number of defects per roll of material made by the machine is now 4, calculate the probability of a Type II error. **(2)**

(Total 7 marks)

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2. The number of tornadoes per year to hit a particular town follows a Poisson distribution with mean λ . A weatherman claims that due to climate changes the mean number of tornadoes per year has decreased. He records the number of tornadoes x to hit the town last year.

To test the hypotheses $H_0 : \lambda = 7$ and $H_1 : \lambda < 7$, a critical region of $x \leq 3$ is used.

(a) Find, in terms λ the power function of this test. **(3)**

(b) Find the size of this test. **(2)**

(c) Find the probability of a Type II error when $\lambda = 4$. **(2)**

(Total 7 marks)

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3. A manufacturer has a machine that produces lollipop sticks.

The length of a lollipop stick produced by the machine is normally distributed with unknown mean μ and standard deviation 0.2

Farhan believes that the machine is not working properly and the mean length of the lollipop sticks has decreased.

He takes a random sample of size n to test, at the 1% level of significance, the hypotheses

$$H_0 : \mu = 15 \quad H_1 : \mu < 15$$

(a) Write down the size of this test. **(1)**

Given that the actual value of μ is 14.9

- (b)** **(i)** calculate the minimum value of n such that the probability of a Type II error is less than 0.05
Show your working clearly. **(6)**
- (ii)** Farhan uses the same sample size, n , but now carries out the test at a 5% level of significance. Without doing any further calculations, state how this would affect the probability of a Type II error. **(1)**

(Total 8 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks
(i) Type I – H_0 rejected when it is true	B1
(ii) Type II – H_0 is accepted when it is false	B1
	(2)

Mark scheme (b)

Scheme	Marks
$P(X < c \mid \lambda = 6) \approx 0.05$	M1
$P(X \leq 2) = 0.0620$	
$P(X \leq 1) = 0.0174$	
Critical region = $X \leq 2$	A1
$P(\text{Type 1 error}) = P(X \leq 2 \mid \lambda = 6) = 0.062$	Alcao
	(3)

Notes

M1 use of Po(6)

A1 correct CR. May be implied by correct probability. Allow if written as part of a 2 tailed CR

A1 awrt 0.062

Mark scheme (c)

Scheme	Marks
$P(\text{Type 2 error}) = P(X \geq 3 \mid \lambda = 4)$	M1
$= 1 - 0.2381$	
$= 0.7619$	A1
	(2)

(7 marks)

Notes

M1 using $Po(4)$ and $1 - P(X \leq 2)$, ft their CR in (b) if one tail

A1 awrt 0.762

2.

Mark scheme (a)

Scheme	Marks
Power = $P(X \leq 3/\lambda)$	M1
$= e^{-\lambda} + e^{-\lambda}\lambda + \frac{e^{-\lambda}\lambda^2}{2} + \frac{e^{-\lambda}\lambda^3}{6}$	A1
$= \frac{e^{-\lambda}}{6}(6 + 6\lambda + 3\lambda^2 + \lambda^3)$	A1
	(3)

Mark scheme (b)

Scheme	Marks
CR is $X \leq 3$	M1
Size = $P[X \leq 3/\lambda = 7]$ $= 0.0818$	A1
	(2)

Mark scheme (c)

Scheme	Marks
$P(\text{Type II error}) = 1 - \text{power}$ $= 1 - \frac{e^{-4}}{6}(6 + 6 \times 4 + 3 \times 4^2 + 4^3)$	M1
$= 0.5665..$	A1
	(2)

3.

Mark scheme (a)

Scheme	Marks	AO
Size of the test = 0.01	B1	1.2
	(1)	

Notes

B1: 0.01

Mark scheme (b)

Scheme	Marks	AO
(i) Let CR be $\bar{L} < k$		
$\frac{k - 15}{\frac{0.2}{\sqrt{n}}} = -2.3263$	M1	3.4
$k = 15 - \frac{0.46526}{\sqrt{n}}$	A1	1.1b
$\frac{“15 - \frac{0.46526}{\sqrt{n}}” - 14.9}{\frac{0.2}{\sqrt{n}}} > 1.6449$	M1d A1ft	3.4 1.1b
$\frac{0.79424}{\sqrt{n}} < 0.1 \quad \sqrt{n} > 7.9424 \text{ oe}$	M1d	1.1b
$n = 64$	A1cso	2.1
	(6)	
(ii) The probability of a Type II error would decrease.	B1	2.2a
	(1)	
	(8 marks)	

Notes

(i) M1: Finding the CR using the Normal distribution must have $1.5 < |z| < 3.5$

A1: A correct equation in the form $k = \dots$ and for use of awrt 2.326 (implied by awrt 0.46526 or awrt 0.46527)

M1d: Dependent on previous M being awarded. Standardising using their k and equating to a z value $1.5 < |z| < 3$ to form an equation to able n to be found. May use = rather than >

A1ft: Ft their k for a correct equation with awrt 1.645

M1d: Dependent on previous M being awarded. Isolating $\sqrt{n}$ or squaring both sides leading to a value for n . Condone $n = 7.9424$

A1cso: 64 with correct working

(ii) B1: Suitable comment

ALT (b)(i)

Scheme	Marks	AO
$\frac{k - 14.9}{\frac{0.2}{\sqrt{n}}} = 1.6449$	M1	3.4
$k = 14.9 + \frac{0.32898}{\sqrt{n}}$	A1	1.1b
$\frac{"14.9 + \frac{0.32898}{\sqrt{n}} - 15}{\frac{0.2}{\sqrt{n}}} > -2.3263$	M1d A1ft	3.4 1.1b
$\frac{0.79424}{\sqrt{n}} < 0.1 \quad \sqrt{n} > 7.9424 \text{ oe}$	M1d	1.1b
$n = 64$	A1cso	2.1
	(6)	