

QQQ – Further Statistics 1 – Chapter 5 – Central Limit Theorem (Version 2)

Total Marks: 22

(22 = Platinum, 20 = Gold, 18 = Silver, 15 = Bronze)

1. A sample of size 5 is taken from a population that is normally distributed with mean 10 and standard deviation 3. Find the probability that the sample mean lies between 7 and 10. **(6)**

(Total 6 marks)

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2. There are 32 students in a class.

Each student rolls a fair die repeatedly, stopping when their total number of sixes is 4

Each student records the total number of times they rolled the die.

- Estimate the probability that the mean number of rolls for the class is less than 27.2 **(6)**

(Total 6 marks)

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3. A football team scores goals at an average rate of 1.8 goals per match.

- (a) Give two assumptions that would be necessary to use a Poisson distribution to model the number of goals scored in a match by the team. **(2)**

Given that in their next match the team scores exactly 3 goals,

- (b) find the exact probability that 2 of these goals were scored in the first half of the match. **(4)**

A hockey team plays 2 games each week during a 40-week season.

Each game consists of 2 periods.

The probability that the team concedes no goals in a period is 0.55

The random variable X represents the number of **periods** in a week in which the team concedes no goals.

- (c) (i) Write down a suitable distribution for X

- (ii) For this 40-week season, use the Central Limit Theorem to estimate $P(\bar{X} > 2)$ **(4)**

(Total 10 marks)

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Solutions

1.

Scheme	Marks
$X \sim N(10, 3^2) \quad \therefore \quad \bar{X} \sim N\left(10, \frac{9}{5}\right)$ Can be implied 10; $\frac{9}{5}$	B1; B1
$P(7 \leq \bar{X} \leq 10) = P\left(\frac{7-10}{\sqrt{9/5}} < Z < 0\right)$ Standardising with 10 & their σ	M1 A1
$= P(-2.236 < Z < 0)$	
$= \Phi(0) - \{1 - \Phi(2.24)\}$	M1 ($p < 0.5$)
$= \underline{0.4875}$	A1
	(6)
	(6 marks)

2.

Scheme	Marks	AO
$[X = \text{no. of rolls to 4 sixes}] \quad X \sim \text{NegBin}\left(4, \frac{1}{6}\right)$	M1	3.3
$\mu \left[= \frac{r}{p} \right] = \underline{24}, \quad \sigma^2 \left[= \frac{r(1-p)}{p^2} \right] = \frac{4 \times \frac{5}{6}}{\frac{1}{36}} = \underline{120}$	A1, A1	1.1b(x2)
$[\bar{X} \approx \sim] N\left(\text{"24"}, \sqrt{\frac{\text{"120"}^2}{32}}\right)$	M1 M1	2.1,3.4
$P(\bar{X} < 27.2) = 0.95078 \dots$ awrt <u>0.951</u>	A1	1.1b
	(6 marks)	

Notes

1st M1 for selecting the correct negative binomial model. May be implied by correct mean or variance
NegBin on its own is M0

1st A1 for mean = 24

2nd A1 for variance = 120 $\sigma = 120$ is A0 unless recovered

2nd M1 for writing or using of normal with mean 24 (may be implied by correct answer)
ft their mean which may come from any distribution

3rd M1 for writing or using normal with standard deviation = $\sqrt{\frac{120}{32}}$ [= $\sqrt{3.75}$]

ft $\frac{\text{their } \sigma}{\sqrt{32}}$ where σ may come from any distribution

(may be implied by correct answer)

2nd A1 for awrt 0.951 (correct answer scores 6 out of 6)

3.

Mark scheme (a)

Scheme	Marks	AO
One correct assumption in context from below	B1	3.5b
Two correct assumptions from below, with goals or matches mentioned at least once - Goals are scored at a <u>constant rate</u> - Goals are scored <u>independently/randomly</u> or matches are <u>independent</u> - Goals are scored not scored <u>simultaneously</u> or Goals are scored <u>singly</u>	B1	3.5b
	(2)	

Notes

B1: One contextual correct assumption relating to independence/constant rate/singly.

Must mention goals or match, or equivalent context.

B1: Two correct contextual assumptions with goals/matches mentioned at least once.

Assumptions relating to probability or number of goals are B0, but do not penalise if two correct assumptions also given.

Mark scheme (b)

Scheme	Marks	AO
$A_{0.5} \sim \text{Po}(0.9)$ and $A_1 \sim \text{Po}(1.8)$	B1	3.3

$P(A_{0.5} = 2 \mid A_1 = 3) = \frac{P(A_{0.5} = 2) \times P(A_{0.5} = 1)}{P(A_1 = 3)}$	M1	3.4
$P(A_{0.5} = 2 \mid A_1 = 3) = \frac{\frac{0.9^2 e^{-0.9}}{2!} \times \frac{0.9 e^{-0.9}}{1!}}{\frac{1.8^3 e^{-1.8}}{3!}} \text{ or }$ $\frac{0.164 \dots \times 0.365 \dots}{0.160 \dots}$	M1	1.1b
$P(A_{0.5} = 2 \mid A_1 = 3) = \frac{3}{8} \quad \frac{3}{8} \text{ or exact equivalent}$	A1	1.1b
	(4)	

Notes

B1: Stating or using both correct Poisson distributions [0.164... or 0.365... implies Po(0.9), 0.160... implies Po(1.8). Both are required for this mark]

or writing or using B(3, 0.5)

M1: Correct use of conditional probability formula. Implied by $\frac{0.164 \dots \times 0.365 \dots}{0.160 \dots}$

Or writing or using **any** binomial distribution **and** $P(X = 2)$

M1: Correct expression with probabilities **or** writing or using B(3, 0.5) **and** $P(X = 2)$

A1: Correct exact answer. Do not award if there is clear evidence of rounding.

Mark scheme (c)

Scheme	Marks	AO
(i) $X \sim B(4, 0.55)$	B1	3.3
(ii) $E(X) = '4' \times '0.55' [= 2.2] \text{ or }$ $\text{Var}(X) = '4' \times '0.55' \times (1 - '0.55') [= 0.99]$	M1	2.1
$\bar{X} \simeq N\left(2.2, \frac{0.99}{40}\right)$	M1	3.3
$P(\bar{X} > 2) \text{ awrt } 0.898$	A1	3.4
	(4)	
	(10 marks)	

Notes

(i) **B1:** Stating correct binomial distribution

(ii) **M1:** Correct method to find the mean **or** variance of X (a mean of 2.2 from Poisson is M1)
ft their n and p

M1: Correct approximate normal distribution [condone using X not $\bar{X}$]. Allow variance awrt 0.0248

A1: awrt 0.898

NB: awrt 0.898 implies the M1M1A1