

QQQ – Further Statistics 1 – Chapter 5 – Central Limit Theorem (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. Explain what you understand by the Central Limit Theorem. **(3)**

(Total 3 marks)

Edexcel S3 June 2011 Q1

2. A courier delivers parcels. The random variable X represents the number of parcels delivered successfully each day by the courier where $X \sim B(400, 0.64)$

A random sample $X_1, X_2, \dots, X_{100}$ is taken.

- Estimate the probability that the mean number of parcels delivered each day by the courier is greater than 257 **(4)**

(Total 4 marks)

Edexcel A2 October 2021 Q3

3. A random sample of 150 observations is taken from a geometric distribution with parameter 0.3

- Estimate the probability that the mean of the sample is less than 3.45 **(5)**

(Total 5 marks)

Edexcel A2 June 2022 Q5

4. A report on the health and nutrition of a population stated that the mean height of three-year old children is 90 cm and the standard deviation is 5 cm. A sample of 100 three-year old children was chosen from the population.

- (a) Write down the approximate distribution of the sample mean height. Give a reason for your answer. **(3)**

- (b) Hence find the probability that the sample mean height is at least 91 cm. **(3)**

(Total 6 marks)

Edexcel S3 June 2006 Q2

5. A biased spinner can land on the numbers 1, 2, 3, 4 or 5 with the following probabilities.

Number on spinner	1	2	3	4	5
Probability	0.3	0.1	0.2	0.1	0.3

The spinner will be spun 80 times and the mean of the numbers it lands on will be calculated.

Find an estimate of the probability that this mean will be greater than 3.25 **(6)**

(Total 6 marks)

Solutions

1.

Scheme	Marks
$X_1, X_2, \dots, X_n$ is a random sample of size n , for large n ,	B1
drawn from a population of any distribution with mean μ and variance σ^2	B1
then $\bar{X}$ is (approximately) $N\left(\mu, \frac{\sigma^2}{n}\right)$	B1
	(3)
	(3 marks)

Notes

1st B for large sample or equivalent

2nd B for 'population of any distribution' or 'any population'

3rd B require mean or symbol and normal (parameters not required)

2.

Scheme	Marks	AO
$\bar{X} \approx N(256, \dots)$ oe	M1	3.1a
$\bar{X} \approx N(256, 0.9216)$	A1	1.1b
$P(\bar{X} > 257) = P\left(Z > \frac{257 - 256}{\sqrt{0.9216}}\right)$ [= awrt 1.04]	dM1	3.4
$p = 0.1492 \dots$	A1	1.1b
	(4)	
	(4 marks)	

Notes

M1: For realising the need to use the CLT with correct mean

A1: For a correct normal stated

dM1: Dep on previous Method mark. Use of the normal model to find $P(\bar{X} > 257)$ If final answer is incorrect then we need to see the standardisation using their σ .

A1: awrt 0.149 (0.14878... from calculator)

NB Allow awrt 0.148 if a continuity correction is used.

3.

Scheme	Marks	AO
$\text{Geo}(0.3) \quad \mu = \frac{1}{0.3} \left[\text{or exact equivalent e.g. } \frac{10}{3} \right]$	B1	1.1b
$\sigma^2 = \frac{1 - 0.3}{0.3^2} \left[\text{or exact equivalent e.g. } \frac{70}{9} \right]$	B1	1.1b
$\text{CLT} \Rightarrow \bar{X} \approx N \left(\frac{10}{3}, \dots \right) \text{ oe}$	M1	2.1
$\Rightarrow \bar{X} \approx N \left(\frac{10}{3}, \frac{7}{135} \right) \text{ and attempt (sight of) } P(\bar{X} < 3.45)$	M1	3.4
$= 0.69579 \dots \text{ awrt } \underline{\mathbf{0.696}}$	A1	1.1b
	(5)	
	Total 5	

Notes

1st B1 correct mean

2nd B1 correct Var may be implied by sight of $\frac{7}{135}$ in distribution of $\bar{X}$

1st M1 For use of CLT (must see $\bar{X}$ and Normal with mean correct ft) or sight of $N \left(\frac{10}{3}, \frac{7}{135} \right)$ or

$N \left(\frac{10}{3}, \frac{70}{9 \times 150} \right)$ with any letter

Allow 3.33 or better for $\frac{10}{3}$ and 7.78 or better for $\frac{70}{9}$

May be implied by **2nd M1**

2nd M1 Using the normal distribution to find $P(\bar{X} < 3.45)$ ft their $\frac{10}{3}$ and $\frac{70}{150}$

May be implied by correct answer.

A1 awrt 0.696

Correct answer with no incorrect working scores 5/5

Alternative (Use of $Y = \sum X$)

Scheme	Marks
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$\mu = \frac{150}{0.3} [= 500]$	B1
$\sigma^2 = \frac{150 \times 0.7}{0.3^2} \left[\frac{3500}{3} \right] = 1166.\dot{6}$	B1
$\Rightarrow Y \approx N \left(500, \frac{3500}{3} \right)$	M1
$P(Y < 517.5)$	M1
$= 0.69579 \dots$	A1

4.

Mark scheme (a)

Scheme	Marks
$\bar{X} \sim N \left(90, \frac{5^2}{100} \right)$ i.e. $N(90, 0.25)$	M1 A1
Application of <u>central limit theorem</u> as (sample large)	B1
	(3)

Mark scheme (b)

Scheme	Marks
$P(\bar{X} \geq 91) = 1 - P \left(Z < \frac{91 - 90}{0.5} \right)$ Stand.	M1 A1
$= 1 - P(Z < 2)$ $= 1 - 0.9772$ $= 0.0228$ awrt 0.0228	A1
	(3)
	(6 marks)

5.

Scheme	Marks	AO
{Let X = the number when the spinner is spun} $\mu = \underline{3}$	B1	1.1b
$[E(X^2) =] 0.3 + 4 \times 0.1 + 9 \times 0.2 + 16 \times 0.1 + 25 \times 0.3$ $[= 11.6 \text{ or } \frac{58}{5}]$	M1	1.1b

$\sigma^2 [= 11.6 - 3^2 =] \underline{2.6}$	A1	1.1b
$\bar{X} \approx N\left(“3”, \sqrt{\frac{“2.6”}{80}}^2\right)$	M1 A1ft	2.1 1.1b
$P(\bar{X} > 3.25) = [P(Z > 1.3867 \dots) =] 0.0827589 \dots$ (calc) awrt 0.0828	A1	3.4
	(6 marks)	

Notes

B1 for stating or using mean = 3

1st M1 for using the given model to attempt $E(X^2)$ with at least 3 correct products seen

1st A1 for $\text{Var}(X) = 2.6$ or $\sigma = \sqrt{2.6} = 1.6124 \dots$ (awrt 1.61)

ALT Use of pgf (B1 when mean = 3 seen) (M1 when correct $G''(t)$ seen with attempt at $G''(1)$)

$$\begin{aligned}
 G(t) &= 0.3t + 0.1t^2 + 0.2t^3 + 0.1t^4 + 0.3t^5 \\
 G'(t) &= 0.3 + 0.2t + 0.6t^2 + 0.4t^3 + 1.5t^4 \\
 G''(t) &= 0.2 + 1.2t + 1.2t^2 + 6t^3 \quad \text{leading to } G''(1) = 8.6
 \end{aligned}$$

2nd M1 for use of CLT – must use $\bar{X}$ and normal or sight of $N\left(“3”, \sqrt{\frac{“2.6”}{80}}^2\right)$ with any letter

2nd A1ft for a correct mean and variance, ft their 3 and their 2.6

This M1A1ft may be implied by sight of correct st. dev. used in a standardisation leading to $P(Z > 1.39)$

Must see correct use of Z

NB $\frac{2.6}{80} = 0.0325$ and $\sqrt{\frac{2.6}{80}} = 0.18027 \dots$ so allow e.g. $N(3, \text{awrt } (0.180)^2)$

3rd A1 for using the normal model to find probability awrt 0.0828

ALT Use of $\sum X$ (If see clear attempt at $P(\sum X > 260)$ condone $P(\sum X > 260.5)$) then:

2nd M1 for $\sum X \sim N(\dots)$ or any letter $\sim N\left(“240”, \sqrt{“2.6” \times 80}^2\right)$

2nd A1ft for mean = “3” $\times 80 = 240$ and variance = “2.6” $\times 80 = 208$

May see $P(\sum X > 260.5) = 0.077597 \dots$ but it will only score **2nd M1** **2nd A1ft** and **3rd A0**