

QQQ – Further Statistics 1 – Chapter 4 – Hypothesis Testing (Version 2)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. Every morning Geethaka repeatedly rolls a fair, six-sided die until he rolls a 3 and then he stops. The random variable X represents the number of times he rolls the die each morning.

(a) Suggest a suitable model for the random variable X (1)

(b) Show that $P(X \leq 3) = \frac{91}{216}$ (2)

After 64 mornings Geethaka will calculate the mean number of times he rolled the die.

(c) Estimate the probability that the mean number of rolls is between 5.6 and 7.2 (5)

Nira wants to check Geethaka's die to decide whether or not the probability of rolling a 3 with his die is less than $\frac{1}{6}$

Nira rolls the die repeatedly until she rolls a 3

She obtains $x = 16$

(d) By carrying out a suitable test, determine what Nira's conclusion should be. You should state your hypotheses clearly and use a 5% level of significance. (4)

(Total 12 marks)

2. Andreia's secretary makes random errors in his work at an average rate of 1.7 errors every 100 words.

- (a)** Find the probability that the secretary makes fewer than 2 errors in the next 100-word piece of work. **(2)**

Andreia asks the secretary to produce a 250-word article for a magazine.

- (b)** Find the probability that there are exactly 5 errors in this article. **(2)**

Andreia offers the secretary a choice of one of two bonus schemes, based on a random sample of 40 pieces of work each consisting of 100 words.

In scheme **A** the secretary will receive the bonus if more than 10 of the 40 pieces of work contain no errors.

In scheme **B** the bonus is awarded if the total number of errors in all 40 pieces of work is fewer than 56

- (c)** Showing your calculations clearly, explain which bonus scheme you would advise the secretary to choose. **(5)**

Following the bonus scheme, Andreia randomly selects a single 500-word piece of work from the secretary to test if there is any evidence that the secretary's rate of errors has decreased.

- (d)** Stating your hypotheses clearly and using a 5% level of significance, find the critical region for this test. **(4)**

(Total 13 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
$X \sim \text{Geo}\left(\frac{1}{6}\right)$ accept in words: <u>geometric</u> distribution with $p = \frac{1}{6}$	B1	3.3
	(1)	

Notes

B1 for both “geometric” or “Geo” **and** correct parameter of $\frac{1}{6}$. Probability must be seen in (a)

Mark scheme (b)

Scheme	Marks	AO
$P(X \leq 3) = 1 - P(X > 3)$ <u>or</u> $P(X = 1) + P(X = 2) + P(X = 3)$	M1	1.1b
$= 1 - \left(\frac{5}{6}\right)^3$ or $\frac{1}{6} + \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^2 \times \frac{1}{6} = \frac{91}{216}$ (*)	Alcso	1.1b
	(2)	

Notes

M1 for a correct method, may be implied by a correct expression

Alcso* for a correct solution leading to printed answer with no incorrect working seen

Mark scheme (c)

Scheme	Marks	AO
$E(X) = 6$ $\text{Var}(X) = \frac{\frac{5}{6}}{\left(\frac{1}{6}\right)^2} [= 30]$	M1 A1	3.4 1.1b

$\bar{X} \approx \sim N \left(6, \left(\sqrt{\frac{30}{64}} \right)^2 \right)$	M1 A1	3.3 1.1b
$P(5.6 < \bar{X} < 7.2) = 0.68064 \dots$	A1	1.1b
	(5)	

Notes

1st M1 for correct use of formula for $E(X)$ or $\text{Var}(X)$

1st A1 for both correct must see value of 6 for $E(X)$ and at least correct formula used for $\text{Var}(X)$ the value of 30 may be implied by 2nd A1

2nd M1 for use of CLT to get a normal distribution with correct mean and variance $\neq$ their 30

Condone use of X (or their letter) instead of $\bar{X}$ or seeing $N \left(6, \sqrt{\frac{30}{64}} \right)$

2nd A1 for a correct normal distribution stated or used. May be implied by correct answer.

3rd A1 for awrt 0.681 but allow 0.68 or 0.680 if correct distribution is seen

Mark scheme (d)

Scheme	Marks	A0
$H_0 : p = \frac{1}{6} \quad H_1 : p < \frac{1}{6}$	B1	2.5
<u>Probability approach</u> : $P(X \geq 16)$ <u>CR approach</u> : $\left(\frac{5}{6}\right)^{c-1} < 0.05$ or $\left(\frac{5}{6}\right)^d < 0.05$ with use of logs	M1	3.4
<u>Probability approach</u> : $= P(X > 15) = \left(\frac{5}{6}\right)^{15} = 0.0649 \dots$ <u>CR approach</u> : CR: $X \geq 18$ or $X > 17$	A1	1.1b
(Not significant) insufficient evidence that <u>probability</u> is $< \frac{1}{6}$ or (Not significant) insufficient evidence that dice is <u>biased</u>	A1	2.2b
	(4)	
	(12 marks)	

Notes

B1 for both hypotheses correct in terms of p

M1 for sight or use of $P(X \geq 16)$ (may be implied)

or correct expression with logs (use of logs may be implied by 17.43...)

1st A1 for 0.065 or better but accept 0.06 if a correct expression is seen, **or** correct CR

2nd A1 for a correct conclusion mentioning probability or biased

Condone 'evidence suggests probability is equal to $\frac{1}{6}$ '

NB: Must have correct probability or correct CR to gain final A1.

NB: Send responses comparing 0.935 with 0.95 to review

2.

Mark scheme (a)

Scheme	Marks	AO
$[X = \text{number of errors in 100-word piece}] X \sim \text{Po}(1.7)$	M1	3.3
$P(X < 2) = P(X \leq 1) = 0.49324 \dots$ awrt 0.493	A1	1.1b
	(2)	

Notes

M1 for selecting the correct Poisson distribution

A1 for awrt 0.493

Mark scheme (b)

Scheme	Marks	AO
$[R = \text{number of errors in the article}] R \sim \text{Po}(4.25)$	M1	3.3
$P(R = 5) = 0.16482 \dots$ awrt 0.165	A1	1.1b
	(2)	

Notes

M1 for selecting the correct Poisson distribution

A1 for awrt 0.165

Mark scheme (c)

Scheme	Marks	A0
Scheme A: Let $A \sim B(40, e^{-1.7})$ or $B(40, 0.18268 \dots)$	M1	3.3
$P(A > 10) = 1 - P(A \leq 10)$	M1	1.1b
$= 0.0995591 \dots$ awrt 0.0996	A1	1.1b
Scheme B: Let $B \sim \text{Po}(40 \times 1.7)$ or $\text{Po}(68)$ $P(B < 56) = P(B \leq 55) = 0.061133 \dots$	M1	3.3
So choose scheme A (since the probability of a bonus is greater)	A1	2.4
	(5)	

Notes

1st M1 for choosing a correct model for scheme A i.e. $B(40, P(X = 0))$, where $X \sim \text{Po}(1.7)$

Allow use of awrt 0.183 for $P(X = 0) \dots 0.183$ gives answer awrt 0.101

Condone $B(0.183, 40)$ (o.e.) if it leads to a prob rounding to range (0.09~0.1) otherwise M0

2nd M1 for $1 - P(A \leq 10)$

1st A1 for awrt 0.0996 [NB use of 0.183 will give awrt 0.101 and scores M1M1A0]

3rd M1 for selecting a correct Poisson model for scheme B i.e. $\text{Po}(40 \times 1.7)$ or better

2nd A1 for a correct conclusion based on comparing two probs: awrt 0.1 vs 0.061 or better

So can allow $0.1 > 0.061$ leading to choosing A [Probably scores M1M1A0M1A1]

NB [Normal approx.(not on spec) leading to 0.06477... might score 3rd M1 if $\text{Po}(68)$ seen but 2nd A0]

Mark scheme (d)

Scheme	Marks	A0
$H_0 : \lambda = 1.7$ (or $\mu = 8.5$) $H_1 : \lambda < 1.7$ (or $\mu < 8.5$)	B1	2.5
[E = no. of errors in the piece of work] $E \sim \text{Po}(8.5)$	M1	3.3
$P(E \leq 3) = 0.0301$ or $P(E \leq 4) = 0.0744$	A1	1.1b
So critical region is $E \leq 3$	A1	2.2a
	(4)	
	(13 marks)	

Notes

B1 for both hypotheses in terms of λ or μ (can be interchanged)

M1 for selecting $\text{Po}(8.5)$ (sight of or use of e.g. may be implied by 1st A1)

1st A1 for some evidence of correct use of $\text{Po}(8.5)$ i.e. either of these probs (2dp or better)

May be implied by a correct critical region

2nd A1 for a correct critical region. Allow $E < 4$ and allow any letter for E .

Two different regions (e.g. from 2 tail test) is 2nd A0

SC Use of binomial throughout: (with hypotheses $H_0 : p = 0.017$ and $H_1 : p < 0.017$ in (d))

Scores 0 in (a) 0 in (b) possibly just 2nd M1 in (c) But allow all 4 marks in (d): B1 hypotheses, M1 for

$Y \sim B(500, 0.017)$, 1st A1 for $P(Y \leq 3) = 0.02913 \dots$ or $P(Y \leq 4) = 0.07266 \dots$ 2nd A1 $Y \leq 3$

Allow probs to be to 2dp or better so 0.03 and 0.07 as in main scheme.