

QQQ – Further Statistics 1 – Chapter 4 – Hypothesis Testing (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. An effect of a certain disease is that a small number of the red blood cells are deformed. Emily has this disease and the deformed blood cells occur randomly at a rate of 2.5 per ml of her blood. Following a course of treatment, a random sample of 2 ml of Emily's blood is found to contain only 1 deformed red blood cell.

Stating your hypotheses clearly and using a 5% level of significance, test whether or not there has been a decrease in the number of deformed red blood cells in Emily's blood. **(6)**

(Total 6 marks)

2. Asha, Davinda and Jerry each have a bag containing a large number of counters, some of which are white and the rest are red.

Each person draws counters from their bag one at a time, notes the colour of the counter and returns it to their bag.

The probability of Asha getting a red counter on any one draw is 0.07

(a) Find the probability that Asha will draw at least 3 white counters before a red counter is drawn. **(2)**

(b) Find the probability that Asha gets a red counter for the second time on her 9th draw. **(2)**

The probability of Davinda getting a red counter on any one draw is p .

Davinda draws counters until she gets n red counters. The random variable D is the number of counters Davinda draws.

Given that the mean and the standard deviation of D are 4400 and 660 respectively,

(c) find the value of p . **(4)**

Jerry believes that his bag contains a smaller proportion of red counters than Asha's bag. To test his belief, Jerry draws counters from his bag until he gets a red counter. Jerry defines the random variable J to be the number of counters drawn up to and including the first red counter.

(d) Stating your hypotheses clearly and using a 10% level of significance, find the critical region for this test. **(5)**

Jerry gets a red counter for the first time on his 34th draw.

(e) Giving a reason for your answer, state whether or not there is evidence that Jerry's bag contains a smaller proportion of red counters than Asha's bag. **(2)**

Given that the probability of Jerry getting a red counter on any one draw is 0.011

(f) show that the power of the test is 0.702 to 3 significant figures. **(3)**

(Total 18 marks)

Solutions

1.

Scheme	Marks
$H_0 : \lambda = 2.5$ (or $\lambda = 5$) $H_1 : \lambda < 2.5$ (or $\lambda < 5$) λ or μ	B1B1
$X \sim \text{Po}(5)$	M1
$P(X \leq 1) = 0.0404$ or CR $X \leq 1$	A1
[0.0404<0.05] this is significant or reject H_0 or it is in the critical region	M1
There is evidence of a <u>decrease</u> in the (mean) <u>number/rate</u> of <u>deformed blood cells</u>	A1
	(6)
	(6 marks)

Notes

1st B1 for H_0 must use lambda or mu; 5 or 2.5.

2nd B1 for H_1 must use lambda or mu; 5 or 2.5

1st M1 for use of Po(5) may be implied by probability(must be used not just seen) eg.

$P(X = 1) = 0.0404 - \dots$ would score M1 A0

1st A1 for 0.0404 seen or correct CR

2nd M1 for a correct statement (this may be contextual) comparing their probability and 0.05 (or comparing 1 with their critical region). Do not allow conflicting statements.

2nd A1 is not a follow through. Need the word decrease, number or rate and deformed blood cells for contextual mark.

If they have used $\neq$ in H_1 they could get B1 B0 M1 A1 M1A0

mark as above except they gain the

1st A1 for $P(X \leq 1) = 0.0404$ or CR $X \leq 0$

2nd M1 for a correct statement (this may be contextual) comparing their probability and 0.025 (or comparing 1 with their critical region)

They may compare with 0.95 (one tail method) or 0.975 (two tail method)

Probability is 0.9596.

(corrected from the printed mark scheme: printed “0.975 (one tail method)”)

2.

Mark scheme (a)

Scheme	Marks	AO
$P(\text{at least 3 whites}) = (1 - 0.07)^3$ or $1 - 0.07 - 0.93 \times 0.07 - 0.93^2 \times 0.07$	M1	1.1b
$= 0.8043 \dots$ awrt 0.804	A1	1.1b
	(2)	

Notes

M1: A correct method to find $P(X \geq 3)$

A1: awrt 0.804

Mark scheme (b)

Scheme	Marks	AO
$P(\text{2nd red on 9}^{\text{th}} \text{ draw}) = \binom{8}{1} 0.93^7 \times 0.07^2$	M1	3.3
$= 0.02358 \dots$ awrt 0.0236	A1	1.1b
	(2)	

Notes

M1: For selecting the appropriate model negative binomial or binomial with an extra trial

A1: awrt 0.0236

Mark scheme (c)

Scheme	Marks	AO
$\frac{n}{p} = 4400$ and $\frac{n(1-p)}{p^2} = 660^2$	M1 A1	3.1b 1.1b
$1 - p = 99p$ oe	M1	1.1b
$p = 0.01$	A1	1.1b
	(4)	

Notes

M1: Forming an equation for the mean and variance. At least one correct.

A1: Both equations correct

Allow M1 A1 if both equations correct with the same number subst for n

M1: Solving the 2 equations leading to $1 - p = 99p$ or $p - p^2 = 99p^2$ ft their 4400 and 660 Allow $1 - p = 0.15p$

A1: 0.01

Mark scheme (d)

Scheme	Marks	AO
$H_0 : p = 0.07 \quad H_1 : p < 0.07$	B1	2.5
$J \sim \text{Geo}(0.07)$	M1	3.3
$P(J \geq c) < 0.1 \Rightarrow (1 - 0.07)^{c-1} < 0.1$	M1	3.4
$c - 1 > \frac{\log 0.1}{\log 0.93}$	M1	1.1b
$c > 32.72 \dots \therefore \text{CR } J \geq 33$	A1	1.1b
	(5)	

Notes

B1: Both hypotheses correct using correct notation allow eg $p > 0.93$ (corrected from the printed mark scheme, whose notes label this mark M1; the scheme awards it as B1)

M1: Realising the need to use Geo(0.07) ft their Hypotheses

M1: Using the model to find $P(J \geq c)$ Condone $(1 - 0.07)^c < 0.1$ ft their $0.07 \neq 0.93$

ALT $P(J \geq 32) = 0.1[054 \dots]$ or $P(J \geq 33) = 0.09[8 \dots]$ Implied by correct CR

M1: For a valid method to solve the inequality or $P(J \geq 32) = 0.1[054]$ and $P(J \geq 33) = 0.09[81]$ Implied by correct CR

A1: Correct CR (any letter) AO if given as a probability statement. Must be integer

Mark scheme (e)

Scheme	Marks	AO
34 is in the Critical region	M1	1.1b
There is evidence to suggest that Jerry's bag contains a smaller proportion of red counters than Asha's bag.	A1	2.2b

	(2)	
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Notes

M1: Comparing 34 with their CR eg $34 > 33$ $34 \geq 33$ or $P(J \geq 34) = 0.09[12]$

A1: Fully correct conclusion in context. Allow Jerry's belief is true. Allow probability for proportion

Mark scheme (f)

Scheme	Marks	AO
Power of test = $P(J \geq 33 \mid p = 0.011)$	M1	2.1
$= (1 - 0.011)^{32}$ oe	M1	1.1b
$= 0.7019 \dots^*$	A1*	1.1b
	(3)	
	(18 marks)	

Notes

M1: Realising they need to find $P(\text{their CR in (d)})$ Allow $1 - P(J \leq 32)$

M1: For a Correct method. Allow $1 - 0.2981 \dots$ May be implied by $0.7019 \dots$ If the CR is incorrect $(1 - 0.011)^{\text{CR}-1}$ or $1 - \{1 - (1 - 0.011)^{\text{CR}-1}\}$ must be seen

A1*: Only award if both method marks awarded.