

QQQ – Further Statistics 1 – Chapter 3 – Geometric and Negative Binomial Distributions (Version 2)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. A spinner can land on red or blue. When the spinner is spun, there is a probability of $\frac{1}{3}$ that it lands on blue. The spinner is spun repeatedly.

The random variable B represents the number of the spin when the spinner first lands on blue.

(a) Find

(i) $P(B = 4)$

(ii) $P(B \leq 5)$

(4)

(b) Find $E(B^2)$

(3)

Steve invites Tamara to play a game with this spinner.

Tamara must choose a colour, either red or blue.

Steve will spin the spinner repeatedly until the spinner first lands on the colour Tamara has chosen. The random variable X represents the number of the spin when this occurs.

If Tamara chooses red, her score is e^X

If Tamara chooses blue, her score is X^2

(c) State, giving your reasons and showing any calculations you have made, which colour you would recommend that Tamara chooses. (5)

(Total 12 marks)

2. In a game a spinner is spun repeatedly. When the spinner is spun, the probability of it landing on blue is 0.11

(a) Find the probability that the spinner lands on blue

(i) for the first time on the 6th spin, **(2)**

(ii) for the first time before the 6th spin, **(2)**

(iii) exactly 4 times during the first 6 spins, **(2)**

(iv) for the 4th time on or before the 6th spin. **(4)**

Zac and Izana play the game. They take turns to spin the spinner. The winner is the first one to have the spinner land on blue. Izana spins the spinner first.

(b) Show that the probability of Zac winning is 0.471 to 3 significant figures. **(3)**

(Total 13 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
(i) $[B \sim \text{Geo}(\frac{1}{3})] \quad P(B = 4) = (\frac{2}{3})^3 \times \frac{1}{3}$	M1	3.3
$= \frac{8}{81}$	A1	1.1b
(ii) $P(B \leq 5) = 1 - P(B > 5) \text{ or } 1 - (\frac{2}{3})^5$	M1	2.1
$= \frac{211}{243}$	A1	1.1b
	(4)	

Notes

(i) **M1** for selecting the correct model i.e. $\text{Geo}(p)$ (May be implied by a correct expression)

A1 for $\frac{8}{81}$ (= 0.098765... accept awrt 0.0988)

(ii) **M1** for a suitable strategy to use the geometric model to find a correct expression

A1 for $\frac{211}{243}$ (= 0.868312... accept awrt 0.868)

Mark scheme (b)

Scheme	Marks	AO
$E(B^2) = \text{Var}(B) + [E(B)]^2$	M1	2.1
From formula booklet: $E(B) = \frac{1}{\frac{1}{3}} = 3$ and $\text{Var}(B) = \frac{1 - \frac{1}{3}}{(\frac{1}{3})^2} = 6$	B1	1.1b
So $E(B^2) = 6 + 9 = \underline{15}$	A1	1.1b
	(3)	

Notes

M1 for a suitable strategy to find $E(B^2)$ [allow $G''(1) + G'(1)$]

B1 for use of the correct formulae to find $E(B) = 3$ and $\text{Var}(B) = 6$ or $G''(1) = 12$

A1 for 15

SC Formula for $E(B^2)$ Allow M1B1A0 for $E(B^2) = \frac{2-p}{p^2}$ (o.e.)

Mark scheme (c)

Scheme	Marks	AO
[Let R = no. of the spin when it first lands on red] $X = R \sim \text{Geo}\left(\frac{2}{3}\right)$	M1	3.3
Require $E(e^X) = \sum_{x=1}^{\infty} e^x \left(\frac{1}{3}\right)^{x-1} \frac{2}{3}$	M1	3.1a
$= \frac{2e}{3} \sum_{x=1}^{\infty} \left(\frac{e}{3}\right)^{x-1}$	M1	2.1
$= \frac{2e}{3} \times \frac{1}{1 - \frac{e}{3}} \text{ or } \frac{2e}{3-e}$	A1	1.1b
$E(e^X) = 19.297 \dots \{ > 15 = E(B^2) \}$ so Tamara should choose red since it has the greater expected score	A1	2.2a
	(5)	
	(12 marks)	

Notes

1st M1 for choosing a suitable geometric model (sight of $\text{Geo}\left(\frac{2}{3}\right)$ or at least 3 correct probabilities)

2nd M1 for realising the need for appropriate expected value and using $E(g(X))$ [Need sum and $f(x)$]

NB simply finding $e^{E(X)} = e^{1.5} = \text{awrt } 4.48$ is M0 and probably no more marks.

3rd M1 for a suitable strategy to turn the expression into a sum that can be found

1st A1 for correct use of sum to infinity of geometric series

2nd A1 for interpreting the outcome of the calculations in terms of a solution to the problem must choose red and see the awrt 19.3 (and allow ft of their $E(B^2) < 19$)

2.

Mark scheme (a)

Scheme	Marks	A0
(i) $[W \sim \text{Geo}(0.11)] \quad P(W = 6) = (0.89)^5(0.11)$	M1	3.3
$= 0.06142 \dots$ awrt <u>0.0614</u>	A1	1.1b
	(2)	
(ii) $P(W \leq 5) = 1 - (0.89)^5$	M1	3.1b
$= 0.44159 \dots$ awrt <u>0.442</u>	A1	1.1b
	(2)	
(iii) $X \sim B(6, 0.11)$	M1	3.3
$P(X = 4) = 0.001739 \dots$ awrt <u>0.00174</u>	A1	1.1b
	(2)	
(iv) $[Y \sim \text{NB}(4, 0.11)]$ using a neg bin or $V \sim B(6, 0.11)$ and $P(V \geq 4)$ for M2	M1	3.3
$P(Y \leq 6) = P(Y = 4) + P(Y = 5) + P(Y = 6)$	M1	3.1b
$= (0.11)^4 + \binom{4}{3}(0.11)^3(0.89)^1 \times 0.11 +$ $\binom{5}{3}(0.11)^3(0.89)^2 \times 0.11$	M1	3.4
$= 0.001827$ awrt <u>0.00183</u>	A1	1.1b
	(4)	

Notes

(i) **M1** Correct method to find $P(W = 6)$ eg $(p)^5(1 - p)$ for $p = 0.11$ or 0.89

A1 awrt 0.0614 (Correct ans with no incorrect working 2/2)

(ii) **M1** Correct method to find $P(W \leq 5)$

A1 awrt 0.442 (Correct ans with no incorrect working 2/2)

(iii) **M1** For using the model $B(6, 0.11)$ allow $B(6, 0.89)$ [Implied by 0.0017 or awrt 0.114]

A1 awrt 0.00174 (Correct ans with no incorrect working 2/2)

(iv) In part (iv) we can accept correct expressions or values for probabilities

1st M1 For using a negative binomial model implied by correct $P(Y = 5)$ or $P(Y = 6)$

2nd M1 Correct method to find $P(Y \leq 6)$

3rd M1 At least two correct terms or $1 - 0.99817\dots$ from $1 - P(V \leq 3)$

A1 awrt 0.00183

a	4	5	6
$P(Y = a)$	1.46×10^{-4}	5.21×10^{-4}	1.16×10^{-3}
$P(V = a)$	1.74×10^{-3}	8.60×10^{-5}	1.77×10^{-6}

Mark scheme (b)

Scheme	Marks	AO
$P(\text{Zac wins}) = 0.89 \times 0.11 + (0.89)^3 \times 0.11 + (0.89)^5 \times 0.11 + \dots$	M1	3.1b
$= \frac{0.89 \times 0.11}{1 - (0.89)^2}$ oe	M1	1.1b
$= 0.47089\dots = 0.471^*$	Alcso*	2.1
	(3)	
	Total 13	

Notes

1st M1 Forming the correct probability of Zac winning or identify a and r of GP

Allow for $p = (0.11) \times 0 + (1 - 0.11)(1 - p)$

2nd M1 Using sum to infinity of a GP

Allow for $p = \frac{0.89}{1 + 0.89}$

A1* Previous method marks must be seen leading to an answer 0.471 (NOT awrt 0.471)