

QQQ – Further Statistics 1 – Chapter 3 – Geometric and Negative Binomial Distributions (Version 1)

Total Marks: 22

(22 = Platinum, 20 = Gold, 18 = Silver, 15 = Bronze)

1. A chocolate manufacturer places special tokens in 2% of the bars it produces so that each bar contains at most one token. Anyone who collects 3 of these tokens can claim a prize.

Andreia buys a box of 40 bars of the chocolate.

(a) Find the probability that Andreia can claim a prize. (2)

Barney intends to buy bars of the chocolate, one at a time, until he can claim a prize.

(b) Find the probability that Barney can claim a prize when he buys his 40th bar of chocolate. (3)

(c) Find the expected number of bars that Barney must buy to claim a prize. (1)

(Total 6 marks)

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2. Irina is practising her serves in badminton and counts the number of her serves that are faults. She finds that 15% of her serves are faults.

Assuming that each serve is independent,

(a) find the probability that

(i) Irina's 4th fault comes on her 20th serve, (2)

(ii) in 18 serves, Irina has 4 faults. (2)

With practice, Irina reduces her proportion of faults, p , so that the mean number of serves until her 4th fault is at least 32

(b) Find the maximum value of p (3)

(Total 7 marks)

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3. Suzanne and Jon are playing a game.

They put 4 red counters and 1 blue counter in a bag.

Suzanne reaches into the bag and selects one of the counters at random. If the counter she selects is blue, she wins the game. Otherwise she puts it back in the bag and Jon selects one at random.

If the counter he selects is blue, he wins the game. Otherwise he puts it back in the bag and they repeat this process until one of them selects the blue counter.

- (a)** Find the probability that Suzanne selects the blue counter on her 4th selection. **(2)**
- (b)** Find the probability that the blue counter is first selected on or after Jon's third selection. **(2)**
- (c)** Find the mean and standard deviation of the number of selections made until the blue counter is selected. **(2)**
- (d)** Find the probability that Suzanne wins the game. **(3)**

(Total 9 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
[Let X = no. of prizes Andreia wins] $X \sim B(40, 0.02)$	M1	3.3
[Require $P(X \geq 3) = 1 - P(X \leq 2) = 0.04567 \dots$ awrt <u>0.0457</u>	A1	1.1b
	(2)	

Notes

M1 for selecting a suitable model i.e. $B(40, p)$ where p is any probability

Written or used, may be implied by a correct ans or $0.037429 \dots$ from $P(X = 3)$

A1 for awrt 0.0457 (correct answer only 2/2)

Mark scheme (b)

Scheme	Marks	AO
[Let Y = no. of the bar when Barney wins] $Y \sim \text{NegBin}(3, 0.02)$	M1	3.3
$[P(Y = 40) =] \binom{39}{2} \times 0.02^2 \times 0.98^{37} \times 0.02$	M1	3.4
$= 0.0028071 \dots$ awrt <u>0.00281</u>	A1	1.1b
	(3)	

Notes

1st M1 for selecting a suitable model ($NB(3, 0.02)$) May be implied by a correct expression

2nd M1 for use of model to form a correct expression

SC $p \neq 0.02$ Allow prob of the form $\binom{39}{2} p^3 (1 - p)^{37}$ where $0 < p < 1$ scores M0M1

A1 for awrt 0.00281 (accept awrt 2.81×10^{-3}) [correct answer with no working scores 3/3]

Mark scheme (c)

Scheme	Marks	AO
$E(Y) = \frac{3}{0.02} = \underline{150}$	B1	1.1b
	(1)	
	(6 marks)	

Notes

B1 for 150

2.

Mark scheme (a)

Scheme	Marks	AO
(i) $X \sim \text{NB}(4, 0.15)$	M1	3.3
$\binom{19}{3} (0.15)^4 (1 - 0.15)^{16} = 0.03642$ awrt 0.0364	A1	1.1b
	(2)	
(ii) $X \sim \text{B}(18, 0.15)$	M1	3.3
$\binom{18}{4} (0.15)^4 (1 - 0.15)^{14} = 0.15920 \dots$ awrt 0.159	A1	1.1b
	(2)	

Notes

(i) **M1**: selecting NB(4, 0.15) or use of a correct method, may be implied by a correct answer

A1: awrt 0.0364

(ii) **M1**: selecting B(18, 0.15) or use of a correct method, may be implied by a correct answer

A1: awrt 0.159

Mark scheme (b)

Scheme	Marks	AO
Uses $\frac{4}{p}$ as an expression for the mean	B1	3.4

$\frac{4}{p} \geq 32$	M1	1.1b
$p = \frac{1}{8}$ or 0.125 or 12.5%	A1	1.1b
	(3)	
	(7 marks)	

Notes

B1: $E(X) = \frac{4}{p}$

M1: for an equation or inequality with **their** expression for mean and 32

allow mean = 32 or mean > 32

mean $\leq$ 32 or mean < 32 is M0

A1: 0.125 or exact equivalent

Must not come from clearly incorrect working

NB: $p \leq \frac{1}{8}$ is B1M1A0

M0 implies A0 (method mark cannot be implied here)

3.

Mark scheme (a)

Scheme	Marks	A0
$[X \sim \text{Geo}(0.2)]$ Suzanne's 4 th selection is the 7 th selection overall] $P(X = 7) = (0.8)^6(0.2)$ or $(0.64)^3(0.2)$	M1	3.3
$= 0.05242 \dots$ awrt <u>0.0524</u>	A1	1.1b
	(2)	

Notes

M1: Selecting geometric distribution with $p = 0.2$ and attempting required probability.

Allow $(0.8)^n(0.2)$ to imply M1 with $n = 6$ or $n = 3$

A1: awrt 0.0524 Allow exact fraction $\frac{4096}{78125}$

Mark scheme (b)

Scheme	Marks	AO
$P(X \geq 6) [= (1 - 0.2)^5]$	M1	1.1b
$= 0.32768$ awrt 0.328	A1	1.1b
	(2)	

Notes

M1: $P(X \geq 6)$ may be implied by $(1 - p)^5$ or $1 - (p + pq + pq^2 + pq^3 + pq^4)$

A1: awrt 0.328 Allow exact fraction $\frac{1024}{3125}$

Mark scheme (c)

Scheme	Marks	AO
Mean = 5	B1	1.1b
Standard deviation $\left[= \sqrt{\frac{1 - 0.2}{0.2^2}} \right] = \sqrt{20}$ awrt 4.47	B1	1.1b
	(2)	

Notes

B1: Mean = 5

B1: Standard deviation = $\sqrt{20}$ o.e. or awrt 4.47

Mark scheme (d)

Scheme	Marks	AO
$P(\text{Suzanne wins}) = 0.2 + (0.8)^2(0.2) + (0.8)^4(0.2) + \dots$	M1	3.1b
Infinite geometric series $= \frac{0.2}{1 - 0.8^2}$ (oe)	M1	2.1
$= \frac{5}{9}$	A1	1.1b
	(3)	
	(9 marks)	

Notes

M1: Determining the probability that Suzanne wins with at least three terms seen (may be implied by 2nd M1)

M1: Recognising need to sum terms of an infinite geometric series with correct $r = 0.8^2$ (with numerator less than denominator)

A1: $\frac{5}{9}$ (allow awrt 0.556)