

QQQ – Further Statistics 1 – Chapter 2 – Poisson Distributions (Version 2)

Total Marks: 21

(21 = Platinum, 19 = Gold, 17 = Silver, 14 = Bronze)

1. A plumbing company receives call-outs during the working day at an average rate of 2.4 per hour.

- (a)** Find the probability that the company receives exactly 7 call-outs in a randomly selected 3-hour period of a working day. **(2)**

The company has enough staff to respond to 28 call-outs in an 8-hour working day.

- (b)** Show that the probability that the company receives more than 28 call-outs in a randomly selected 8-hour working day is 0.022 to 3 decimal places. **(2)**

In a random sample of 100 working days each of 8 hours,

- (c)** **(i)** find the expected number of days that the company receives more than 28 call-outs, **(1)**
- (ii)** find the standard deviation of the number of days that the company receives more than 28 call-outs, **(2)**
- (iii)** use a Poisson approximation to estimate the probability that the company receives more than 28 call-outs on at least 6 of these days. **(3)**

(Total 10 marks)

2. A botanist is studying the distribution of daisies in a field. The field is divided into a number of equal sized squares. The mean number of daisies per square is assumed to be 3. The daisies are distributed randomly throughout the field.

Find the probability that, in a randomly chosen square there will be

- (a)** more than 2 daisies, **(3)**
- (b)** either 5 or 6 daisies. **(2)**

The botanist decides to count the number of daisies, x , in each of 80 randomly selected squares within the field. The results are summarised below

$$\sum x = 295 \quad \sum x^2 = 1386$$

- (c)** Calculate the mean and the variance of the number of daisies per square for the 80 squares. Give your answers to 2 decimal places. **(3)**
- (d)** Explain how the answers from part (c) support the choice of a Poisson distribution as a model. **(1)**
- (e)** Using your mean from part (c), estimate the probability that exactly 4 daisies will be found in a randomly selected square. **(2)**

(Total 11 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
$X \sim \text{Po}(7.2)$	M1	3.4
$P(X = 7) = 0.14858 \dots$ awrt 0.149	A1	1.1b
	(2)	

Notes

M1: Writing or using $\text{Po}(7.2)$

A1: awrt 0.149

Mark scheme (b)

Scheme	Marks	AO
$Y \sim \text{Po}(19.2)$	M1	3.3
$[P(Y > 28) =]$ $1 - P(Y \leq 28) = 1 - 0.9780 \dots = 0.02199 \dots$ 0.022*	A1*	1.1b
	(2)	

Notes

M1: Writing or using $\text{Po}(19.2)$

A1*: cso given answer with correct probability statement (e.g. $1 - P(Y \leq 28)$) and no incorrect working seen

Mark scheme (c)

Scheme	Marks	AO
(i) $[100 \times 0.022]$ awrt 2.2	B1	1.1b
	(1)	
(ii) $\sqrt{100(0.022)(1 - 0.022)}$	M1	1.1b

$= 1.466 \dots$ awrt 1.47	A1	1.1b
	(2)	
(iii) $B(100, 0.022) \rightarrow \text{Po}(2.2)$	M1	3.4
$P(W \geq 6) = 1 - P(W \leq 5) [= 1 - 0.9750 \dots]$	M1	1.1b
$= 0.02490 \dots$ awrt 0.0249	A1	1.1b
	(3)	
	(10 marks)	

Notes

(c)(i) B1: awrt 2.2 (isw once awrt 2.2 is seen)

(c)(ii) M1: Correct expression including square root

A1: awrt 1.47

Watch out for $\sqrt{2.2} = 1.483 \dots$ which is MOA0

(iii) M1: Approximating binomial (100, 0.022) with $\text{Po}(2.2)$ [may be seen in (i) or (ii)]

M1: Using $1 - P(W \leq 5)$ from Poisson distribution

A1: awrt 0.0249

Note: Using Binomial distribution $1 - 0.9765588 \dots = 0.02344 \dots$ scores M0M0A0

2.

Mark scheme (a)

Scheme	Marks
The random variable X is the number of daisies in a square. Poisson(3)	B1
$1 - P(X \leq 2) = 1 - 0.4232 \quad 1 - e^{-3} \left(1 + 3 + \frac{3^2}{2!} \right)$	M1
$= 0.5768$	A1
	(3)

Mark scheme (b)

Scheme	Marks
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$P(X \leq 6) - P(X \leq 4) = 0.9665 - 0.8153 \quad e^{-3} \left(\frac{3^5}{5!} + \frac{3^6}{6!} \right)$	M1
$= 0.1512$	A1
	(2)

Mark scheme (c)

Scheme	Marks
$\mu = 3.69$	B1
$\text{Var}(X) = \frac{1386}{80} - \left(\frac{295}{80} \right)^2$	M1
$= 3.73/3.72/3.71 \quad \text{accept } s^2 = 3.77$	A1
	(3)

Mark scheme (d)

Scheme	Marks
For a Poisson model, Mean = Variance ; For these data $3.69 \approx 3.73$ $\Rightarrow$ Poisson model	B1
	(1)

Mark scheme (e)

Scheme	Marks
$\frac{e^{-3.6875} 3.6875^4}{4!} = 0.193 \quad \text{allow their mean or var}$	M1
Awrt 0.193 or 0.194	A1 ft
	(2)
	(11 marks)